Propositional Logic: Pro & Con

Propositional logic: pro and con

- **PL Advantages**
  - **Simple** KR language good for many problems
  - **Lays foundation** for higher logics (e.g., FOL)
  - **Reasoning is decidable**, though NP complete; efficient techniques exist for many problems
- **PL Disadvantages**
  - **Not expressive** enough for many problems
  - Even when it is, it can very **unconcise**

PL is a weak KR language

- Hard to identify ***individuals*** (e.g., Mary, 3)
- Can't directly represent **properties** of individuals or **relations** between them (e.g., "Bill age 24")
- **Generalizations**, patterns, regularities hard to represent (e.g., "all triangles have 3 sides")
- First-Order Logic (FOL) represents this information via **relations**, **terms**, **variables** & **qualifiers**, e.g.,
  - *John loves Mary:* loves(John, Mary)
  - *Every elephant is gray:* $\forall$ X (elephant(X) $\rightarrow$ gray(X))
  - *There is a black swan:* $\exists$ X (swan(X) ^ black(X))

Hunt the Wumpus domain

- Some atomic propositions:

  A12 = agent is in cell (1,2)

  S12 = There's a stench in cell (1,2)

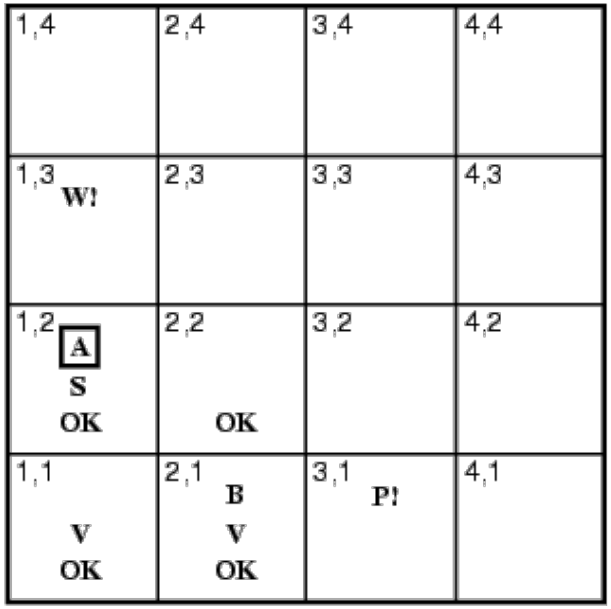
  B34 = There's a breeze in cell (3,4)

  W22 = Wumpus is in cell (2,2)

  V11 = We've visited cell (1,1)

  OK11 = cell (1,1) is safe

  …



- Some rules:

  $\neg S22 \rightarrow \neg W12 \wedge \neg W23 \wedge \neg W32 \wedge \neg W21$

  $S22 \rightarrow W12 \vee W23 \vee W32 \vee W21$

  $B22 \rightarrow P12 \vee P23 \vee P32 \vee P21$

  $W22 \rightarrow S12 \wedge S23 \wedge S32 \wedge W21$

  $W22 \rightarrow \neg W11 \wedge \neg W21 \wedge \ldots \neg W44$

  $A22 \rightarrow V22$

  $A22 \rightarrow \neg W11 \wedge \neg W21 \wedge \ldots \neg W44$

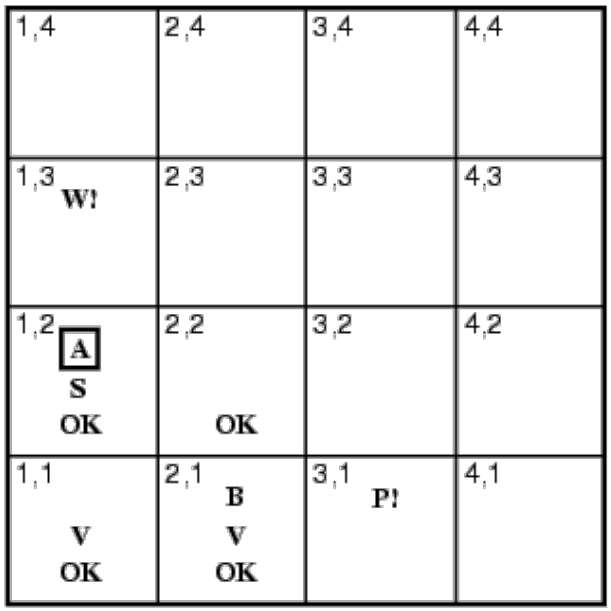
  $V22 \rightarrow OK22$

If there's no stench in cell 2,2 then the Wumpus isn't in cell 21, 23 32 or 21

Hunt the Wumpus domain

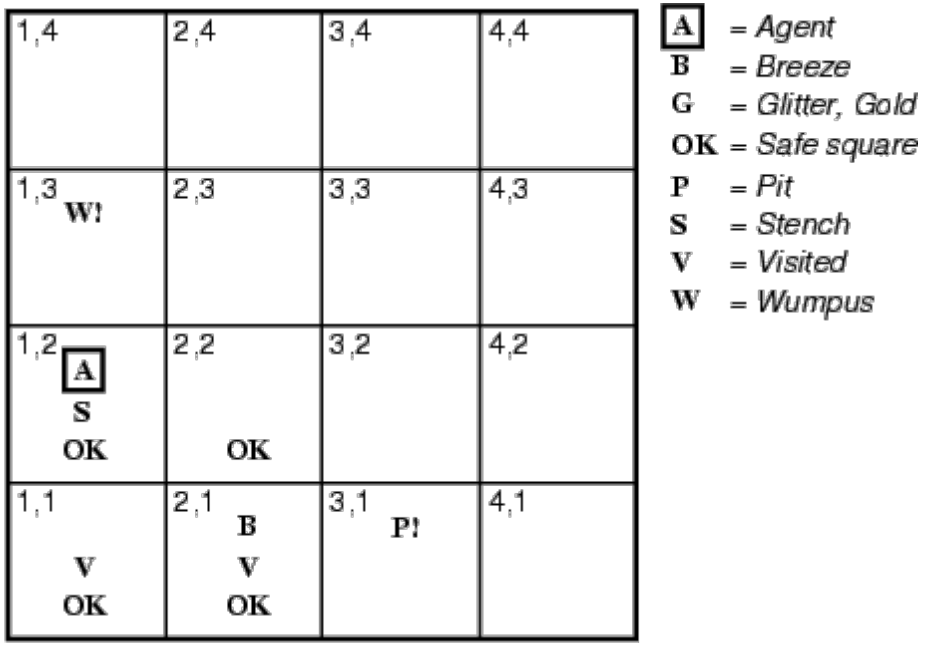
- Eight symbols for each cell, i.e.: A11, B11, G11, OK11, P11, S11, V11, W11
- Lack of variables requires giving similar rules for each cell!
- Ten rules (I think) for each

$A11 \rightarrow \ldots$
$V11 \rightarrow \ldots$
$P11 \rightarrow \ldots$
$\neg P11 \rightarrow \ldots$
$W11 \rightarrow \ldots$
$\neg W11 \rightarrow \ldots$
$S11 \rightarrow \ldots$
$\neg S11 \rightarrow \ldots$
$B11 \rightarrow \ldots$
$\neg B11 \rightarrow \ldots$



- 8 symbols for 16 cells => 128 symbols
- 2^{128} possible models ☹
- Must do better than brute force

After third move



- We can prove that the Wumpus is in (1,3) using these four rules
- See R&N section 7.5

(R1) $\neg S11 \rightarrow \neg W11 \wedge \neg W12 \wedge \neg W21$

(R2) $\neg S21 \rightarrow \neg W11 \wedge \neg W21 \wedge \neg W22 \wedge \neg W31$

(R3) $\neg S12 \rightarrow \neg W11 \wedge \neg W12 \wedge \neg W22 \wedge \neg W13$

(R4) $S12 \rightarrow W13 \vee W12 \vee W22 \vee W11$

Proving W13: Wumpus is in cell 1,3

(R1) $\neg S11 \rightarrow \neg W11 \wedge \neg W12 \wedge \neg W21$

(R2) $\neg S21 \rightarrow \neg W11 \wedge \neg W21 \wedge \neg W22 \wedge \neg W31$

(R3) $\neg S12 \rightarrow \neg W11 \wedge \neg W12 \wedge \neg W22 \wedge \neg W13$

(R4) $S12 \rightarrow W13 \vee W12 \vee W22 \vee W11$

Apply **MP** with $\neg S11$ and R1:

$\neg W11 \wedge \neg W12 \wedge \neg W21$

Apply **AE**, yielding three sentences:

$\neg W11, \neg W12, \neg W21$

Apply **MP** to ~S21 and R2, then apply **AE**:

$\neg W22, \neg W21, \neg W31$

Apply **MP** to S12 and R4 to obtain:

$W13 \vee W12 \vee W22 \vee W11$

Apply **UR** on $(W13 \vee W12 \vee W22 \vee W11)$ and $\neg W11$:

$W13 \vee W12 \vee W22$

Apply **UR** with $(W13 \vee W12 \vee W22)$ and $\neg W22$:

$W13 \vee W12$

Apply **UR** with $(W13 \vee W12)$ and $\neg W12$:

$W13$

QED

Rule Abbreviation

MP: modes ponens

AE: and elimination

R: unit resolution

Propositional Wumpus problems

#1 Lack of variables prevents general rules

- Encoding that any cell we've visited is safe just requires one FOL sentence:

  $\forall$ x, y V(x,y) $\rightarrow$ OK(x,y)

- Encoding that a stench implies the Wumpus is nearby is also simple

  $\forall$ x, y S(x,y) $\rightarrow$ W(x-1,y) $\vee$ W(x+1,y) $\vee$ …

- Though adjusting for the world edges complicates it, but that's easy to fix

Propositional Wumpus problems

#2 Change of KB over time hard to represent

- In classic logic; a fact is true or false for all time
- A standard FOL technique is to index **dynamic facts** with the time when they're true
  - A(1, 1, 0) *# agent was in cell 1,1 at time 0*
  - A(2, 1, 1) *# agent was in cell 2,1 at time 1*
- For propositional logic, we need a separate KB for every time point

Propositional logic summary

- **Inference**: deriving new sentences from old
  - **Sound** inference derives true conclusions given true premises
  - **Complete** inference derives all true conclusions from premises
- Different logics make different **commitments** about what world is made of and kinds of beliefs we can have
- **Propositional logic** commits only to existence of facts that may or may not be the case
  - Simple syntax & semantics illustrates inference process
  - Sound, complete and fast proof procedures
  - It can be impractical or cumbersome for many worlds

Fin