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A Note on Navigation



- Navigation is hard!
- Encompasses (at least) four components:
 1. Perception: based on sensor data, what do I know about my environment?
 2. Localization: Where am I in that environment?
 3. Cognition: What should I do now?
 4. Motion Control: How do I do that?



Navigation is a hard problem, but many tasks depend on it





www.youtube.com/watch?v=Is4JZqhA8-M
ucsdnews.ucsd.edu/feature/real_life_toy_story

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Localization ≠ Mapping



- **Mapping:** creating a map of an environment
- **Localization:** sensor and odometry data are used to figure out where robot is *in* a map
- Needs some kind of info about environment
 - Typically a map of some kind
 - Physical, semantic, topological
- Doesn't care about **source** of information
 - Could be given a map beforehand
 - Or constructing the map as you go
- Localization often includes mapping as a step



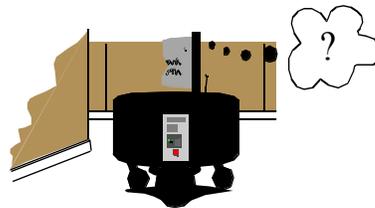
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Localization + Map Building



- To localize or not to localize
 - When is hard-coding better?
- Belief representation
 - How do I represent the environment and my state?
- Map representation
 - What does a map contain?
- And more...
 - Probabilistic map-based localization
 - Autonomous map building



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Image: Ibrahim, Omar. (2011). Extended Kalman Filter Simultaneous Localization and Mapping (Graduation Project)

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Challenges of Localization



- Knowing absolute position (e.g. GPS) isn't enough

Lat 40.7127, Long -74.0059 \neq



- Localization in human-scale
 - "Give or take 5 meters" – in a building? On the street?
- Many sources of uncertainty
 - Sensor noise, sensor aliasing, effector noise, odometric position estimation, ...
- Aliasing (coming soon)



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Sensor Noise



- Sensors give "noisy" (uncertain, imperfect) readings
- Source of sensor noise may be environmental
 - Surfaces, illumination, background noise...
 - Glass walls 😞
- Or the nature of the sensor
 - Interference between ultrasonic sensors
 - Cameras in high dynamic range lighting (like outside)
- Or may just be because sensors are imperfect



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vivairailines.com/portfolio/page/aluminum-glass-railine-sunvl

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Challenge 1: Sensor Noise



- Sensor noise drastically reduces useful readings
- Solutions:
 - Improve sensors
 - Change assumptions
 - "So, if we have glass walls, we'll see readings like..."
 - Use multiple readings
 - Employ temporal and/or multi-sensor fusion



"Is my environment made of dots?"



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www.photoreview.com/a/tips/shooting/how-to-control-in-ave-noise

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Challenge 2: Sensor Aliasing

- **Different** positions give the **same** sensor readings
- Robots: non-uniqueness of sensors readings is normal
 - What does that mean?
- To people, unique places look unique
 - We're really good at picking up on differences
 - We have really good sensors
- To robots, different places often look the same
 - There is a many-to-one mapping from environmental state to robot's perceptual inputs



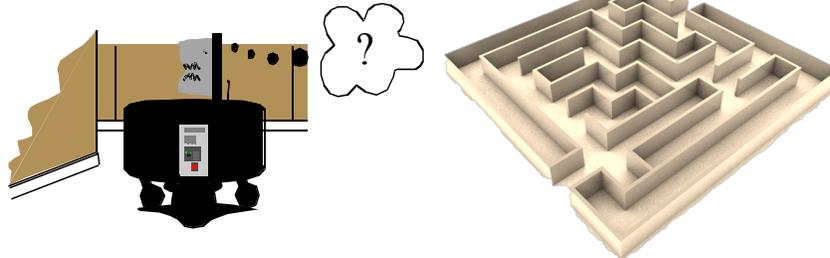
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Sensor Aliasing (2)

- Different places give the same readings



- Information from sensors often not enough to identify position from a single reading
 - Solution: localization usually based on a series of readings
 - Enough information is recovered by the robot over time

