These are some review questions to test your understanding of the material. Some of these questions may appear on an exam.

1 Red-Black Tree

Please see the diagrams:

- flow-chart for bottom-up insertion in a RedBlack tree on page 2
- bottom-up deletion on page 3
- top-down insertion on page 4

1.1 Define *Red-Black tree.*

1.2 Show the result of inserting `2,1,4,5,9,3,6,7` into an initially empty Red-Black tree (show the tree at the end of **each** insertion). Do this using bottom-up and using top-down insertion rules.

1.3 Show the result of removing a given element from the tree. Do this using bottom-up and using top-down deletion rules.

1.4 What is the "Big-Oh" performance (in terms of the number of nodes in the tree) for each operation `find`, `insert`, and `remove` for Red-Black trees in the best, worst, and average cases?

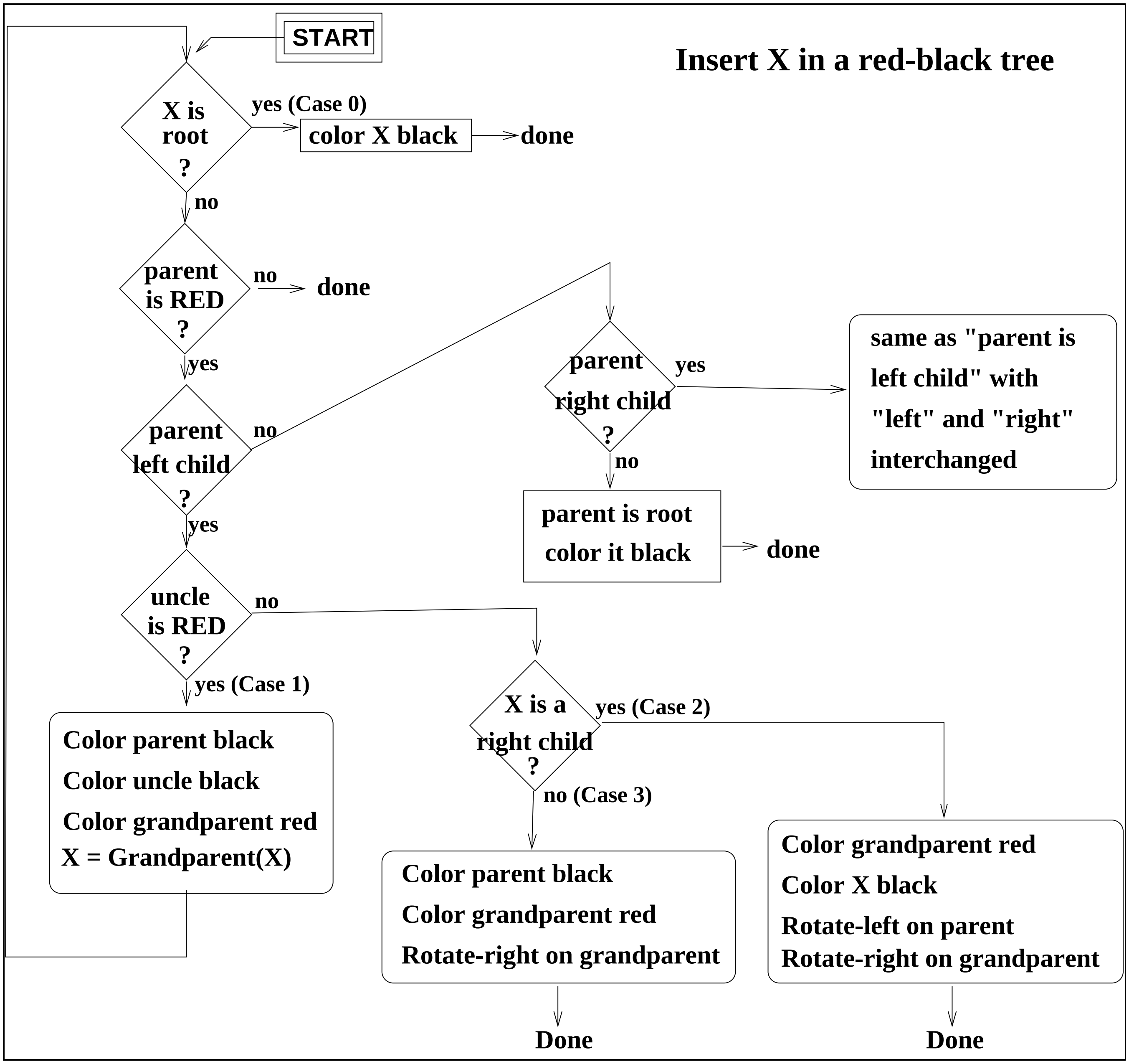
1.5 What property of Red-Black trees is most significant in explaining their "Big-Oh" behavior for the operations `find`, `insert`, and `remove`.

