CMSC 341

Splay Trees

Splay Trees

Concept

- adjust tree in response to accesses to make common operations (insert, find, remove) efficient
- after access node is moved to root by *splaying*

Performance

- amortized such that m operations take $O(m \lg n)$ where n is the number of insertions (nodes in the tree)

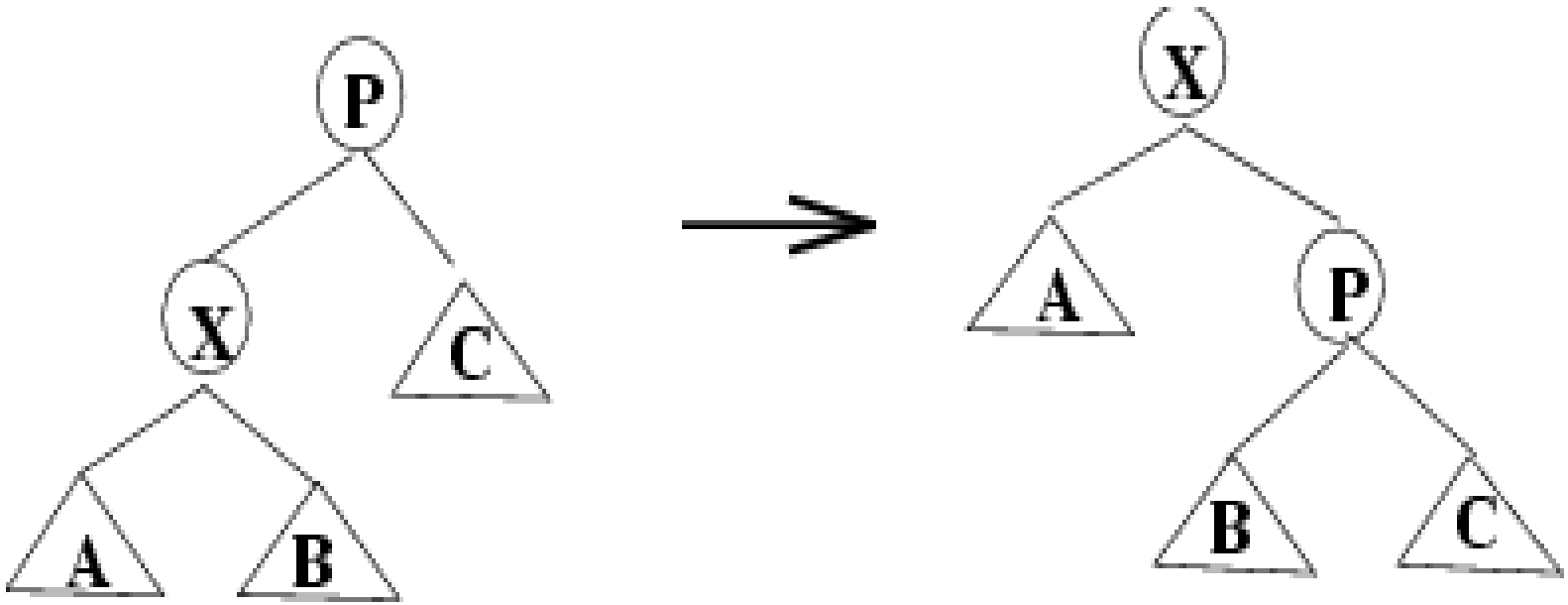
Splay Operation

Traverse tree from node x to root, rotating along the way until x is the root

