CMSC 341

Hashing

Readings: Chapter 5

Announcements

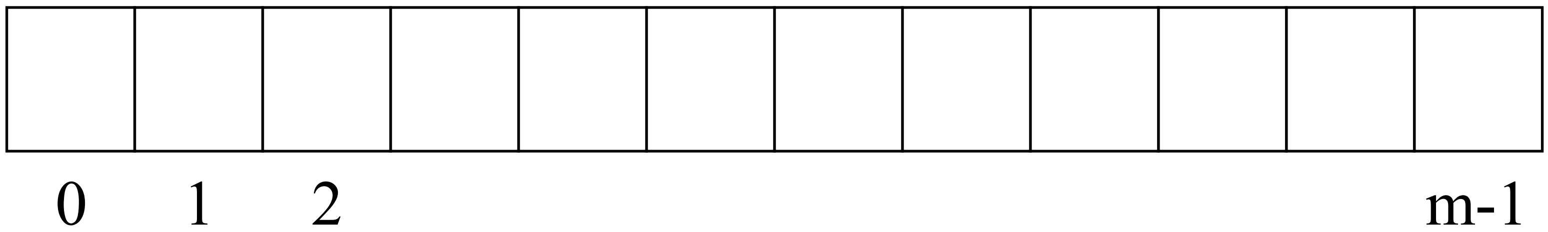
- Midterm II on Nov 7
- Review out Oct 29
- HW 5 due Thursday

- Project due Nov 5
- Midterm II review posted on Tuesday

Motivations

- We have lots of data to store.
- We desire efficient – O(1) – performance for insertion, deletion and searching.
- Too much (wasted) memory is required if we use an array indexed by the data's key.
- The solution is a "hash table".

Hash Table



- Basic Idea
  - The hash table is an array of size 'm'
  - The storage index for an item determined by a *hash function* h(k): U → {0, 1, …, m-1}

Exercise: A Simple Example

Example: insert 89, 18, 49, 58, 69 to a table size of 10.

Hash function: h(k) = k mod m where m is the table size.

```
Public static int hash(String key, int tableSize)
{
   hashVal %= tableSize;

   return hasVal;
}
```

What is the problem here? How to resolve it?
Hints:
(1) How should we choose m?
(2) How to pick a hashing function?

Getting a better hash function; make a table (instead we make a linked list); pick a better table size (prime number)

Hashing function: F(i) = i

Example: h'(k) = k mod 10 in a table of size 10 (not prime, but easy to calculate)

U={89,18,49,58,69}

f(I) = I

1. 89 hashes to 9
2. 18 hashes to 8
3. 49 hashes to 9, collides with 89

   h(k,1) = (49%10+1)%10=0
4. 58 hashes to 8, collides with 18

   h(k,1)=(58 % 10 + 1) % 10=9, collides with 89

   h(k,2)=(58 % 10+2)%10=0, collides with 49

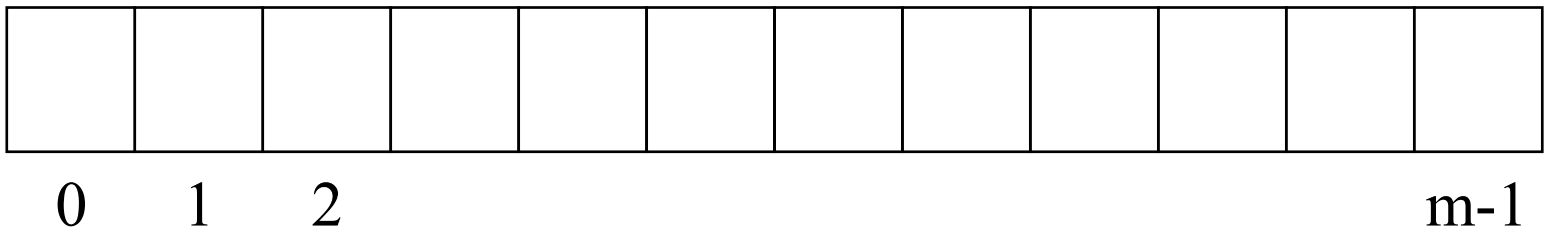
   h(k,3)=(58 % 10+3)%10=1
5. 69 hashes to 9, collides with 89

