These are some review questions to test your understanding of the material. Some of these questions may appear on an exam.

1 Binary Trees

1.1 Define *binary tree*, *full* binary tree, *complete* binary tree, and *perfect* binary tree.

1.2 Define *internal node* and *external node* of a rooted tree.

1.3 Define *internal path length* and *external path length* of a rooted tree.

1.4 Define *augmented* binary tree.

1.5 Prove: A perfect binary tree of height h has 2^h leaf nodes.

1.6 Prove: A perfect binary tree of height h has $2^{h+1} - 1$ vertices.

1.7 Prove: A full binary tree with n_i internal nodes has $n_i + 1$ leaf nodes.

1.8 Prove: In any binary tree of n nodes, there are $n + 1$ "null pointers."

1.9 Consider a binary tree that has n_i *internal* nodes. Let $E(n_i)$ and $I(n_i)$ denote the external and internal path lengths of the tree, respectively. Show that for a full binary tree, $E(n_i) = I(n_i) + 2n_i$.

1.10 Prove: the internal path length of an augmented binary tree T with n internal nodes is

$$I(n) = I(n_L) + I(n - n_L - 1) + n - 1$$

where n_L is the number of internal nodes in the left subtree.

1.11 Suppose you have the following two traversal sequences from the same binary tree:

pre-order: `A D F G H K L P Q R W Z`

in-order: `G F H K D L A W R Q P Z`

Draw the tree.

1.12 Suppose you have the following two traversal sequences from the same binary tree:

post-order: `F G H D A L P Q R Z W K`

in-order: `G F H K D L A W R Q P Z`

Draw the tree.