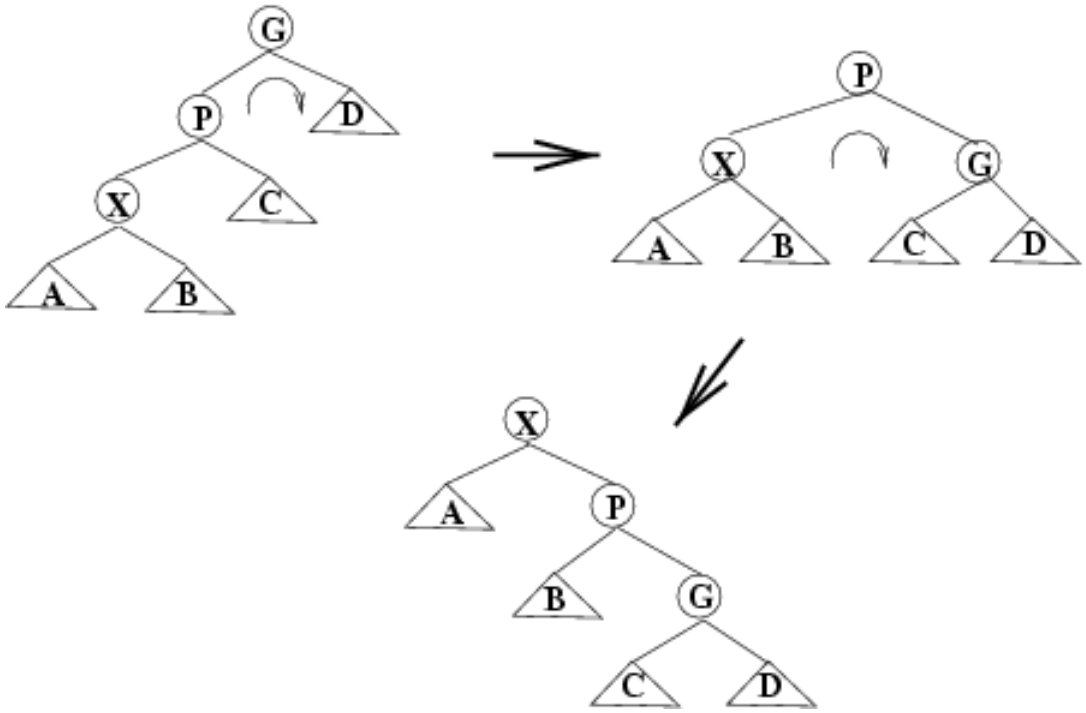
Each rotation

- If x is root, do nothing.
- If x has no grandparent, rotate x about its parent.
- If x has a grandparent,
  - if x and its parent are both left children or both right children, rotate the parent about the grandparent, then rotate x about its parent
  - if x and its parent are opposite type children (one left and the other right), rotate x about its parent, then rotate x about its new parent (former grandparent)

