

## Belief and Map Representations

## Last Time...

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- ◆ Localization
  - ◆ Concepts
  - ◆ Error
  - ◆ Error variance
  - ◆ Error sources
- ◆ Movement error
  - ◆ Error accumulation
  - ◆ Deriving error description (differential drive)
  - ◆ Calibration
- ◆ Behavior-Based Navigation
  - ◆ Is...
- ◆ Belief State
  - ◆ Belief about...

## Belief Representation

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- ◆ So what "belief" are we talking about?
- ◆ **Where the robot believes it is during localization.**
  - ◆ Lots of ways to think about "where something is"
- ◆ "Belief" = Hypothesis (or hypotheses) about location
  - ◆ With uncertainty, variance, probability
- ◆ AI 101: any stored information about the world must be **represented** in some machine-usable way

## Characterizing Belief Representations

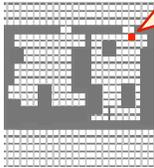
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- ◆ **Discrete vs. continuous**
  - ◆ Is the robot's location fixed to a grid, or can it be anywhere?
- ◆ **Single vs. multiple** hypotheses
  - ◆ At any given time, how many possible locations is the robot hypothesizing it might be in?
- ◆ **Probabilistic vs. bounded vs. point**

## Discrete/Continuous

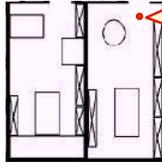
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- ◆ Discrete vs. continuous
  - ◆ Fixed to a grid vs. infinitely fine resolution



I think I'm in this cell

Discrete



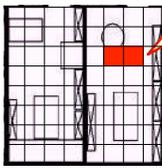
I think I'm at {x = 81.1, y = 14.2}

Continuous

## Discrete/Continuous (2)

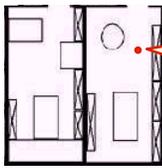
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- ◆ Discrete vs. continuous
- ◆ **Belief** can be discretized on a **continuous** map



In one of these

Discrete



{x = 81.1, y = 14.2}

Continuous

## Single/Multiple Hypothesis

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- ◆ Single hypothesis vs. multiple hypothesis
  - ◆ "Best guess, I'm here" vs
  - ◆ "Best guess, I'm either here or here"

Single Multiple

- ◆ Multiple hypothesis: maintaining a set of "best" locations

## Location Representation

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- ◆ Probabilistic vs. bounded vs. point

Point Bounded Polygon Probabilistic

- ◆ You are here
- ◆ Somewhere in here (undifferentiated)
- ◆ Spread of likelihood

## Belief Rep. Characteristics

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- ◆ Beliefs about where robot is can be:
  - ◆ Continuous or discretized
  - ◆ Single or multiple hypothesis
    - ◆ Single hypothesis: "Best guess, I am here"
    - ◆ Multiple hypotheses: "Here or here?"
  - ◆ Point, bounded, or probability distributions
    - ◆ In practice we almost always use probabilities.

## Possible Combinations

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- ◆ These are orthogonal choices
- ◆ Many combinations are possible

Single Hypothesis, Continuous, Probabilistic Multiple Hypothesis, Continuous, Point Single, Discrete, Probabilistic

## Probability with Multiple Hypotheses

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- ◆ Robot's belief in its location shown as probability "cloud"
- ◆ Darker coloring = higher certainty

Path of the robot Belief states at positions 2, 3 and 4

## Belief Representation

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- Continuous map with single hypothesis
- Continuous map with multiple hypotheses
- Discretized map with probability distribution
- Discretized topological map with probability distribution

## Belief Representation

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- a) Continuous map with single hypothesis
- b) Continuous map with multiple hypotheses
- c) Discretized map with probability distribution
- d) Discretized topological map with probability distribution

## Continuous vs. Discrete

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Belief Representation can be...

<ul style="list-style-type: none"> <li>◆ Continuous</li> <li>◆ Precision bound by sensor data</li> <li>◆ Often single hypothesis pose estimate</li> <li>◆ Lost when diverging (for single hypothesis)</li> <li>◆ Compact representation</li> <li>◆ Typically reasonable in processing power</li> </ul>	<ul style="list-style-type: none"> <li>◆ Discrete</li> <li>◆ Precision bound by resolution of discretization</li> <li>◆ Often multiple hypothesis pose estimates</li> <li>◆ Never lost (when diverges, converges to another cell)</li> <li>◆ Less memory and processing power</li> </ul>
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## Belief and Map Representations

## Map Representations

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- ◆ We use "Map" to mean the robot's representation of the space it's moving through
  - ◆ Any stored information about the world must be represented in some machine-usable way
- ◆ What does a (robot-usable) "map" look like?