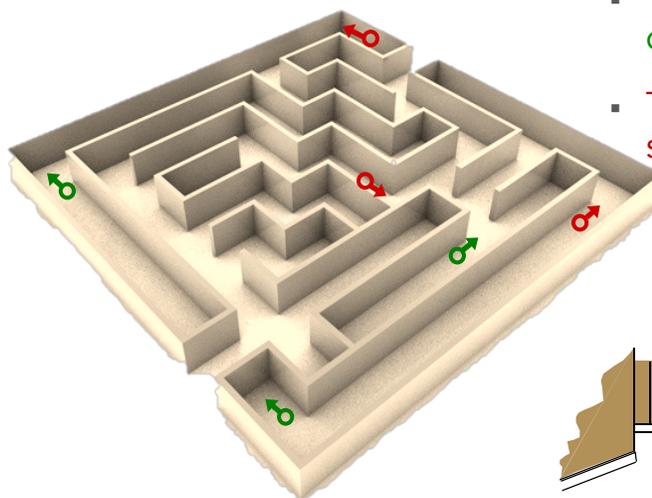


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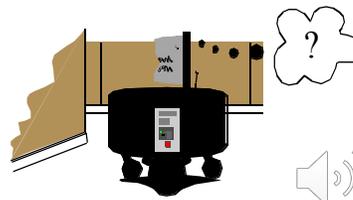
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Sensor Aliasing (3)



- These look different.
- These look the same.
 - Wall in front, wall on the right, opening on left.



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Odometry, Dead Reckoning

- Odometry: wheel sensors only
 - E.g., go 5 cm, turn 10 degrees – where are you?
- Dead reckoning: also use heading sensors
 - Add a compass or gyroscope
- Position update is partly based on proprioceptive sensors
 - Sensing movement: wheel encoders + heading sensors
 - Integrate that into model of environment to get the position
 - Pros: Straightforward, easy
 - Cons: Errors are integrated → accumulative



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Effector (Actuator) Noise



- Causes:
 - Inexact actuation + noise in sensors: Probably not exactly 5 cm
 - Environment: Duct tape is slippery!
- This error is cumulative over time, but reduced with additional sensors (not eliminated)
- Errors exist on a spectrum:

Deterministic
(systematic):
“This servo always
turns 2% too far”



Non-deterministic
(random):
“Sometimes this servo goes
too far or not far enough”



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Odometry: dealing with errors



- Deterministic errors can be eliminated with calibration
- Random errors can be described by error models
 - Will always lead to uncertain position estimate
- More major sources of error:
 - Limited resolution during integration (time increments, measurement resolution ...)
 - Misalignment of the wheels (deterministic)
 - Unequal wheel diameter (deterministic)
 - Variation in the contact point of the wheel
 - Unequal floor contact (slipping, not planar ...)



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Some movement errors



- **Range error:** Integrated path length (distance) of movement is wrong
 - How far has robot moved in heading direction?
- **Turn error:** similar to range error, but for turns
 - What's robot's θ from starting position?
 - Accumulated error over multiple turns
- **Drift error:** difference in wheels \rightarrow error in angular orientation
 - Right wheel turns 90° , left turns 89.8° . What happens?



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Error Severity



- Over long periods of time, turn and drift errors **far** outweigh range errors.
 - As θ grows linearly, change in location grows nonlinearly
- Why?
- Imagine moving forward a distance d on a straight line along axis x .
- As $\Delta\theta$ (angular error) grows, error in y will have a component of $d \sin \Delta\theta$.



