CMSC 341

Lecture 14

Announcements

Proj3 due today

Proj4 up by midnight

Splay Trees

Concept

- adjust tree in response to accesses to make common operations efficient
- after access node is moved to root by *splaying*

Performance

- amortized such that m operations take O(m lg n) where n is the number of insertions

Splay Operation

Traverse tree from node x to root, rotating along the way until x is the root

Each rotation

- If x is root, do nothing.
- If x has no grandparent, rotate x about its parent.
- If x has a grandparent,
  - if x and its parent are both left children or both right children, rotate the parent about the grandparent, then rotate x about its parent
  - if x and its parent are opposite type children (one left and the other right), rotate x about its parent, then rotate x about its new parent (former grandparent)

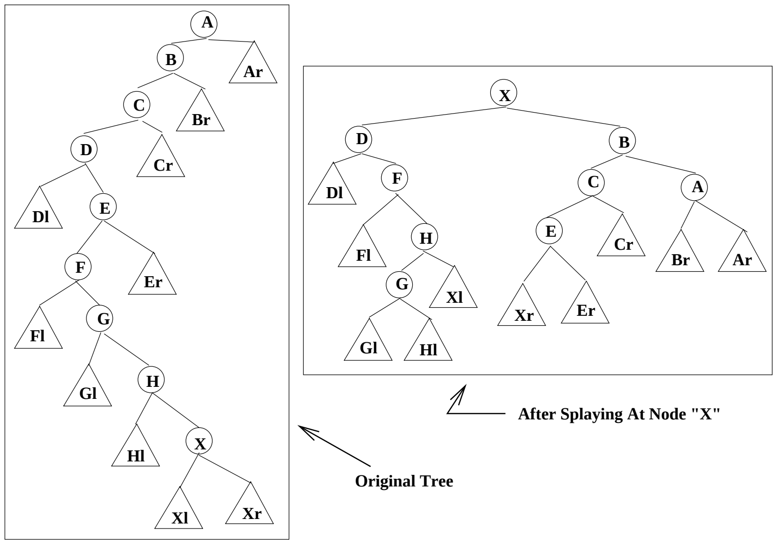
Operations in Splay Trees

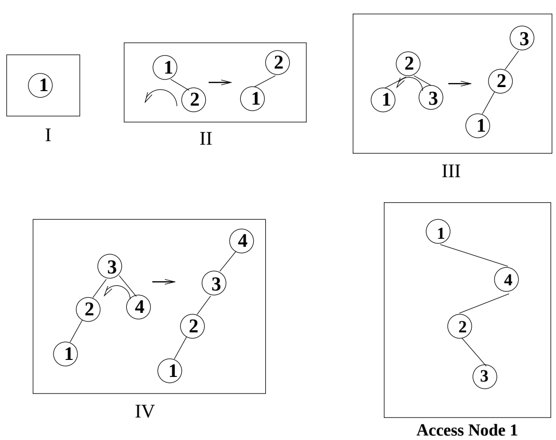
insert
- first insert as in normal binary search tree
- than splay inserted node

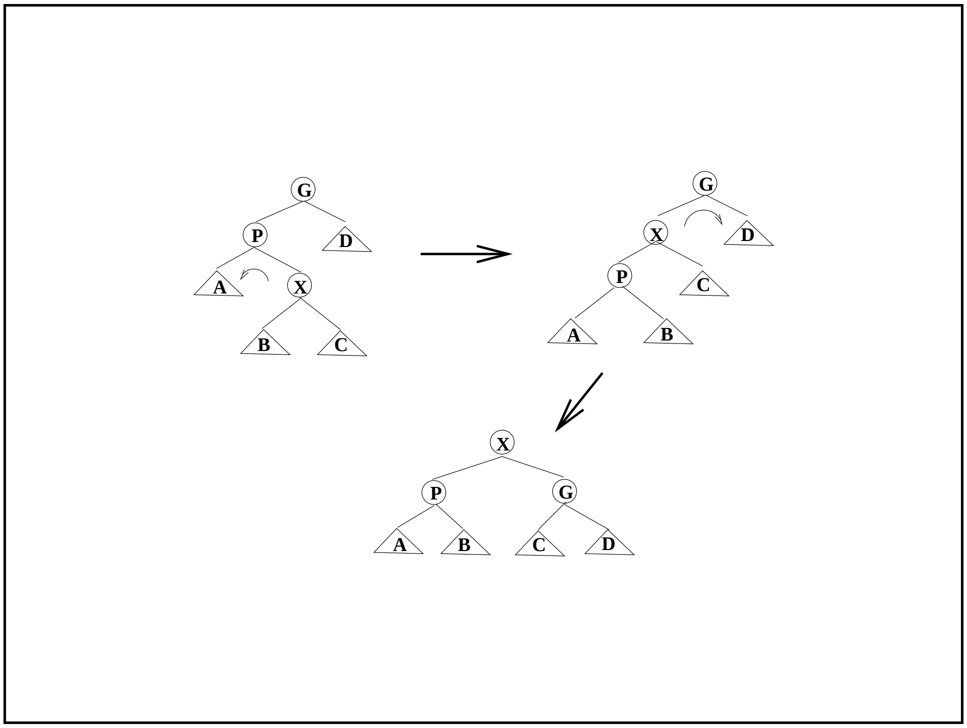
find
- search for node
- if found, splay to root; otherwise splay last node on path

