

CMSC 479/679: Assignment 3

Kinematics and Error Propagation

Turnin: Blackboard.

Please submit all parts together as a **single PDF file** named *lastname_hw3.pdf*, with parts clearly marked and delineated. **All** files must start with your last name and have your full name(s) in the file, at/near the top. This assignment should be done **individually**.

PART I. 2D TRANSFORMS (20 PTS.)

Consider a 1.2m x 1.2m x 1.2m cube-shaped mobile robot with recessed omni wheels. We define the following P as the geometric center of the robot and the origin of the robot's frame of reference. The robot starts with its back left bottom corner on the global frame's origin (see figure) and the x , y , and z axes of its frame of reference parallel to, and positive in the same direction as, those of the world frame.

Initially, the robot's back left bottom corner is on the (x,y,z) origin. After the initial timestep, the robot moves forward 3 meters, rotates 15° clockwise, moves to its right 2 meters, rotates 30° counterclockwise, and then moves 2 meters at a 45° angle in the front-right direction (equal movement in $+x$ and $+y$ in its frame).

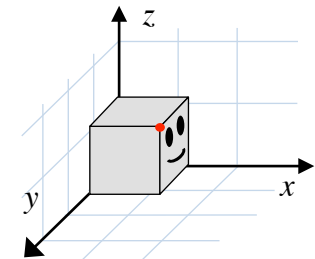


Figure 1. A mobile robot with recessed omni wheels. Initial "forward" direction is $+x$ (smiley shows orientation).

Assignment:

1. Give the robot's initial 3-element ξ vector.
2. Give a series of transformation matrices for the movement described.
3. From these, create a single transformation matrix. Show all work.
4. Find the robot's 3-element ξ after the movement (ξ'). Show all work.
5. Consider the robot's top right front corner (red dot in the figure). Find the 4-element ξ vector for this point after the robot has moved. Show all work. (Hint: what matrix describes the relationship between this point and P ?)

PART II. 3D TRANSFORMS (20 PTS.)

Consider the back-driveable robot arm in Figure 2. The first joint on this arm is continuous rotation and the second has a maximum rotation of 180°. The link lengths are 50cm and 40cm. The end effector is fixed, so there are no joints on the tool frame.

Initially, the joints are set to $q = \langle 0^\circ, 0^\circ \rangle$. Someone grasps the robot and moves it to q' . According to the joint encoders, this rotates the first joint 76° clockwise and the second joint 49° counterclockwise.

Assignment:

6. Give q' .

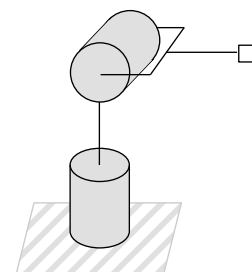


Figure 2. A 2-DoF manipulator robot with fixed gripper.

7. Define axes for each frame on this robot. (Don't forget the tool frame!)
8. Write the initial 4-element ξ vector for the robot's end effector.
9. Give transformation matrices from the first joint to the second joint (R_1^0) and the second joint to the end effector (R_2^1). (Use rotation and translation matrices, not DH parameters.)
10. Find the transformation matrix from the global frame of reference to the end effector's frame of reference. Show all work.
11. Use this matrix to find the end effector's ξ' vector.

PART III. DENAVIT-HARTENBERG TRANSFORMS (20 PTS.)

Using the same problem setup as in Part II (same robot, same movement):

12. Define axes that obey Denavit-Hartenberg conventions for each frame on this robot (these may be the same as in 7.)
13. Calculate all DH parameters, and give a populated DH parameter table.
14. Find the DH transformation matrices for all frame transitions.
15. Find the transformation matrix from the global frame of reference to the end effector's frame of reference. Show all work.
16. Use this matrix to find the end effector's position and orientation after being moved.

PART IV. DH TRANSFORMS FOR PRISMATIC JOINTS (20 PTS.)

Consider the robot arm in Figure 3. The first joint on this arm is continuous rotation, the second has a rotation of 180° , and the third has a maximum distance of travel of 10cm. The link lengths are 20cm, 30cm, and 25cm. The end effector is fixed.

Initially, the joints are set to $q = \langle 0^\circ, 0^\circ, 0\text{cm} \rangle$. The robot executes a series of motion commands that result in the configuration $q' = \langle 60^\circ, 15^\circ, 5\text{cm} \rangle$.

Assignment:

17. Define axes that obey Denavit-Hartenberg conventions for each frame on this robot.
18. Calculate all DH parameters, and give a populated DH parameter table. Find the DH transformation matrices for all frame transitions.
19. Find the transformation matrix from the global frame of reference to the end effector's frame of reference. Show all work.
20. Use this matrix to find the end effector's ξ' vector. Show all work.

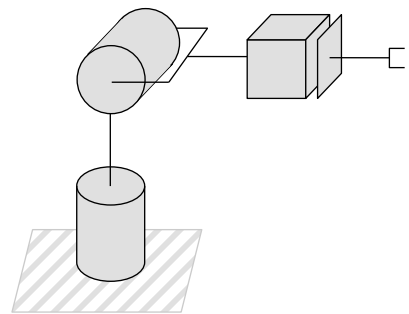


Figure 3. A 3-DoF manipulator robot with two revolute joints, a prismatic joint, and a fixed gripper.

PART V. DH TRANSFORMS WITH OFFSETS (20 PTS.)

Consider the manipulator in Figure 4. The first joint on this arm is continuous rotation and the second has a maximum travel of 10cm. The link connecting the second joint to the end effector has two components, connected at right angles. All measurements are as shown.

Assignment:

21. Define axes that obey Denavit-Hartenberg conventions for each frame on this robot.
22. Calculate all DH parameters, and give a populated DH parameter table. Find the DH transformation matrices for all frame transitions.
23. Find the transformation matrix from the global frame of reference to the end effector's frame of reference. Show all work.

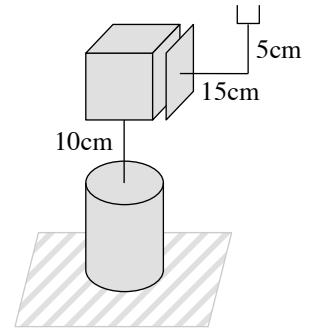


Figure 4. A 2-DoF robot with an offset link connecting a fixed end effector.