Maps can be...

- ◆ Geometric
- ◆ Topological
- ◆ Semantic

} Many, many variations

## Map Representation Types

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- ◆ **Geometric:** Tied to layout of the world
  - ◆ This is what we think of as a "map:"
- ◆ **Topological:** How areas connect / adjacencies
  - ◆ "The set of places and edges linking them"

```

            graph LR
            Z1((Zone 1)) -- Door 1 --> Z2((Zone 2))
            Z2 -- Door 2 --> Z3((Zone 3))
            Z3 -- Door 2 --> Z2
            Z2 -- Door 1 --> Z1
            
```
- ◆ **Semantic:** What areas actually "are"
 

```

            graph LR
            K[Kitchen] <--> H[Hall]
            H <--> LR[Living Room]
            
```

## Discrete vs. Continuous

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- ◆ Actually easier to understand in maps than beliefs...

Continuous

Discretized

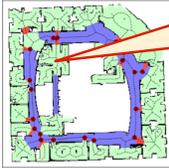
Obstacles represented as polygons

Obstacles represented as blocks in a grid

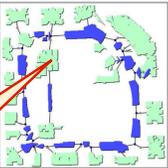
## Geometric vs. Topological

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- ◆ Aligned to the “real world” vs. just “traversable”



Geometric



Topological

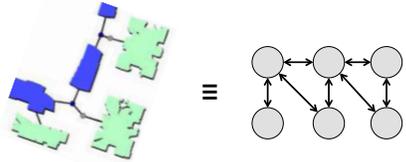
Actual locations of obstacles and areas

Relative locations with adjacencies

## A Note on Topological Maps

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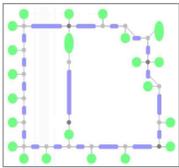
- ◆ Often shown as semi-geometric
- ◆ Doesn't have to be!
- ◆ Areas (nodes) and adjacencies (edges) are what matter



## Semantic Maps

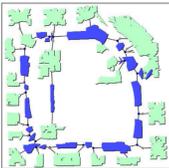
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- ◆ Areas are labeled by meaningful **type**
- ◆ Example: semantically labeled topological map



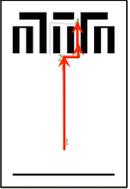
Semantic





## Example

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Path of the robot

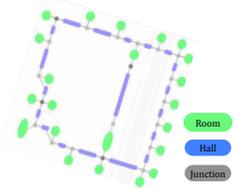
Belief states at positions 2, 3 and 4

- ◆ Belief Representation: Probabilistic? Discrete? **yes, no**
- ◆ Map Representation: Discretized? Topological? Semantic? **maybe, no, no (geometric)**

## The Environment

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- ◆ Can contain:
  - ◆ Static or dynamic **obstacles**
  - ◆ **Features** (e.g., doors, floor tiles)
- ◆ Can be semantically labeled
- ◆ Environment Representation
  - ◆ Continuous Metric →  $\{x,y,\theta\}$
  - ◆ Discrete Metric → metric grid (eg. sq. D76)
  - ◆ Discrete Topological → topological grid



## The Environment: Features

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- ◆ Raw sensor data (ex.: laser range, grayscale images)
  - ◆ Lots of data, low distinctiveness (per reading)
  - ◆ Uses all acquired information
- ◆ Low level features (ex.: line extraction)
  - ◆ Some data, average distinctiveness
  - ◆ Filters out some useful information, still ambiguities
- ◆ High level features (ex.: doors, a car, the Eiffel tower)
  - ◆ Little data, high distinctiveness
  - ◆ Filters out the useful information, few/no ambiguities
  - ◆ (Usually) insufficient environmental information

easy to get

hard to get

## Choosing Map Representations

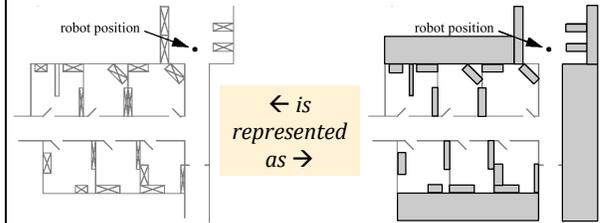
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- ◆ Map precision vs. application
  - ◆ Precision should match goals
  - ◆ How precise does it need to be?
- ◆ Feature precision vs. map precision
  - ◆ Precision of map should match precision of sensors
  - ◆ 20cm. map precision  $\neq$  20cm. obstacle avoidance
- ◆ Precision vs. computational complexity
  - ◆ More capability = more computational complexity
- ◆ Closed-world assumption: if it exists, it's on the map
  - ◆ When is this okay? When is it not?

