These are some review questions to test your understanding of the material. Some of these questions may appear on an exam.

1 Disjoint Set

1.1 Give the value of the expression $lg^*(1024)$, where $lg^*(N)$ is the iterated base-2 logarithm of N.

1.2 Suppose you do a sequence of N^2 union-find operations on a set that contains N elements. What is the worst-case asymptotic Big-Oh time performance of the sequence? Explain your answer.

1.3 Now, suppose you do a sequence of $N \log n$ union-find operations on a set that contains N elements. What is the worst-case asymptotic Big-Oh time performance of the sequence? Explain your answer.

1.4 Define *Union-by-Weight heuristic.*

1.5 Define *Path-compression heuristic.*

1.6 When both the union-by-weight and the path-compression heuristics are applied on disjoint sets totaling N items, a sequence of M union-find operations can be done in $O(M \lg^*(N))$ time. It is sometimes said that under these conditions, union-find is done in constant time per operation. What does this mean? Why is it true?

1.7 In an uptree with root x, let $R(x)$ be the length of the longest path and let $N = W(x)$ be the number of vertices (including x). Assume the uptree was created by means of multiple applications of the "union-by-size" algorithm.
Prove: $R(x) \leq \lg(N)$