   h(69,1) = (h'(69)+f(1))mod 10 = 0, collides with 49

   h(69,2) = (h'(69+f(2))mod 10 =0, collides with 58

   h(69,3) = (h'(69)+f(3))mod 10 = 2

Hash Table



- Basic Idea
  - The hash table is an array of size 'm'
  - The storage index for an item determined by a *hash function* $h(k)\colon U \rightarrow \{0, 1, \ldots, m-1\}$
- Desired Properties of h(k)
  - easy to compute
  - uniform distribution of keys over $\{0, 1, \ldots, m-1\}$
    - when $h(k_1) = h(k_2)$ for $k_1, k_2 \in U$, we have a *collision*

Division Method

- The hash function:

  h(k) = k mod m where m is the table size.
- m must be chosen to spread keys evenly.
  - Poor choice: m = a power of 10
  - Poor choice: $m = 2^b$, b> 1
- A good choice of m is a prime number.
- Table should be no more than 80% full.
  - Choose m as smallest prime number greater than m_{min}, where
    m_{min} = (expected number of entries)/0.8

Handle Non-integer Keys

- In order to have a non-integer key, must first convert to a positive integer:

  $$h(k) = g(f(k)) \text{ with } f: U \rightarrow \text{integer}$$
  $$g: I \rightarrow \{0 \,..\, m-1\}$$

- Suppose the keys are strings.
- How can we convert a string (or characters) into an integer value?

Horner's Rule

```
static int hash(String key, int tableSize)
{
  int hashVal = 0;

  for (int i = 0; i < key.length(); i++)
     hashVal = 37 * hashVal + key.charAt(i);

  hashVal %= tableSize;
  if(hashVal < 0)
     hashVal += tableSize;

  return hashVal;
}
```

Exercise: Hash Function

Which hashFunction is better, when tableSize =10,007?

Method 1:

```
Public static int hash(String key, int tableSize)
{
  int hashVal =0;
  for(int i=0; i<key.length(); i++)
    hashVal += key.charAt (i);
  return hashVal % tableSize
}  // not good: waste a lot of memory
```

Method 2: Assuming three letters

```
Public static int hash(String key, int tableSize)
{ return (key.charAt(0)+27*key.charAt(1)+27^2*key.charAt(2)) %
      tableSize; }
```

Method 3:

```
Public static int hash(String key, int
      tableSize)
{
  int hashVal = 0;
  for(int i=0; i<key.length(); i++)
    hashVal = 37*hashVal + key.charAt(i);
  hashVal %= tableSize;
  if(hashVal < 0)     hashVal += tableSize;
}
```

97	61	141	a	a
98	62	142	b	b
99	63	143	c	c
100	64	144	d	d
101	65	145	e	e
102	66	146	f	f
103	67	147	g	g
104	68	150	h	h
105	69	151	i	i
106	6A	152	j	j
107	6B	153	k	k
108	6C	154	l	l
109	6D	155	m	m
110	6E	156	n	n
111	6F	157	o	o
112	70	160	p	p
113	71	161	q	q
114	72	162	r	r
115	73	163	s	s
116	74	164	t	t
117	75	165	u	u
118	76	166	v	v
119	77	167	w	w
120	78	170	x	x
121	79	171	y	y
122	7A	172	z	z
123	7B	173	{	{
124	7C	174			|
125	7D	175	}	}
126	7E	176	~	~
127	7F	177	&#127;	DEL