## Polygonal Representation

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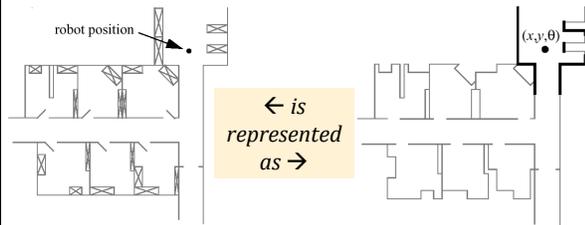
- ◆ Everything in the world is represented as polygons



## Line Representations

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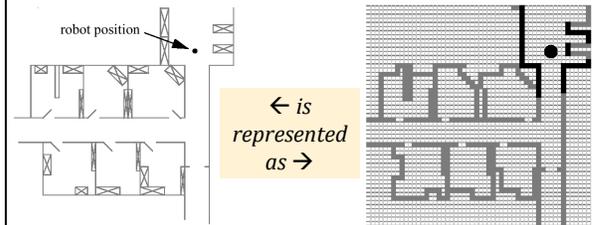
- ◆ Everything is lines (usually from edge detection)



## Discretized Representations

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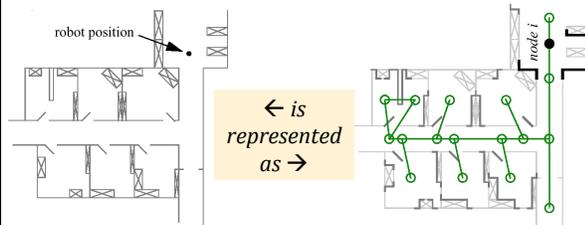
- ◆ This example: Fixed cell size, grid layout



## Topological Maps

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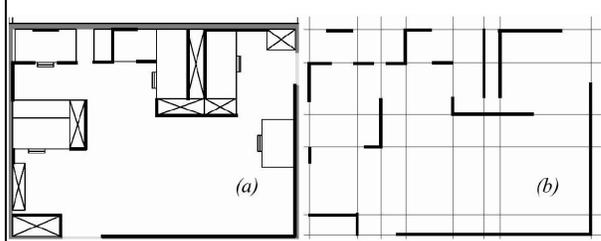
- ◆ Areas (nodes) and traversable adjacencies (edges)



## Continuous Line Based

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- a) Representation with set of infinite lines (line extraction)



## How To Build Maps



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- ◆ All of these map representations are simplifications of reality
  - ◆ Easier to store
  - ◆ More intuitive (sometimes)
  - ◆ Faster to use computationally
- ◆ Simplified maps still derived from sensor data
- ◆ How do we get from rich sensor data to representations?

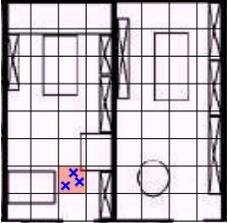
## Decomposing Maps



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- ◆ **Map decomposition**
  - ◆ Break the map into areas we care about (cells)
- ◆ Important warning: In cell decompositions, **→ every point in a cell is the same as every other ←**
  - ◆ Can't differentiate, e.g., "the top half of cell C2"
  - ◆ Would need a new cell



## Exact Cell Decomposition



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- ◆ Subdivide a map into blocks of free space
  - ◆ Find vertices of objects in area
  - ◆ Break up map so all vertices bound cells

Idea:

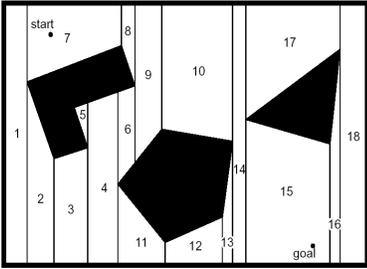
- ◆ It matters how a robot can traverse free cells
- ◆ Exact position doesn't matter

## Exact Cell Decomposition



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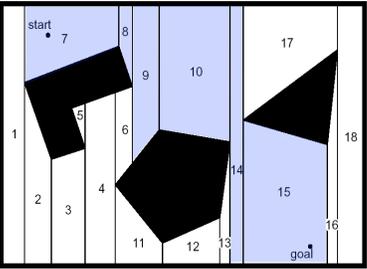
## Navigation through Exact Cells



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- ◆ Sequence: 7, 8, 9, 10, 14, 15



## Navigation through Exact Cells



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- ◆ One possible sequence (of many)