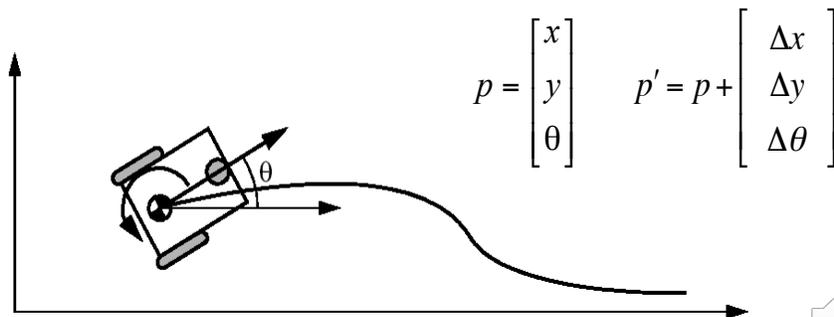
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Odometry & Diff. Drive*



- Discrete sampling rate Δt
- $\Delta s_r, \Delta s_l$: right wheel, left wheel distance travelled
- Changes in pose: $\Delta x, \Delta y, \Delta\theta$
- Distance between wheels (wheel base): b



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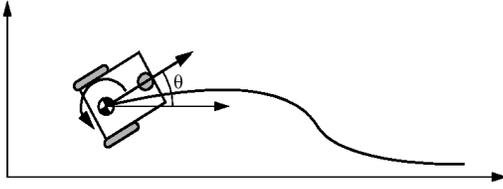
* 2 wheels, shared axis, each can be rotated forward or back

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Odometry & Diff. Drive

- Derivations
 - Δt = Sampling rate
 - $\Delta s_r, \Delta s_l$ = right/left wheel travel
 - b = Wheel base (distance between wheels)



These are given here. These can be derived, but try to make mechanical/intuitive sense of them.

$$p' = p + \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta \theta \end{bmatrix}$$

}

$$\begin{aligned} \Delta s &= (\Delta s_r + \Delta s_l) / 2 \\ \Delta x &= \Delta s \cos(\theta + \Delta \theta / 2) \\ \Delta y &= \Delta s \sin(\theta + \Delta \theta / 2) \\ \Delta \theta &= (\Delta s_r - \Delta s_l) / b \end{aligned}$$

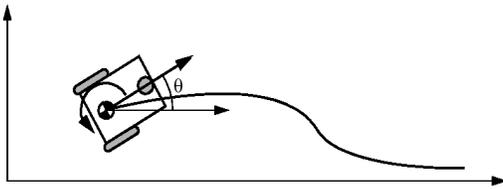


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Odometry & Diff. Drive

- Kinematics



$$\Delta x = \Delta s \cos(\theta + \Delta \theta / 2)$$

$$\Delta y = \Delta s \sin(\theta + \Delta \theta / 2)$$

$$\Delta \theta = \frac{\Delta s_r - \Delta s_l}{b}$$

$$\Delta s = \frac{\Delta s_r + \Delta s_l}{2}$$

$$p' = f(x, y, \theta, \Delta s_r, \Delta s_l) = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{\Delta s_r + \Delta s_l}{2} \cos\left(\theta + \frac{\Delta s_r - \Delta s_l}{2b}\right) \\ \frac{\Delta s_r + \Delta s_l}{2} \sin\left(\theta + \frac{\Delta s_r - \Delta s_l}{2b}\right) \\ \frac{\Delta s_r - \Delta s_l}{b} \end{bmatrix}$$

This is the matrix for finding p'. Don't worry about it too



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Odometry & Diff. Drive



- **How to model error:** Represent uncertainty of location over time
 - Using a covariance matrix of position estimate – how do these parameters vary with respect to each other?
- We can assume:
 - Left and right wheel errors are independent
 - $d \propto \Delta s_r, \Delta s_l$: Variance of wheel errors are proportional to distance traveled
 - An initial matrix Σ_p is known
- So we can get a covariance matrix that describes how error varies as a function of terms.
 - The derivation of this matrix is in the text – for now, make sure you have the general idea.