HashTable Class

```
public class SeparateChainingHashTable<AnyType>
{
    public SeparateChainingHashTable( ){/* Later */}
    public SeparateChainingHashTable(int size){/*Later*/}
    public void insert( AnyType x ){ /*Later*/ }
    public void remove( AnyType x ){ /*Later*/}
    public boolean contains( AnyType x ){/*Later */}
    public void makeEmpty( ){ /* Later */ }
    private static final int DEFAULT_TABLE_SIZE = 101;
    private List<AnyType> [ ] theLists;
    private int currentSize;
    private void rehash( ){ /* Later */ }
    private int myhash( AnyType x ){ /* Later */ }
    private static int nextPrime( int n ){ /* Later */ }
    private static boolean isPrime( int n ){ /* Later */ }
}
```

HashTable Ops

- `boolean contains( AnyType x )`
  - Returns true if x is present in the table.
- `void insert (AnyType x)`
  - If x already in table, do nothing.
  - Otherwise, insert it, using the appropriate hash function.
- `void remove (AnyType x)`
  - Remove the instance of x, if x is present.
  - Otherwise, does nothing
- `void makeEmpty()`

Hash Methods

```
private int myhash( AnyType x )
{
        int hashVal = x.hashCode( );

        hashVal %= theLists.length;
        if( hashVal < 0 )
            hashVal += theLists.length;

        return hashVal;
}
```

Handling Collisions

- Collisions are inevitable. How to handle them?
- Separate chaining hash tables
  - Store colliding items in a list.
  - If m is large enough, list lengths are small.
- Insertion of key k
  - hash(k) to find the proper list.
  - If k is in that list, do nothing, else insert k on that list.
- Asymptotic performance
  - If always inserted at head of list, and no duplicates, insert = O(1) for best, worst and average cases

Hash Class for Separate Chaining

- To implement separate chaining, the private data of the hash table is an array of Lists. The hash functions are written using List functions

```
private List<AnyType> [ ] theLists;
```

Performance of contains( )

- contains
  - Hash k to find the proper list.
  - Call contains( ) on that list which returns a boolean.
- Performance
  - best: selected list is empty or key is first -> O(1)
  - worst: let N be the number of elements in the hash table. All N elements are in one list (all have the same hash value) and key not there -> O(N)
  - Average: suppose there are M buckets and N elements in the table. Then expected list length = N/M -> O (N/M) = O(N) if M is small. = O(1) if M is large.
    - Here $\lambda = N/M$ is called the load factor of the table. It is important to keep the load factor from getting too large. If $N <= M$, $\lambda <= 1$ and O(N/M)-> O(1) where N/M is constant.

Performance of remove( )

- Remove k from table
  - Hash k to find proper list.
  - Remove k from list.
- Performance
  - Best: K is the 1st element on list, or list is empty: O(1)
  - Worst: all elements on one list: O(n)
  - Average: O(N/M)-> O(1) for λ<=1. So what is the big deal? Performance for hash table and list are the same best and worst… But average performance for a well-designed hash table is much better: O(1).

Handling Collisions Revisited

- ***Probing hash tables***
  - All elements stored in the table itself (so table should be large. Rule of thumb: m >= 2N)
  - Upon collision, item is hashed to a new (open) slot.
- Hash function

  h: U x {0,1,2,….} → {0,1,…,m-1}

  h(k, i) = (h'(k) + f(i)) mod m

  for some h': U → { 0, 1,…, m-1}

  and some f(i) such that f(0) = 0
- Each attempt to find an open slot (i.e. calculating h(k, i)) is called a ***probe***

HashEntry Class for Probing Hash Tables

- In this case, the hash table is just an array

```
private static class HashEntry<AnyType>{
   public AnyType  element;  // the element
   public boolean isActive;  // false if deleted
   public HashEntry( AnyType e )
   { this( e, true ); }
   public HashEntry( AnyType e, boolean active )
   { element  = e; isActive = active; }
}
// The array of elements
private HashEntry<AnyType> [ ] array;
// The number of occupied cells
private int currentSize;
```

Linear Probing

- Use a linear function for f(i)

$$f(i) = c * i$$

- Example:

  h'(k) = k mod 10 in a table of size 10 , f(i) = i

So that

  h(k, i) = (k mod 10 + i) mod 10

  Insert the values U={89,18,49,58,69} into the hash table

Linear Probing (cont.)