Operations on Splay Trees (cont)

remove
- splay selected element to root
- disconnect left and right subtrees from root
- do one of:
  - splay max item in T_L (then T_L has no right child)
  - splay min item in T_R (then T_R has no left child)
- connect other subtree to empty child


A
B
Ar
C
Br
D
Cr
Dl
E
F
Er
Fl
G
Gl
H
Hl
X
Xl
Xr
Original Tree
X
D
B
Dl
F
C
A
Fl
H
E
Cr
Br
Ar
G
Xl
Xr
Er
Gl
Hl
After Splaying At Node "X"


1
I
1
2
2
1
II
2
1
3
3
2
1
III
3
2
4
1
4
3
2
1
IV
1
4
2
3
Access Node 1


G
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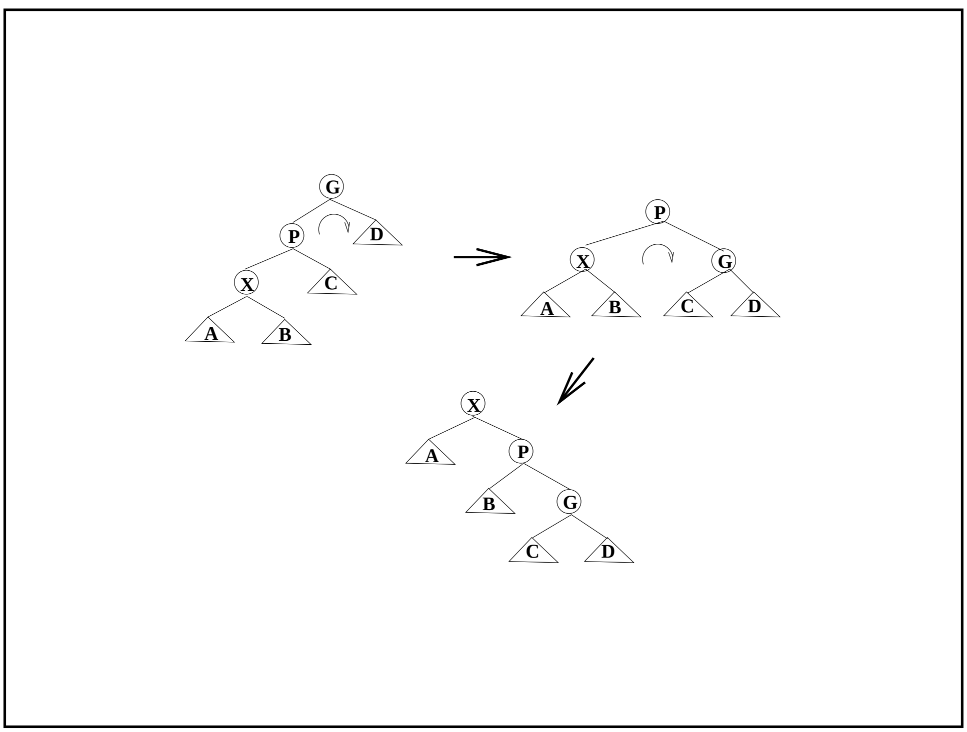
G
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D

Performance of Splay Trees

insert

- regular bst insertion -- O(depth)
- splay: O(1) for each rotation, O(depth) rotations

Red-Black Trees

Concept

- BST with more relaxed notion of balance than AVL trees
- no path from N to leaf is more than twice as long as any other
- for RB tree with n nodes and height h, $h \leq 2 \lg(n+1)$

Definition: A red-black tree is a binary search tree where:

- Every node is either red or black.
- Each NULL pointer is considered to be a black node
- If a node is red, then both of its children are black.
- Every path from a node to a leaf contains the same number of black nodes.

Definition: The height of a node n in a red-black tree is the number of black nodes on any path to a leaf, not counting n.

RedBlackNode

```
template <class Comparable>
class RedBlackNode{
   Comparable  element;
   RedBlackNode *left;
   RedBlackNode *right;
   int color;

   RedBlackNode(const Comparable & theElement =
   Comparable(), RedBlackNode *lt = NULL, RedBlackNode
   *rt =NULL, int c = RedBlackTree<Comparable>::BLACK) :
   element(theElement), left(lt), right(rt), color(c {}

   friend class RedBlackTree<Comparable>;
   };
```

RedBlackTree Class

```
template <class Comparable>
class RedBlackTree {
public:
   explicit RedBlackTree (const Comparable &negInf);
   RedBlackTree (const RedBlackTree &rhs);
   ~RedBlackTree();
   enum {RED, BLACK};
   // usual public member functions
private:
   RedBlackNode<Comparable> *header;
   const Comparable ITEM_NOT_FOUND;
   RedBlackNode<Comparable> *nullNode;
   RedBlackNode<Comparable> *current;
   RedBlackNode<Comparable> *parent;
   RedBlackNode<Comparable> *grand;
   RedBlackNode<Comparable> *great;
```

RedBlackTree (cont.)

```
void handleReorient(const Comparable &item);
RedBlackNode<Comparable> *rotate(const Comparable
&item, RedBlackNode<Comparable> *parent) const;

// additional private member funcs
};
```

Constructor

```
template <class Comparable>
RedBlackTree<Comparable>::RedBlackTree(const Comparable
   &negInf) : ITEM_NOT_FOUND(negInf) {
   nullNode = new RedBlackNode<Comparable>;
   nullNode->left = nullNode->right = nullNode;
   header = newRedBlackNode<Comparable>(negInf);
   header->left = header->right = nullNode;
   }
```