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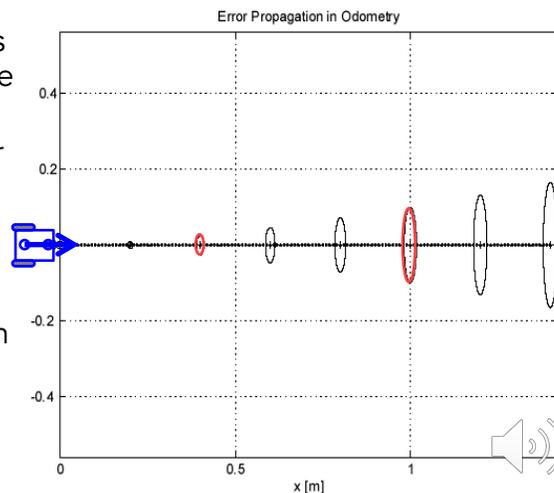
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Odometry:

Growth of Pose Uncertainty for Straight Line Movement



- Intuitively: how do errors grow as the robot moves around?
 - Think of the circles (error) as where the robot *might* be
 - Errors grow slower in x (direction of travel) than in y
 - So, robot is more likely to be off to the side than ahead or behind of where you expect



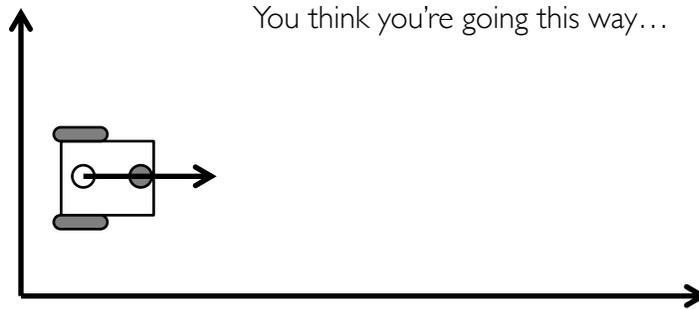
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Error Severity



- Errors grow slower in x (direction of travel) than y



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Error Severity



- Errors grow slower in x (direction of travel) than y



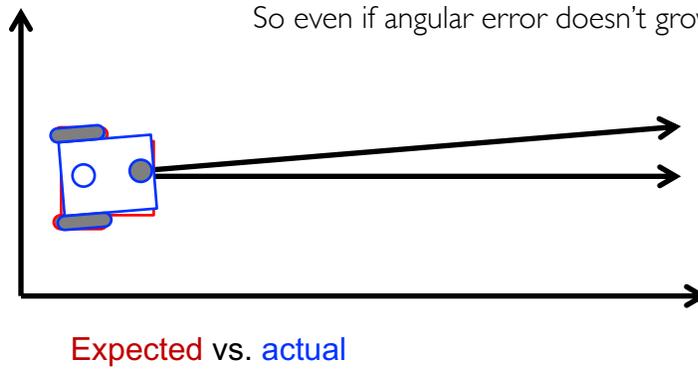
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Error Severity



- Errors grow slower in x (direction of travel) than y



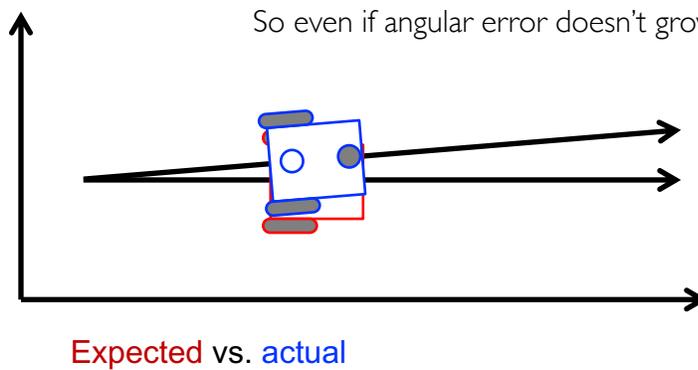
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Error Severity



