Number Systems

Binary

Decimal

Hexadecimal

Bits, Bytes, and Words

- A **bit** is a single **binary digit** (a 1 or 0).
- A **byte** is 8 bits
- A **word** is 32 bits or 4 bytes
- **Long word** = 8 bytes = 64 bits
- **Quad word** = 16 bytes = 128 bits
- Programming languages use these standard number of bits when organizing data storage and access.

Bits, Bytes

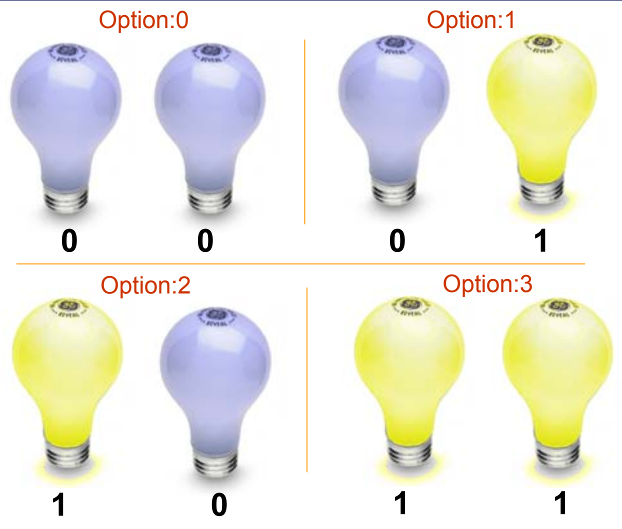
Unit	Symbol	Number of Bytes
kilobyte	KB	2^{10} = 1024
megabyte	MB	2^{20} (over 1 million)
gigabyte	GB	2^{30} (over 1 billion)
terabyte	TB	2^{40} (over 1 trillion)

Bit Permutations

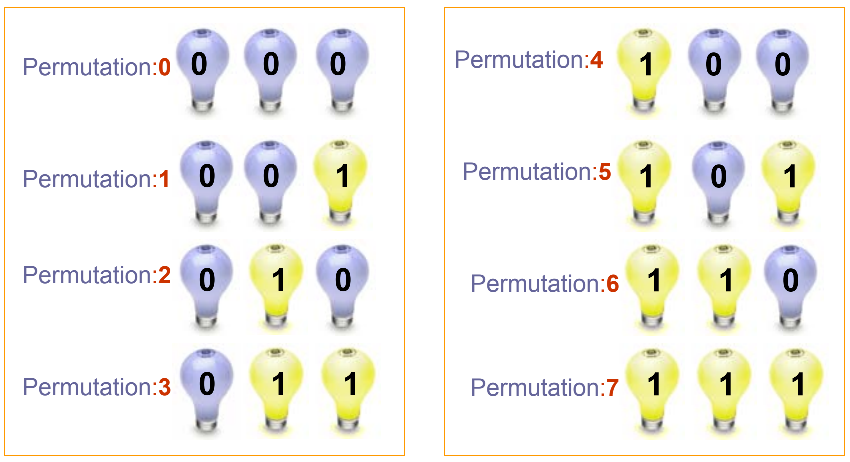
- 1 bit(only 1 light bulb):

Option:1
Option:2
0
1

Bit Permutations - 2 bit



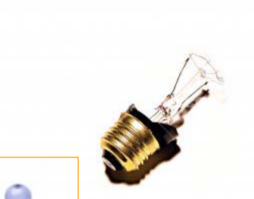
Bit Permutations - 3 bit



Bit Permutations - 4 bit (animation)

Done!!!

Bit Permutations - 4 bit



0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
10	1 0 1 0
11	1 0 1 1
12	1 1 0 0
13	1 1 0 1
14	1 1 1 0
15	1 1 1 1

Bit Permutations

1 bit	2 bits	3 bits	4 bits	
0	00	000	0000	1000
1	01	001	0001	1001
	10	010	0010	1010
	11	011	0011	1011
		100	0100	1100
		101	0101	1101
		110	0110	1110
		111	0111	1111

Each additional bit doubles the number of possible permutations

Number Systems

- The on and off states of the capacitors in RAM can be thought of as the values 1 and 0, respectively.
- Therefore, thinking about how information is stored in RAM requires knowledge of the **binary (base 2) number system.**
- Let's review the **decimal (base 10) number system** first.

The Decimal Number System

- The decimal number system is a positional number system.
- Example:

5 6 2 1

10^3 10^2 10^1 10^0

$1 \times 10^0 = 1$
$2 \times 10^1 = 20$
$6 \times 10^2 = 600$
$5 \times 10^3 = 5000$

The Decimal Number System (con't)

- The decimal number system is also known as **base 10**.
- The values of the positions are calculated by taking 10 to some power.
- Why is the base 10 for decimal numbers?
  - Because we use 10 digits, the digits 0 through 9.

The Binary Number System

- The binary number system is also known as **base 2**. The values of the positions are calculated by taking 2 to some power.
- Why is the base 2 for binary numbers?
  - Because we use 2 digits, the digits 0 and 1.

