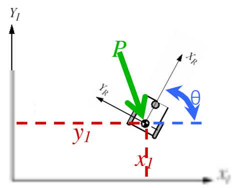
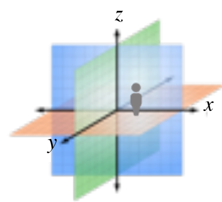
Kinematics: overview, transforms, and wheels





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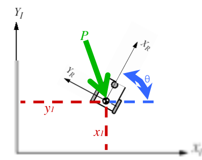
[A final note on] Mobile Kinematics

- Given this setup:



$$\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

- We can map $\{X_I, Y_I\}$ (global) ←→ $\{X_R, Y_R\}$ (robot)
  - Use rotation matrices and velocity vector in x, y, θ
- Why do we care so much?

2

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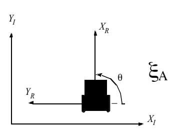
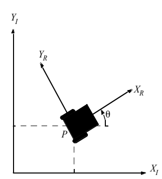
[A final note on] Mobile Kinematics

- Goal: take robot from A_I to B_I
  - We know where we want it in the global setting
  - What do we actually control? (In what frame of reference?)



$$\xi_A = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$



$$\xi_B = \begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix}$$

- Point: Convert from A_I to B_I by changing ξ_R

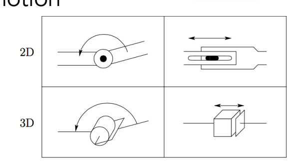
3

3

Manipulator Kinematics

- Kinematics (possible motion of a body) for manipulator robots
  - End effector position and orientation, wrt. an arbitrary initial frame

- A manipulator is moved by changing (sending motion commands to) its…
  - Joints: revolute and prismatic



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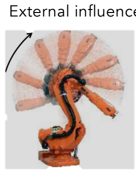
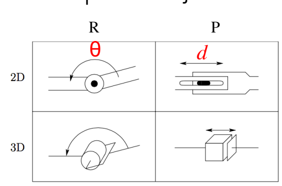
Manipulator State

Configuration: where is every point on a manipulator?
  - **Instantaneous** description of geometry of a manipulator

- State: a set of variables which describe change of configuration over time in response to joint forces
  - Control inputs
  - External influences





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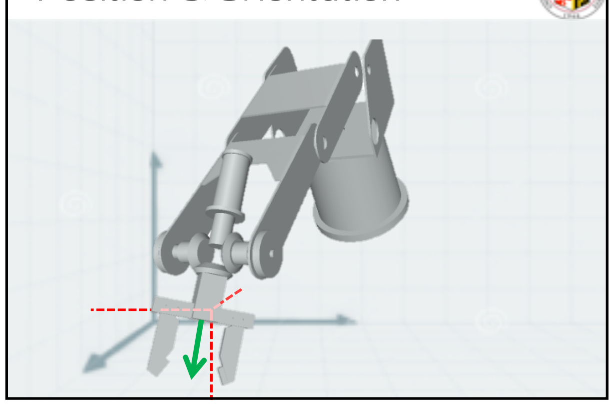
Position & Orientation



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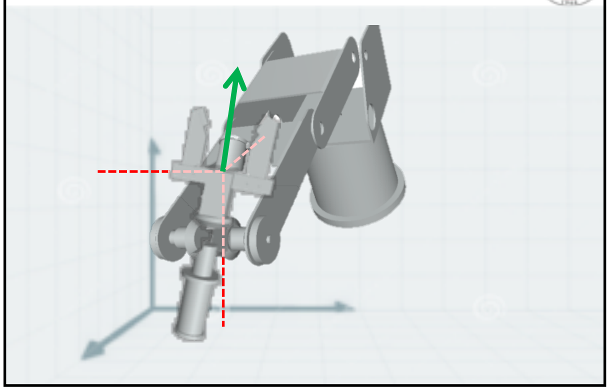
Position & Orientation



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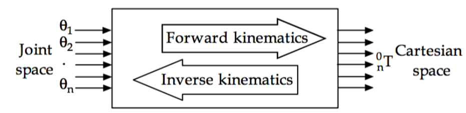
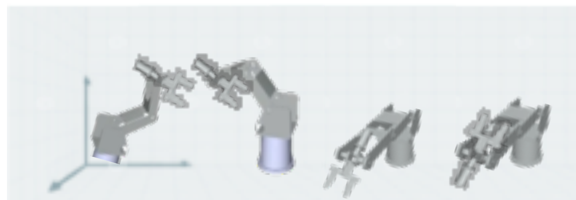
Forward Kinematics & IK





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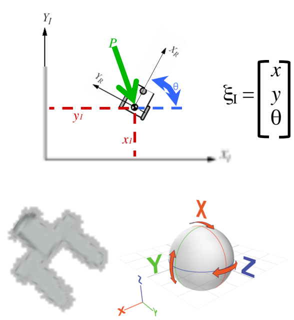
Mobile vs. Manipulator

- Description: how many parameters…
  - …to describe planar position & orientation?
  - …to describe 3D position & orientation?
- In 3D, it's always 6
  - Where is it on x, y, z?
  - What is its x, y, z rotation?