1.6 Prove: Any red-black tree, with root x, has at least $n = 2^{\mathrm{bh}(x)} - 1$ internal nodes, where $\mathrm{bh}(x)$ is the black-height of node x.

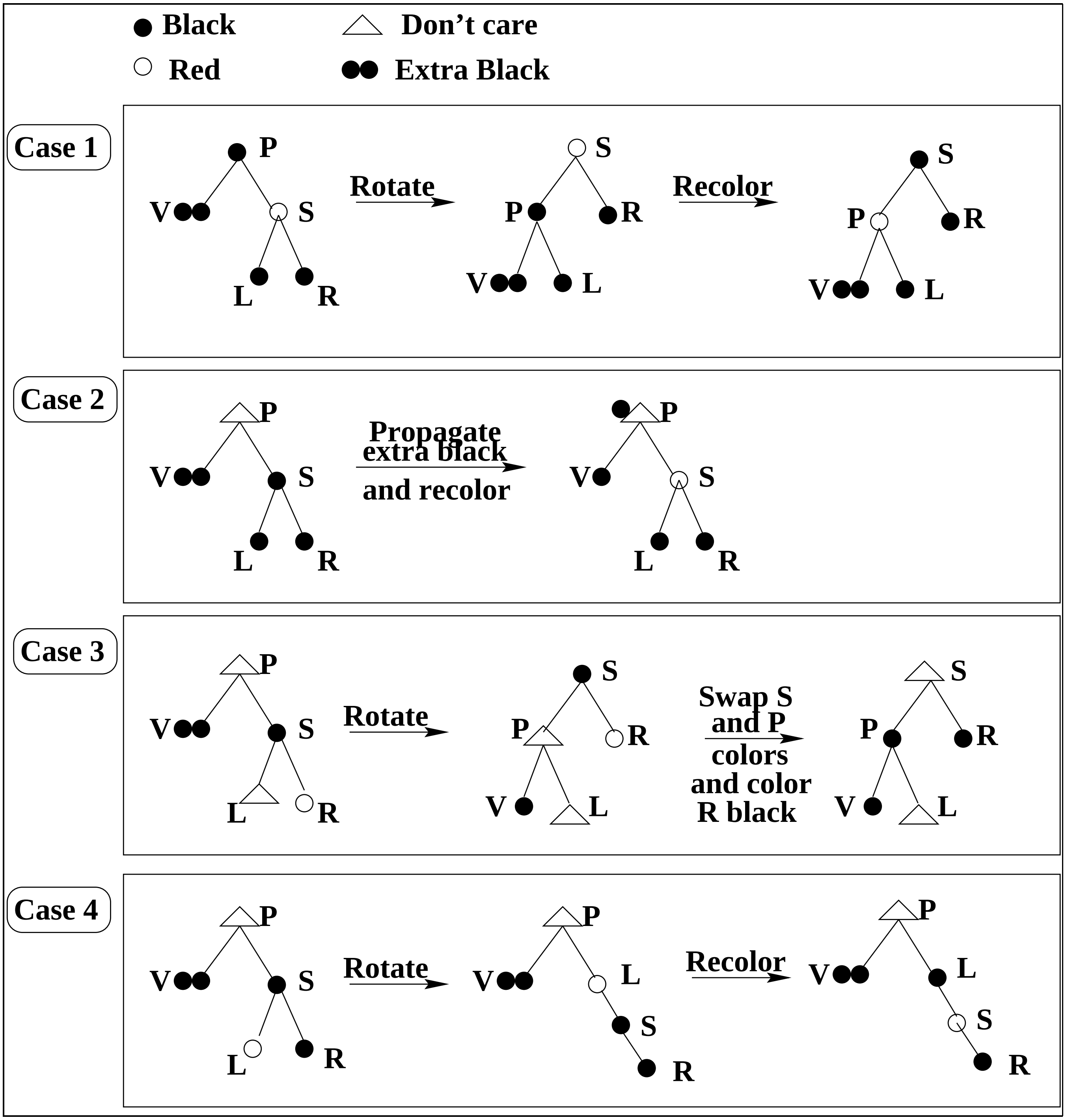
1.7 Prove: In a red-black tree, at least half of the nodes on any path from root to a leaf must be black.

1.8 Prove: In a red-black tree, no path from any node N to a leaf is more than twice as long as any other path from N to any other leaf.

Flow-chart for bottom-up insertion in Red-Black Trees



Cases for Bottom-Up Deletion in Red-Black Trees



Cases for top-down insertion in Red-Black Trees