- Problem: Clustering
  - When the table starts to fill up, performance → O(N)

- Asymptotic Performance
  - Insertion and unsuccessful find, average
    - λ is the "load factor" – what fraction of the table is used
    - Number of probes $\cong$ (½) ($1+1/(1-\lambda)^2$)
    - if $\lambda \cong 1$, the denominator goes to zero and the number of probes goes to infinity

Linear Probing (cont.)

- Remove
  - Can't just use the hash function(s) to find the object and remove it, because objects that were inserted after X were hashed based on X's presence.
  - Can just mark the cell as deleted so it won't be found anymore.
    - Other elements still in right cells
    - Table can fill with lots of deleted junk

Quadratic Probing

- Use a quadratic function for f(i)

$$f(i) = c_2 i^2 + c_1 i + c_0$$

The simplest quadratic function is $f(i) = i^2$

- Example:

Let $f(i) = i^2$ and m = 10

Let h'(k) = k mod 10

So that

$$h(k, i) = (k \bmod 10 + i^2) \bmod 10$$

Insert the value U={89, 18, 49, 58, 69 } into an initially empty hash table

Quadratic Probing (cont.)

- Advantage:
  - Reduced clustering problem
- Disadvantages:
  - Reduced number of sequences
  - No guarantee that empty slot will be found if $\lambda \geq 0.5$, even if m is prime
  - If m is not prime, may not find an empty slot even if $\lambda < 0.5$

Double Hashing

- Let f(i) use another hash function

$$f(i) = i * h_2(k)$$

Then $h(k, I) = (h'(k) + \ * h_2(k)) \bmod m$

And probes are performed at distances of

$h_2(k)$, $2 * h_2(k)$, $3 * h_2(k)$, $4 * h_2(k)$, etc

- Choosing $h_2(k)$
  - Don't allow $h_2(k) = 0$ for any k.
  - A good choice:
    $h_2(k) = R - (k \bmod R)$ with R a prime smaller than m
- Characteristics
  - No clustering problem
  - Requires a second hash function

Rehashing

- If the table gets too full, the running time of the basic operations starts to degrade.
- For hash tables with separate chaining, “too full” means more than one element per list (on average)
- For probing hash tables, “too full” is determined as an arbitrary value of the load factor.
- To rehash, make a copy of the hash table, double the table size, and insert all elements (from the copy) of the old table into the new table
- Rehashing is expensive, but occurs very infrequently.

Multiplication Method

- The hash function:

  $h(k) = \lfloor m(kA - \lfloor kA \rfloor) \rfloor$

  where A is some real positive constant.
- A very good choice of A is the inverse of the "golden ratio."
- Given two positive numbers x and y, the ratio x/y is the "golden ratio" if $\phi = x/y = (x+y)/x$
- The golden ratio:

  $x^2 - xy - y^2 = 0 \quad \Rightarrow \quad \phi^2 - \phi - 1 = 0$

  $\phi = (1 + \text{sqrt}(5))/2 \quad = \quad 1.618033989\ldots$

  $\sim= Fib_i/Fib_{i-1}$

Multiplication Method (cont.)

- Because of the relationship of the golden ratio to Fibonacci numbers, this particular value of A in the multiplication method is called "Fibonacci hashing."
- Some values of

$$h(k) = \lfloor m(k\,\phi^{-1} - \lfloor k\,\phi^{-1} \rfloor) \rfloor$$

= 0 for k = 0

= 0.618m for k = 1 (ϕ^{-1} = 1/ 1.618… = 0.618…)