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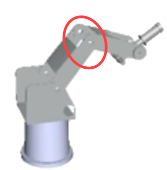
Kinematics Problem

- The **state space** is the set of all possible states
- The **state** of the manipulator is:
  - A set of variables which describe changes in **configuration** over time, in response to joint forces + external forces
- Where do joint forces come from?
  - Controllers!
- So, given some set of joints, what signals do we send?
- In **joint space** vs. **Cartesian space**



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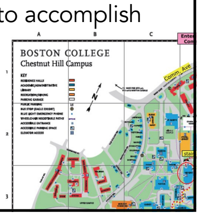
Joint vs. Cartesian space

- Joint space: we **control** the robot's DoFs
  - So we issue commands in terms of those
  - Mobile: "Roll forward 2 meters, rotate 53˚ clockwise"
  - Manipulator: "Rotate joint two 90˚ and joint four 65˚, then slide joint three 17cm"
- Cartesian space: usually we **want** to accomplish things in terms of the world
  - Mobile: Go to the building in B2
  - Manipulator: get the object on the table in front of you
- Kinematics lets us **transform** back and forth



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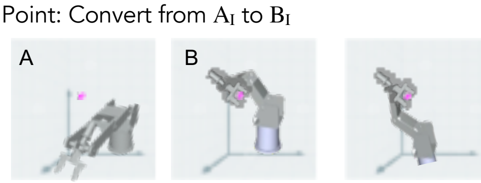
Goal

- Goal: take robot end effector from A_I to B_I
  - We know where we want it in the *global* setting
  - What do we actually control?
- Point: Convert from A_I to B_I



- Now a 6 ←→ 6 transformation

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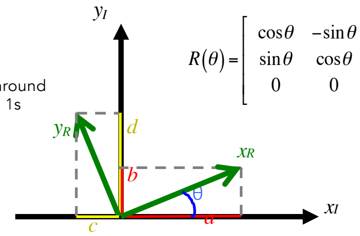
Review: Z Rotation Matrix*

- We derived this geometrically :
  - If we assume frame axes are of length 1
  - $a = \cos\theta$
  - $b = \sin\theta$
  - $c = -\sin\theta$
  - $d = \cos\theta$
  - Rotations around $z \rightarrow$ 0s and 1s



$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

* AKA *orthogonal rotation matrix*

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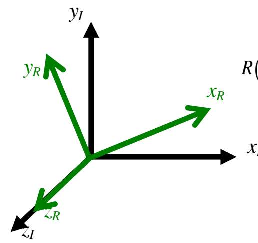
Review: Z Rotation Matrix*

- In practice, it's really this:
- Rotations around $z \rightarrow$ 0s and 1s



$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

* AKA *orthogonal rotation matrix*

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Other Rotation Matrices

- Similarly derived from axis of rotation and trigonometric values of projections

$$R_X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta_X & -sin\theta_X \\ 0 & sin\theta_X & cos\theta_X \end{bmatrix}$$

$$R_Y = \begin{bmatrix} cos\theta_Y & 0 & sin\theta_Y \\ 0 & 1 & 0 \\ -sin\theta_Y & 0 & cos\theta_Y \end{bmatrix}$$

$$R_Z = \begin{bmatrix} cos\theta_Z & -sin\theta_Z & 0 \\ sin\theta_Z & cos\theta_Z & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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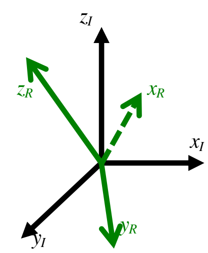
Complex Rotations

- What if we don't just rotate around a single axis?
- Any rotation in 3D space can be broken down into single-axis rotations
  - Given orthogonal axes
- Multiply rotation matrices:

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta_X & -sin\theta_X \\ 0 & sin\theta_X & cos\theta_X \end{bmatrix} \begin{bmatrix} cos\theta_Y & 0 & sin\theta_Y \\ 0 & 1 & 0 \\ -sin\theta_Y & 0 & cos\theta_Y \end{bmatrix}$$



- Can do any number of rotations; just multiply out

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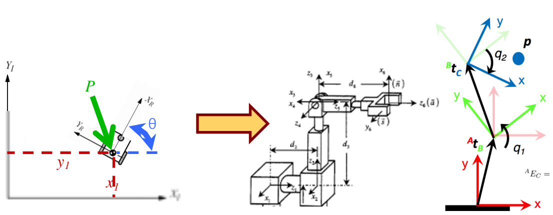
Mobile to Manipulator

- Add a number of chained frames of reference



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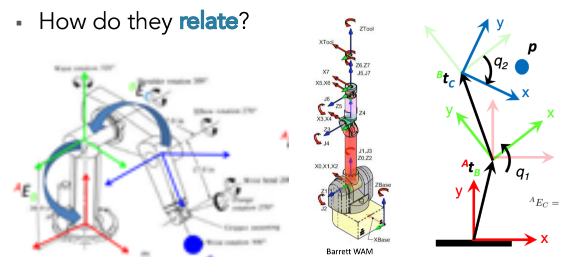
Multiframe Kinematics

- **How many** frames of reference do we have?
  - We've been translating among frames based on possible motion
- How do they **relate**?



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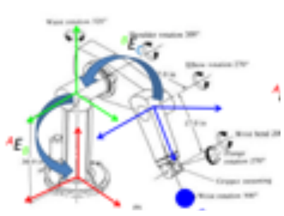
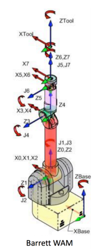
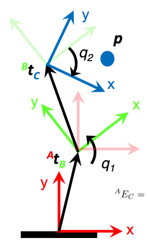
Kinematic Chaining

- Do you need to do every transformation?
- What do we really care about?



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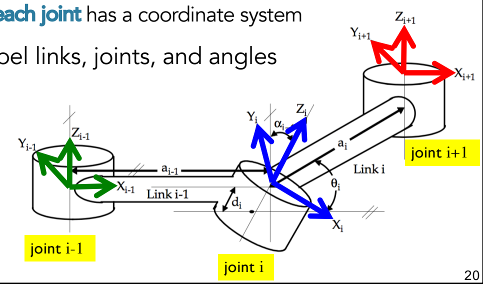
Describing A Manipulator

- Arm made up of links in a chain
- Joints each have <*x,y,z*> and roll/pitch/yaw
  - So, **each joint** has a coordinate system
- We label links, joints, and angles



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Forward Kinematics

- Vector Φ represents the array of M joint values:

$$\mathbf{\Phi} = [\phi_1 \quad \phi_2 \quad \ldots \quad \phi_M]$$

- Vector **e** represents an array of N values that describe the end effector in world space:

$$\mathbf{e} = \left[\; e_1 \quad e_2 \quad \ldots \quad e_N \;\right]$$

- If we need end effector position and orientation, **e** would contain 6 DOFs: 3 translations and 3 rotations. If we only need end effector position, **e** would just contain the 3 translations.

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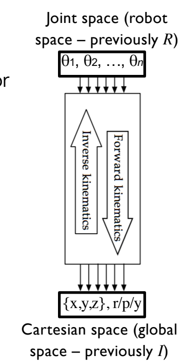
Forward & Inverse

- Forward:
  - Inputs: joint angles
  - Outputs: coordinates of end-effector
- Inverse:
  - Inputs: desired coordinates of end-effector
  - Outputs: joint angles
- Inverse kinematics are tricky
  - Multiple solutions
  - No solutions
  - Dead spots



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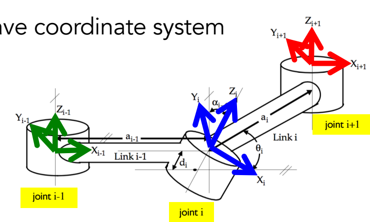
Describing A Manipulator

- Arm made up of links in a chain
  - How to describe each link?
  - Many choices exist
    - DH parameters, quaternions are widely used, Euler angles…
- Joints each have coordinate system
  - {x,y,z}, r/p/y



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Forward: i → i-1

- We are we looking for transformation matrix T, going from frame i to frame i-1:

$$T_i^{i-1} \qquad \text{(also written } {}^{i-1}_{\;\;i}T \text{ or } {}^{i-1}T_i \text{)}$$