- Errors grow slower in x (direction of travel) than y



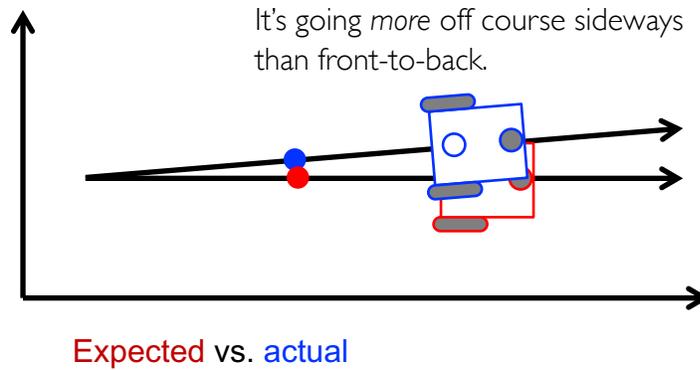
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Error Severity



- Errors grow slower in x (direction of travel) than y



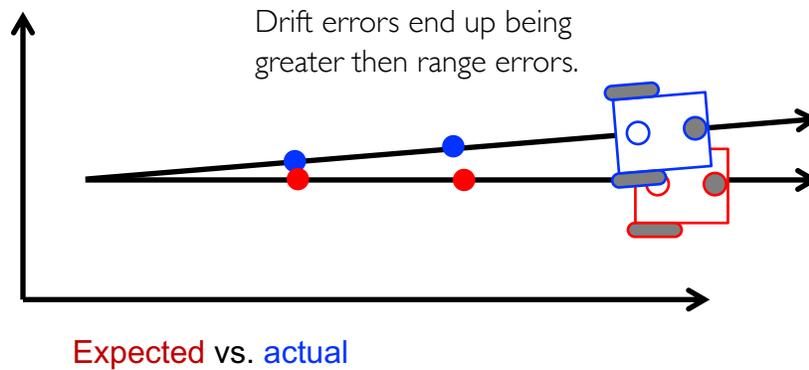
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Error Severity



- Errors grow slower in x (direction of travel) than y



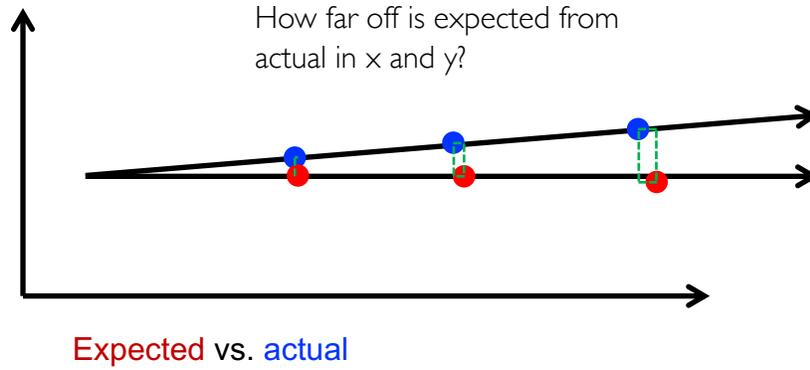
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Error Severity



- Errors grow slower in x (direction of travel) than y



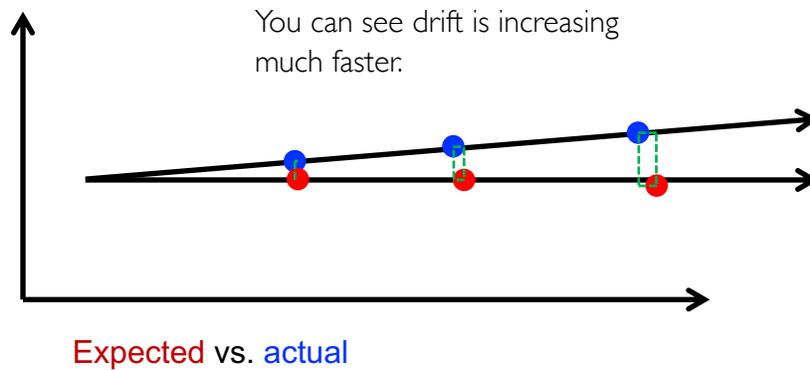
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Error Severity