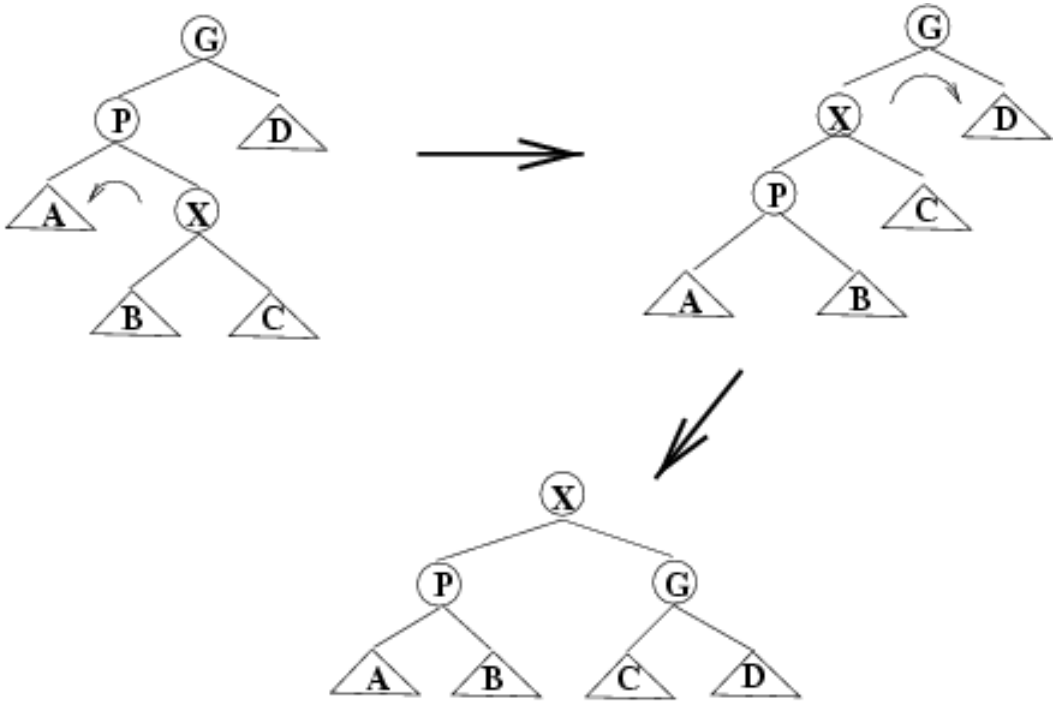
Node has no grandparent



Node and Parent are Same Side
Zig-Zig



Node and Parent are Different Sides
Zig-Zag



Operations in Splay Trees

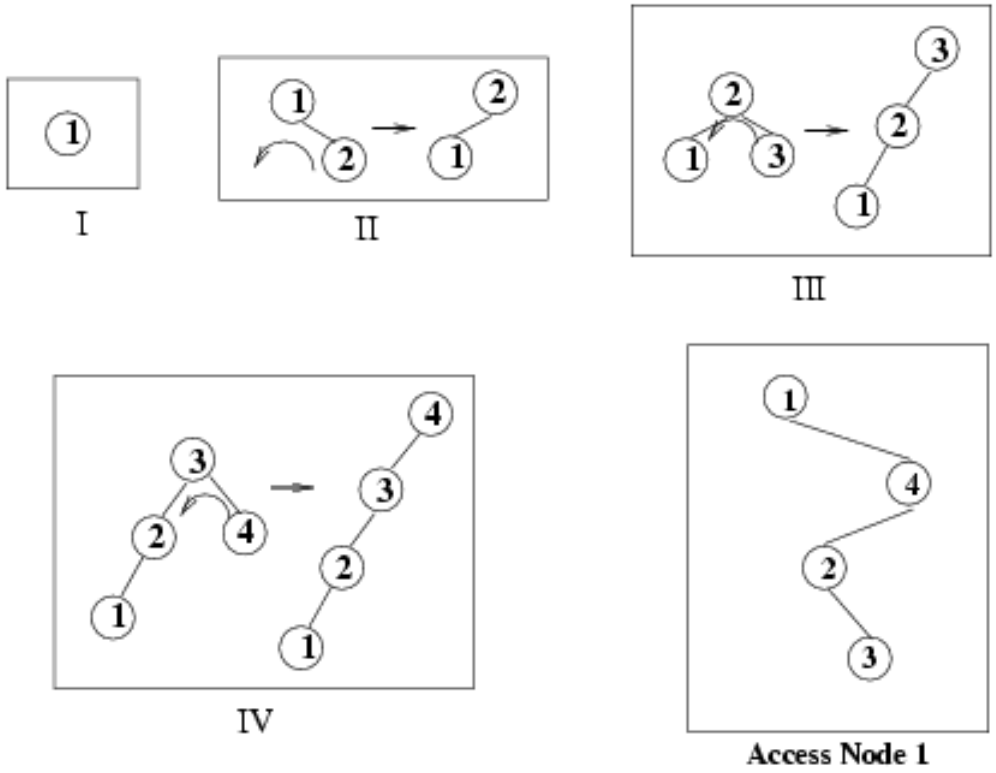
insert

- first insert as in normal binary search tree
- then splay inserted node
- if there is a duplicate, the node holding the duplicate element is splayed

find

- search for node
- if found, splay; otherwise splay last node accessed on the search path

