CMSC 435 / 634 Introduction to Computer Graphics

Homework Assignment 2 (Due Oct 15th before class)

- The work must be all your own.
- Be explicit, define your symbols, and explain your steps. (This will make it a lot easier for us to assign partial credit.)

1. (15 points) Under which conditions do we have C^1 and G^1 continuity for two joined Bezier curves? Write out the condition explicitly as a test on the control points *p0*, *p1*, *p2*, *p3* and *q0*, *q1*, *q2*, *q3* of the two curves. Please provide two examples, with each representing the C^1 or G^1 continuity accordingly.

To get C1 continuity, we need the two tangent vectors to be the same magnitude and opposite direction, i.e., p3-p2 = q1-q0

To get G1 continuity, we still need the two tangent vectors to be in opposite directions, but the magnitudes may be different. That is the condition is the same as one for C1 continuity expect that the vector can be positive multiples of each other: p3-p2 = k(q1-q0) for some real k>0

2. (**634 Only,** 15 points) Compute the normal vector of a Bezier surface patch at the four corners and at the center for a given set of control points.

The normal vector of the Bezier surface is given bythe normalized cross product of its partial derivatives. We write $\mathrm{norm}(\mathbf{n}) = \frac{\mathbf{n}}{\|\mathbf{n}\|}$*. Then*

$$\mathbf{n} = \mathrm{norm}(\frac{\partial \mathbf{B}}{\partial u} \times \frac{\partial \mathbf{B}}{\partial v})$$

(a) *Normal at* $(0,0)$*. We know that* $\frac{\partial}{\partial u}\mathbf{B}(0,0) = 3(\mathbf{p}_{10} - \mathbf{p}_{00})$ *and* $\frac{\partial}{\partial v}\mathbf{B}(0,0) = 3(\mathbf{p}_{01} - \mathbf{p}_{00})$*. Therefore*

$$\mathbf{n} = \mathrm{norm}(3(\mathbf{p}_{10} - \mathbf{p}_{00}) \times 3(\mathbf{p}_{01} - \mathbf{p}_{00})).$$

(b) *Normal at* $(0,1)$*. We know that* $\frac{\partial}{\partial u}\mathbf{B}(0,1) = 3(\mathbf{p}_{13} - \mathbf{p}_{03})$ *and* $\frac{\partial}{\partial v}\mathbf{B}(1,0) = 3(\mathbf{p}_{02} - \mathbf{p}_{03})$*. Therefore*

$$\mathbf{n} = \mathrm{norm}(3(\mathbf{p}_{02} - \mathbf{p}_{03}) \times 3(\mathbf{p}_{13} - \mathbf{p}_{03})).$$

Note the order of the cross product in order to obtain the same directions for all normals.

(c) *Normal at* $(1,1)$*. We know that* $\frac{\partial}{\partial u}\mathbf{B}(1,1) = 3(\mathbf{p}_{23} - \mathbf{p}_{33})$ *and* $\frac{\partial}{\partial v}\mathbf{B}(1,1) = 3(\mathbf{p}_{32} - \mathbf{p}_{33})$*. Therefore*

$$\mathbf{n} = \mathrm{norm}(3(\mathbf{p}_{23} - \mathbf{p}_{33}) \times 3(\mathbf{p}_{32} - \mathbf{p}_{33})).$$

(d) *Normal at* $(1,0)$*. We know that* $\frac{\partial}{\partial u}\mathbf{B}(1,0) = 3(\mathbf{p}_{20} - \mathbf{p}_{30})$ *and* $\frac{\partial}{\partial v}\mathbf{B}(1,0) = 3(\mathbf{p}_{31} - \mathbf{p}_{30})$*. Therefore*

$$\mathbf{n} = \mathrm{norm}(3(\mathbf{p}_{31} - \mathbf{p}_{30}) \times 3(\mathbf{p}_{20} - \mathbf{p}_{30})).$$

Note again the order of the product.

(e) *Normal at* $(\frac{1}{2}, \frac{1}{2})$*. We have* $\mathbf{B}(u,v) = \sum_j \sum_i b_i(u) b_j(v) \mathbf{p}_{ij}$*. We can calculate the partial derivates as*

$$\begin{aligned}
\frac{\partial}{\partial u}\mathbf{B}(u,v) \;=\; & -3(1-u)^2 \textstyle\sum_j b_j(v)\mathbf{p}_{0j} \\
& +3(-2(1-u)u + (1-u)^2) \textstyle\sum_j b_j(v) p_{1j} \\
& +3(u^2 + (1-u)2u) \textstyle\sum_j b_j(v) p_{2j} \\
& +3u^2 \textstyle\sum_j b_j(v) p_{3j}
\end{aligned}$$

and evaluate at $u = v = \frac{1}{2}$*. We obtain the partial derivative* $\frac{\partial}{\partial v}\mathbf{B}(u,v)$ *symmetrically. Then*

$$\mathbf{n} = \mathrm{norm}(\frac{\partial}{\partial u}\mathbf{B}(\frac{1}{2}, \frac{1}{2}) \times \frac{\partial}{\partial v}\mathbf{B}(\frac{1}{2}, \frac{1}{2}))$$