- Errors grow slower in x (direction of travel) than y



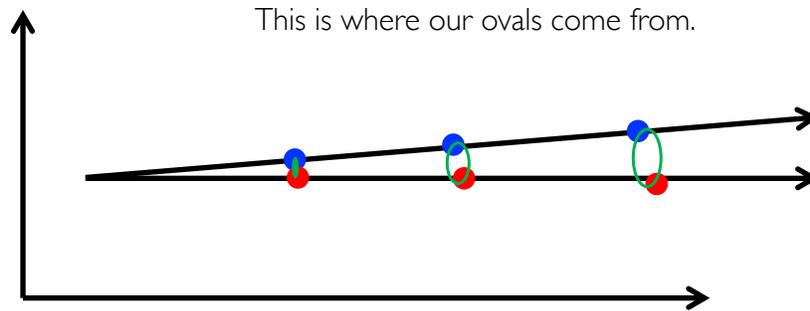
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Error Severity



- As $\Delta\theta$ (angular error) grows, error in y will have a component of $d \sin \Delta\theta$.



- You can do the same thing with a curved line.

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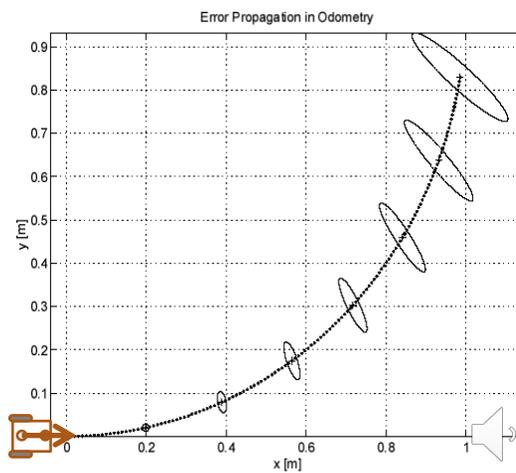
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Odometry:

Growth of Pose uncertainty for Movement on a Circle



- Now imagine moving in a curve
- Errors don't stay perpendicular to direction of movement



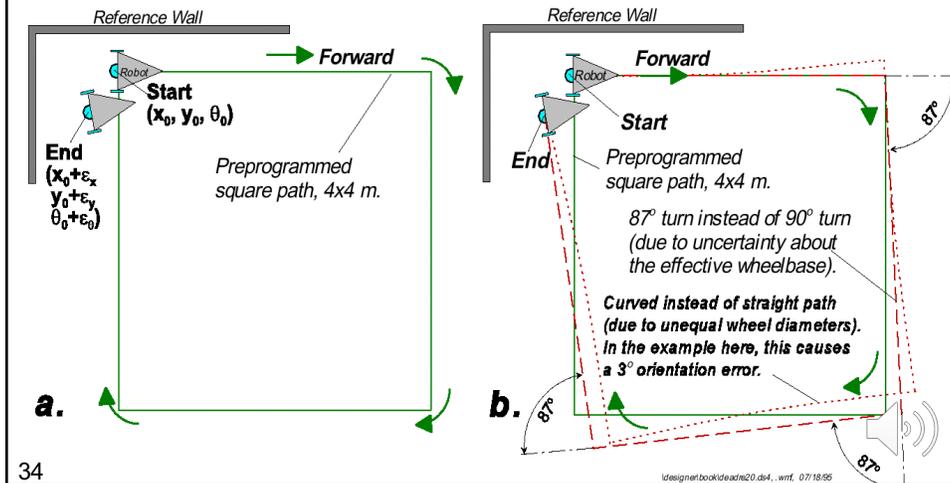
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Calibration of Errors