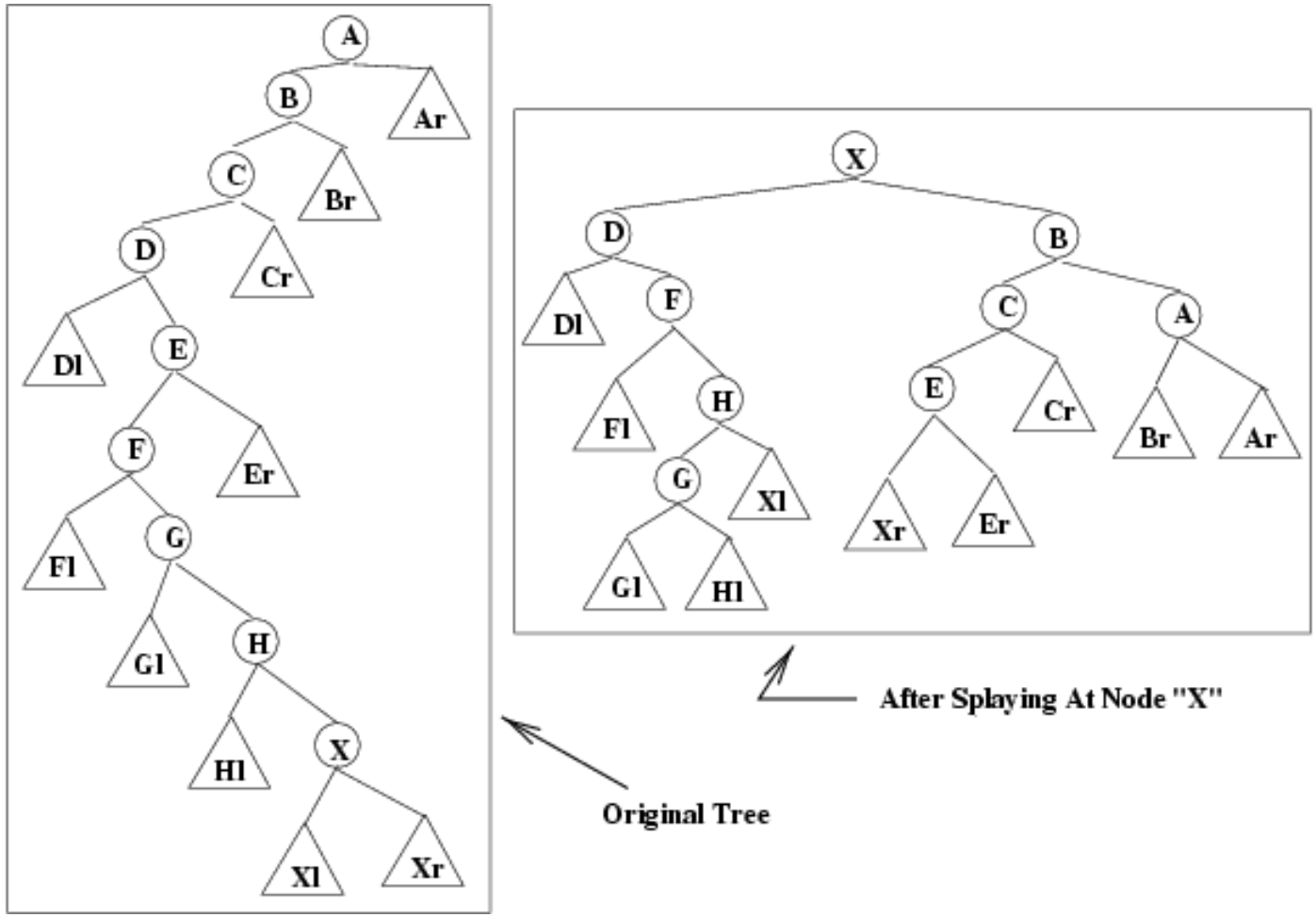
Insertion in order into a Splay Tree



Operations on Splay Trees (cont)

remove

- splay element to be removed
  - if the element to be deleted is not in the tree, the node last visited on the search path is splayed
- disconnect left and right subtrees from root
- do one of:
  - splay max item in T_L (then T_L has no right child)
  - splay min item in T_R (then T_R has no left child)
- connect other subtree to empty child of root


A
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X
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B
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Br
Ar
G
Xl
Xr
Er
Gl
Hl
After Splaying At Node "X"
Original Tree

Performance of Splay Trees

insert

- regular BST insertion -- O(height)
- splay: O(1) for each rotation, O(height) rotations