The Binary Number System

- The binary number system is also a positional numbering system.
- Instead of using ten digits, 0 - 9, the binary system uses only two digits, 0 and 1.
- Example of a binary number and the values of the positions:

1	0	0	1	1	0	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

Converting from Binary to Decimal

1	0	0	1	1	0	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

$1 \times 2^0 = 1$

$0 \times 2^1 = 0$

$1 \times 2^2 = 4$

$1 \times 2^3 = 8$

$0 \times 2^4 = 0$

$0 \times 2^5 = 0$

$1 \times 2^6 = \underline{64}$

77_{10}

$2^0 = 1$
$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$

Converting From Decimal to Binary

- Make a list of the binary place values up to the number being converted.
- Perform successive divisions by 2, placing the remainder of 0 or 1 in each of the positions from right to left.
- Continue until the quotient is zero.
- Example: 42_{10}

2^5	2^4	2^3	2^2	2^1	2^0
32	16	8	4	2	1
1	0	1	0	1	0

Adding Binary

```
carries →     1 1 1 1 1
              1 1 1 0 1 1
            + 1 0 0 1 1 1
overflow →  1 1 0 0 0 1 0
```

```
carries →   1     1
            1 1 0 0 1
          + 0 1 1 0 1
overflow → 1 0 0 1 1 0
```

Working with Large Numbers

0 1 0 1 0 0 0 0 1 0 1 0 0 1 1 1 = ?

- Humans can't work well with binary numbers; there are too many digits to deal with.
- Memory addresses and other data can be quite large. Therefore, we sometimes use the **hexadecimal number system**.

The Hexadecimal Number System

- The hexadecimal number system is also known as **base 16**. The values of the positions are calculated by taking 16 to some power.
- Why is the base 16 for hexadecimal numbers ?
  - Because we use 16 symbols, the digits 0 through 9 and the letters A through F.

The Hexadecimal Number System

Binary	Decimal	Hexadecimal
0	0	0
1	1	1
10	2	2
11	3	3
100	4	4
101	5	5
110	6	6
111	7	7
1000	8	8
1001	9	9

Binary	Decimal	Hexadecimal
1010	10	A
1011	11	B
1100	12	C
1101	13	D
1110	14	E
1111	15	F

The Hexadecimal Number System

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, a, b, c, d, e, f,

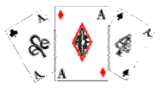
10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1a, 1b, 1c, 1d, 1e, 1f, 20

- Example of a hexadecimal number and the values of the positions:

3	C	8	B	0	5	1
16^6	16^5	16^4	16^3	16^2	16^1	16^0

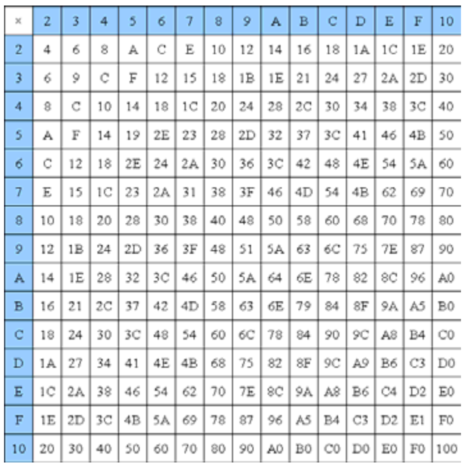
Hex could be fun!

- ACE
- AD0BE
- BEE
- CAB
- CAFE
- C0FFEE
- DECADE



Note: 0 is a zero not and a letter O

Hexadecimal Multiplication Table



×	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10
2	4	6	8	A	C	E	10	12	14	16	18	1A	1C	1E	20
3	6	9	C	F	12	15	18	1B	1E	21	24	27	2A	2D	30
4	8	C	10	14	18	1C	20	24	28	2C	30	34	38	3C	40
5	A	F	14	19	2E	23	28	2D	32	37	3C	41	46	4B	50
6	C	12	18	2E	24	2A	30	36	3C	42	48	4E	54	5A	60
7	E	15	1C	23	2A	31	38	3F	46	4D	54	4B	62	69	70
8	10	18	20	28	30	38	40	48	50	58	60	68	70	78	80
9	12	1B	24	2D	36	3F	48	51	5A	63	6C	75	7E	87	90
A	14	1E	28	32	3C	46	50	5A	64	6E	78	82	8C	96	A0
B	16	21	2C	37	42	4D	58	63	6E	79	84	8F	9A	A5	B0
C	18	24	30	3C	48	54	60	6C	78	84	90	9C	A8	B4	C0
D	1A	27	34	41	4E	4B	68	75	82	8F	9C	A9	B6	C3	D0
E	1C	2A	38	46	54	62	70	7E	8C	9A	A8	B6	C4	D2	E0
F	1E	2D	3C	4B	5A	69	78	87	96	A5	B4	C3	D2	E1	F0
10	20	30	40	50	60	70	80	90	A0	B0	C0	D0	E0	F0	100

Example of Equivalent Numbers

- Binary: **1 0 1 0 0 0 0 1 0 1 0 0 1 1 1** **(2)**
- Decimal: **20647** **(10)**
- Hexadecimal: **50A7** **(16)**
- Notice how the number of digits gets smaller as the base increases.

Converting from Binary to Decimal

Practice conversions: Binary $\rightarrow$ Decimal

11101

1010101

100111

Practice conversions: Decimal $\rightarrow$ Binary:

59

82

175