- The unidirectional square path experiment

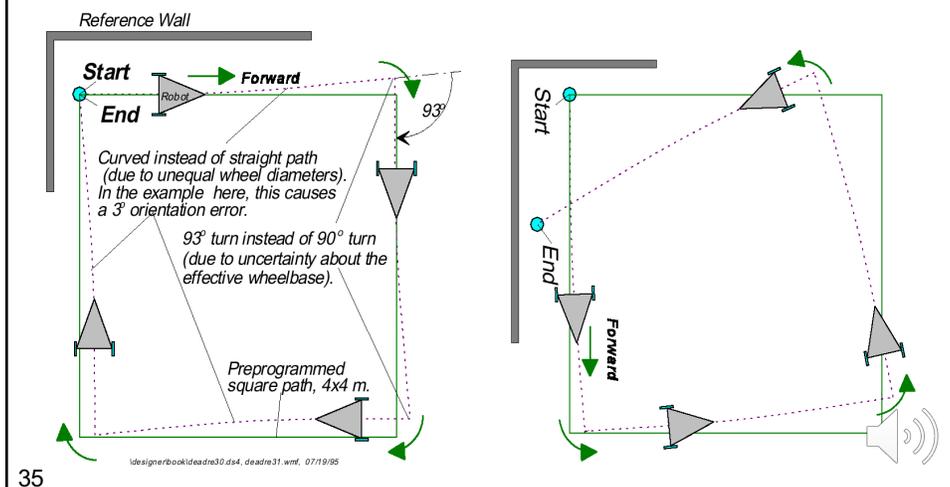


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Calibration of Errors



- Bi-directional square path experiment

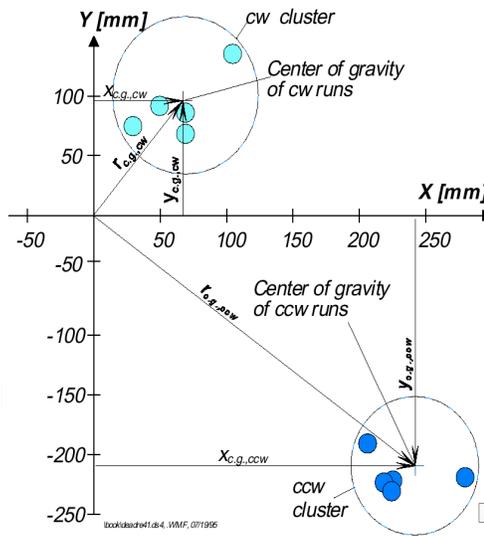


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Calibration of Errors



- Look at actual path traversed: what errors occur?
 - That is, where in the x, y space do we not see the expected location?
- Deterministic and non-deterministic errors



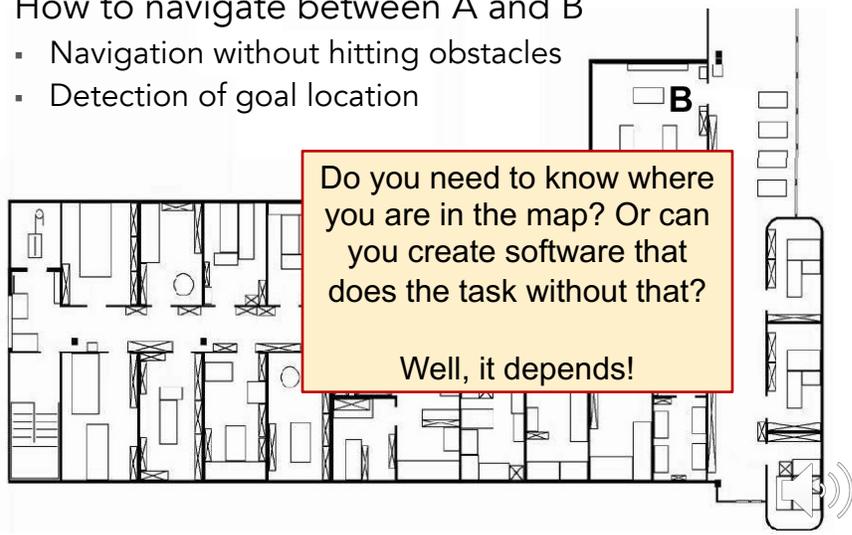
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To Localize, Or Not To...?