- Determine position and orientation of end-effector as function of displacements in joints
- Why?
  - So we can multiply out along all joints

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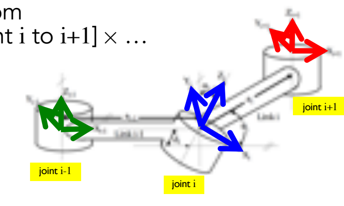
Forward Kinematics and IK

- Joint angles ⇆ end effector configuration
- Can string together rotations with multiplication
  - So, can get end effector rotation by
- Finding rotation from [joint i-1 to i] × [joint i to i+1] × …

$$R_2^0 = R_1^0 R_2^1$$

- **Rotation of end effector frame**, relative to **base frame**



www.youtube.com/watch?v=lVjFhNv2N

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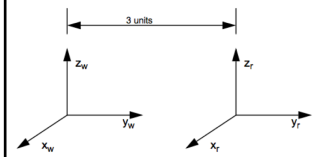
Matrices for Pure Translation



$$\xi_I = \begin{bmatrix} x_I \\ y_I \\ z_I \\ \theta \end{bmatrix} \quad \xi_R = \begin{bmatrix} x_R \\ y_R \\ z_R \\ \theta \end{bmatrix}$$

Origin point of R in I:

$$\begin{bmatrix} 0 \\ 3 \\ 0 \\ 1 \end{bmatrix}$$

In 3D:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Generally:

$$\begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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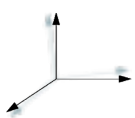
Matrices for Pure Rotation



$$\xi_I = \begin{bmatrix} x \\ y \\ z \\ \theta_I \end{bmatrix} \quad \xi_R = \begin{bmatrix} x \\ y \\ z \\ \theta_R \end{bmatrix}$$

Around z:

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Review?
Introduction to Homogeneous Transformations & Robot Kinematics
Jennifer Kay 2005

ce.aut.ac.ir/~shiry/lecture/robotics/amr/kinematics.p

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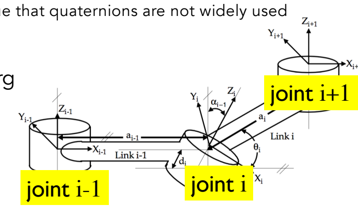
Describing A Manipulator

- Arm made up of links in a chain
  - How to describe each link?
  - Many choices exist
  - DH parameters widely used
    - Although it's not true that quaternions are not widely used
- *DH parameters*
  - Denavit-Hartenberg
  - $a_{i-1}, \alpha_{i-1}, d_i, \theta_2$



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Denavit-Hartenberg Method

- Efficient way to find transformation matrices

1. Set frames for all joints
   - This is actually the tricky part.
2. Calculate all DH parameters from frames
   - 4 DH parameters fully define position and orientation (not 6)
3. Populate DH parameter table
4. Populate joint-to-joint DH transformation matrices
   - Matrix for 0-1, matrix for 1-2, etc.
5. Multiply all matrices together, in order
   - 0-1 × 1-2 × 2-3 × …

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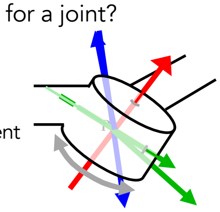
Defining Frames for Joints

- What's the frame of reference for a joint?
  - Actually, completely flexible
- We usually choose:
  - 1 axis through the center of rotation/direction of displacement
  - 2 more perpendicular to that
    - Which can be any orientation!



- We can move the origin
  - P is no longer <0, 0, 0>
- To use DH method, choose frames carefully

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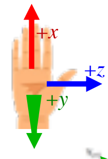
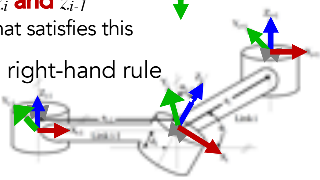
Choosing Frames for DH

- z axis must be axis of motion
  - Rotation around z for revolute
  - Translation along z for prismatic
- x_i axis orthogonal to z_i **and** z_{i-1}
  - There's always a line that satisfies this
- y axis must follow the right-hand rule
  - Fingers point $+x$
  - Thumb points $+z$
  - Palm faces $+y$
- x_i axis must intersect z_{i-1} axis (may mean translating origin)