= 0.236m for k = 2

= 0.854m for k = 3

= 0.472m for k = 4

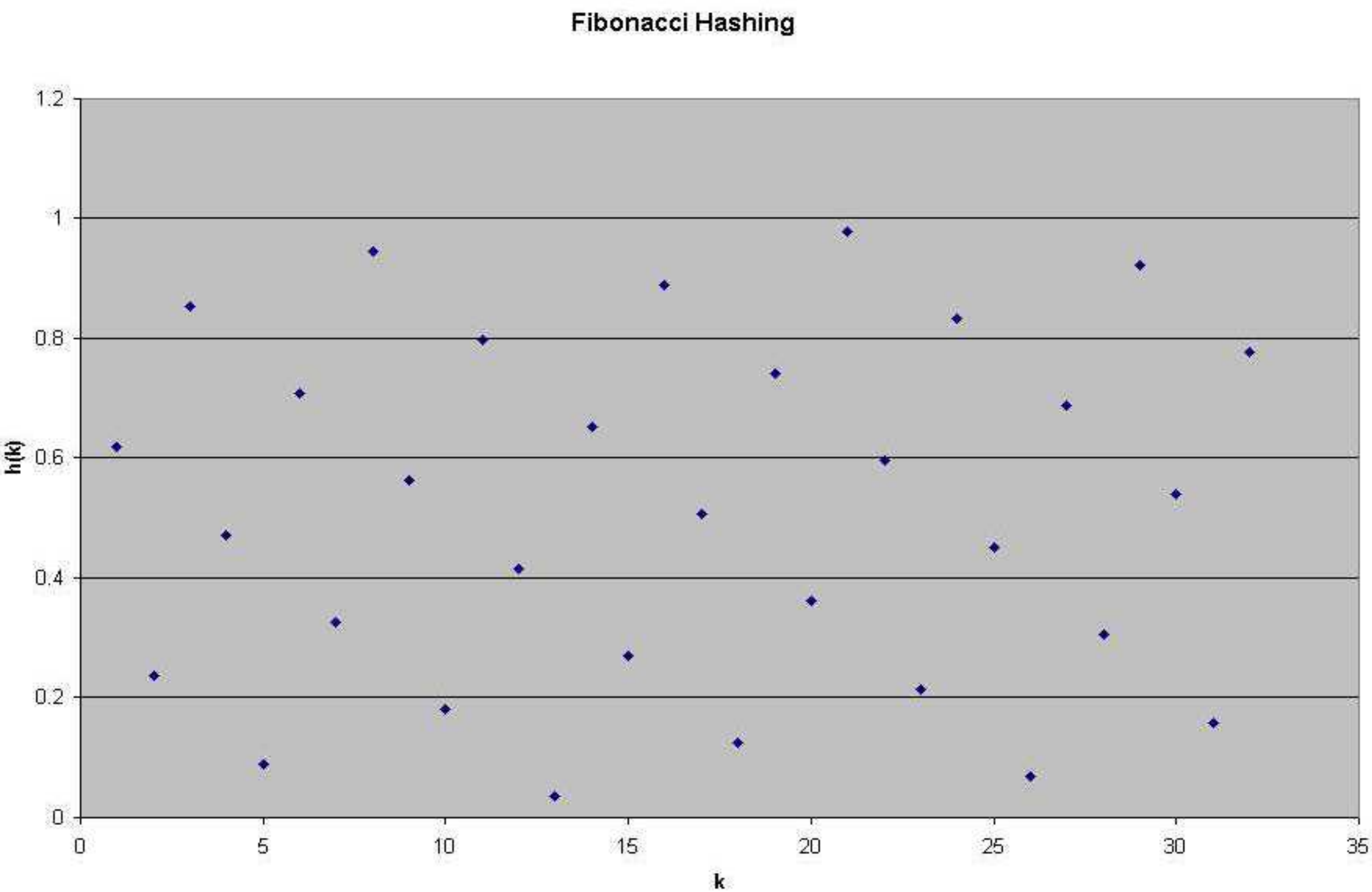
= 0.090m for k = 5

= 0.708m for k = 6

= 0.326m for k = 7

= …

= 0.777m for k = 32


Fibonacci Hashing
h(k)
k
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
30
35