- How to navigate between A and B
 - Navigation without hitting obstacles
 - Detection of goal location



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Localization Summary (1)



- What is localization?
 - Figuring out location wrt. a model of the world
- Purely proprioceptive approaches:
 - Odometry: belief about motion only
 - Wheel encoders, mostly
 - Dead reckoning: belief about motion + heading sensors



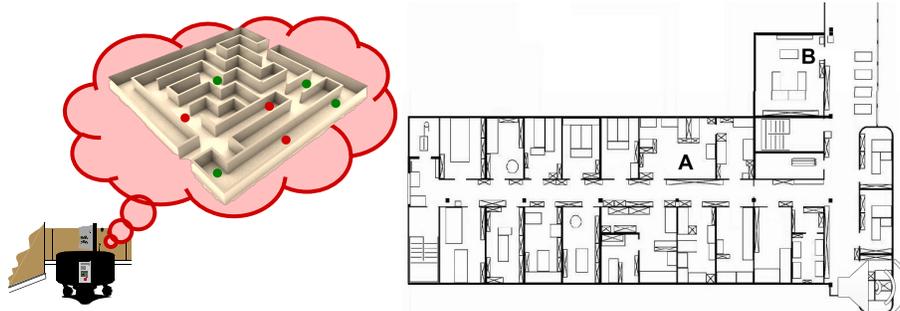
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Localization Summary (2)



- What is sensor aliasing?
 - Different locations giving the same sensor readings
- What is behavior-based navigation?
 - Navigating without localizing



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Behavior based navigation



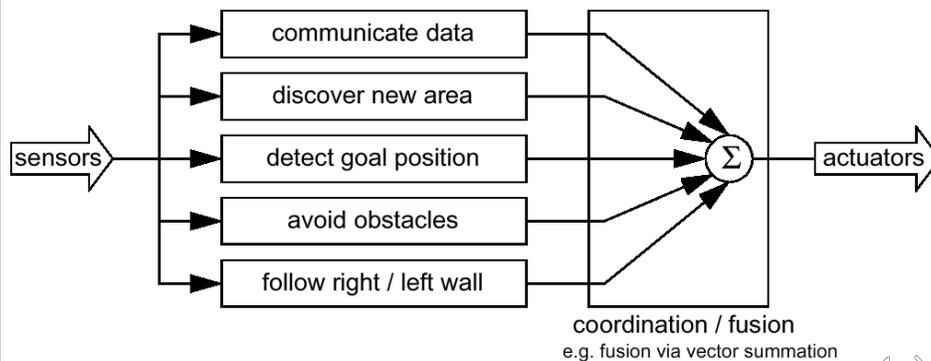
- An alternative to localization
- When you see *<input>*, do *<action>*.
 - Given *these* inputs, behave *this* way.
- When is this a good choice?
 - ✓ Fast to implement
 - ✓ Robust against error accumulation
 - ✓ Effective in unchanging environment
 - ✗ Does not scale to new environments
 - ✗ Behaviors must be designed and debugged
 - ✗ Sensor changes change behavior



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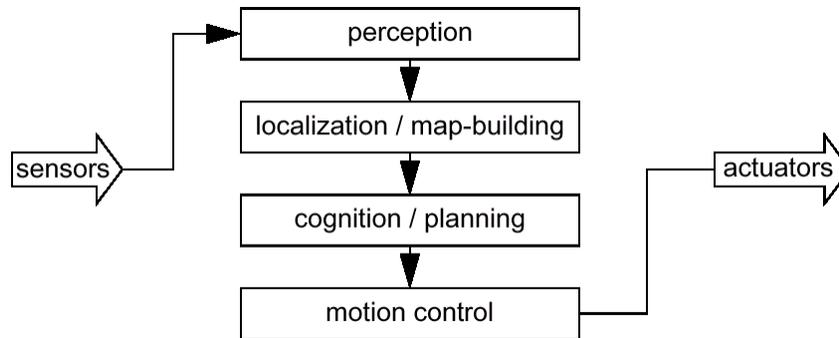
Behavior Based Navigation



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Model Based Navigation



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