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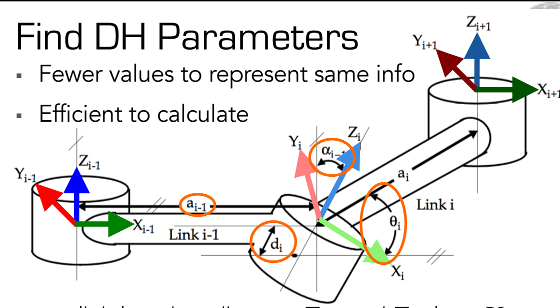
Find DH Parameters

- Fewer values to represent same info
- Efficient to calculate



a_{i-1} : link length – distance Z_{i-1} and Z_i along X_i
α_{i-1} : link twist – angle Z_{i-1} and Z_i around X_i
d_i : link offset – distance X_{i-1} to X_i along Z_i
θ_i : joint angle – angle X_{i-1} and X_i around Z_i

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Denavit-Hartenberg Method

- A way of finding transformation matrix (quickly)

1. Assign DH frames to DoFs (previous slide)
   - This takes practice.
2. Create a parameter table
   - Rows = (# frames – 1)
   - Columns = **4** (always) ← your DH parameters θ, α, a, d

	θ	α	a	d
frame 0-1	$\theta_{0\text{-}1}$	$\alpha_{0\text{-}1}$	$a_{0\text{-}1}$	$d_{0\text{-}1}$
frame 1-2	$\theta_{1\text{-}2}$	$\alpha_{1\text{-}2}$	$a_{1\text{-}2}$	$d_{1\text{-}2}$
frame 2-3	…	…	…	…

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Denavit-Hartenberg Method

- Given parameter table,

3. Fill out transformation matrix* for each transition:

$$R_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_{i,i+1} & \sin\theta_i \sin\alpha_{i,i+1} & a_{i,i+1}\cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_{i,i+1} & -\cos\theta_i \sin\alpha_{i,i+1} & a_{i,i+1}\sin\theta_i \\ 0 & \sin\alpha_{i,i+1} & \cos\alpha_{i,i+1} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. And multiply. Ex: $R_2^0 = R_1^0 R_2^1$

- R_2^0 is the same matrix as would be found by other methods. DH is fast and efficient.

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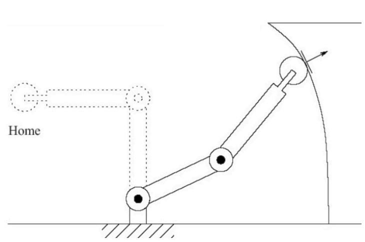
Example: Rotation in Plane



$$x = a_1 \cos\theta_1 + a_2 \cos(\theta_1 + \theta_2)$$
$$y = a_1 \sin\theta_1 + a_2 \sin(\theta_1 + \theta_2)$$
$$a_i = \text{ the length of } i\text{th link}$$

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Transformation i to i-1

a_{i-1} : distance Z_{i-1} and Z_i along X_i
α_{i-1} : angle Z_{i-1} and Z_i around X_i
} together: *screw displacement*

$$[X_i] = \text{Trans}_{X_i}(a_{i,i+1})\,\text{Rot}_{X_i}(\alpha_{i,i+1})$$

d_i : distance X_{i-1} to X_i along Z_i
θ_2 : angle X_{i-1} and X_i around Z_i
} together: *screw displacement*

$$[Z_i] = \text{Trans}_{Z_i}(d_i)\,\text{Rot}_{Z_i}(\theta_i)$$

- Coordinate transformation:

$$^{i-1}T_i = [Z_i][X_i] = \text{Trans}_{Z_i}(d_i)\,\text{Rot}_{Z_i}(\theta_i)\,\text{Trans}_{X_i}(a_{i,i+1})\,\text{Rot}_{X_i}(\alpha_{i,i+1}),$$

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Transformation i to i-1

$$\mathrm{Trans}_{Z_i}(d_i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{Rot}_{Z_i}(\theta_i) = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{Trans}_{X_i}(a_{i,i+1}) = \begin{bmatrix} 1 & 0 & 0 & a_{i,i+1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{Rot}_{X_i}(\alpha_{i,i+1}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_{i,i+1} & -\sin\alpha_{i,i+1} & 0 \\ 0 & \sin\alpha_{i,i+1} & \cos\alpha_{i,i+1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation in DH:

$${}^{i-1}T_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_{i,i+1} & \sin\theta_i\sin\alpha_{i,i+1} & a_{i,i+1}\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_{i,i+1} & -\cos\theta_i\sin\alpha_{i,i+1} & a_{i,i+1}\sin\theta_i \\ 0 & \sin\alpha_{i,i+1} & \cos\alpha_{i,i+1} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

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