

Homework 1: Asymptotic Analysis

1. For each of the following program fragments, give the running time (Big-Oh will suffice) and justify your answers. (2 points each)

1.1. `sum = 0;`
`for (i = 0; i < n; i++)`
`For (j = 0; j < n; j++)`
`sum++;`

1.2. `sum = 0`
`for(i = 0; i < n; i++)`
`for(j = 0; j < n * n; j++)`
`sum++;`

1.3. `sum = 0;`
`for(i = 0; i < n; i++)`
`for(j = 0; j < i; j++)`
`sum++;`

1.4. `sum = 0;`
`for(i = 0; i < n; i++)`
`for(j = 0; j < i * i; j++)`
`for(k = 0; k < j; k++)`
`sum++;`

1.5. `sum = 0;`
`for(i = 1; i <= n; i*=2)`
`for(j = 1; j <= i; j++)`
`sum++;`

2. For each pair of functions given below determine if i) one is in Big-Oh of the other but not vice versa; ii) each is in Big-Oh of the other; and iii) none is in Big-Oh of the other: (2 points each)

2.1. $\log(\log n)$ vs. $\log^2 n$

2.2. 2^n vs. 3^n

2.3. $500 + 10n$ vs. $20n$

2.4. $\sin(n) n$ vs. n

3. Prove the following theorem: (7 points)

If $T(n) = O(f(n))$ and $f(n) = O(g(n))$, then $T(n) = O(g(n))$.