

These are some review questions to test your understanding of the material. Some of these questions may appear on an exam.

## 1 Red-Black Tree

Please see the diagrams:

- flow-chart for bottom-up insertion in a RedBlack tree on page 2
- bottom-up deletion on page 3
- top-down insertion on page 4

1.1 Define *Red-Black tree*.

1.2 Show the result of inserting 2, 1, 4, 5, 9, 3, 6, 7 into an initially empty Red-Black tree (show the tree at the end of **each** insertion). Do this using bottom-up and using top-down insertion rules.

1.3 Show the result of removing a given element from the tree. Do this using bottom-up and using top-down deletion rules.

1.4 What is the “Big-Oh” performance (in terms of the number of nodes in the tree) for each operation **find**, **insert**, and **remove** for Red-Black trees in the best, worst, and average cases?

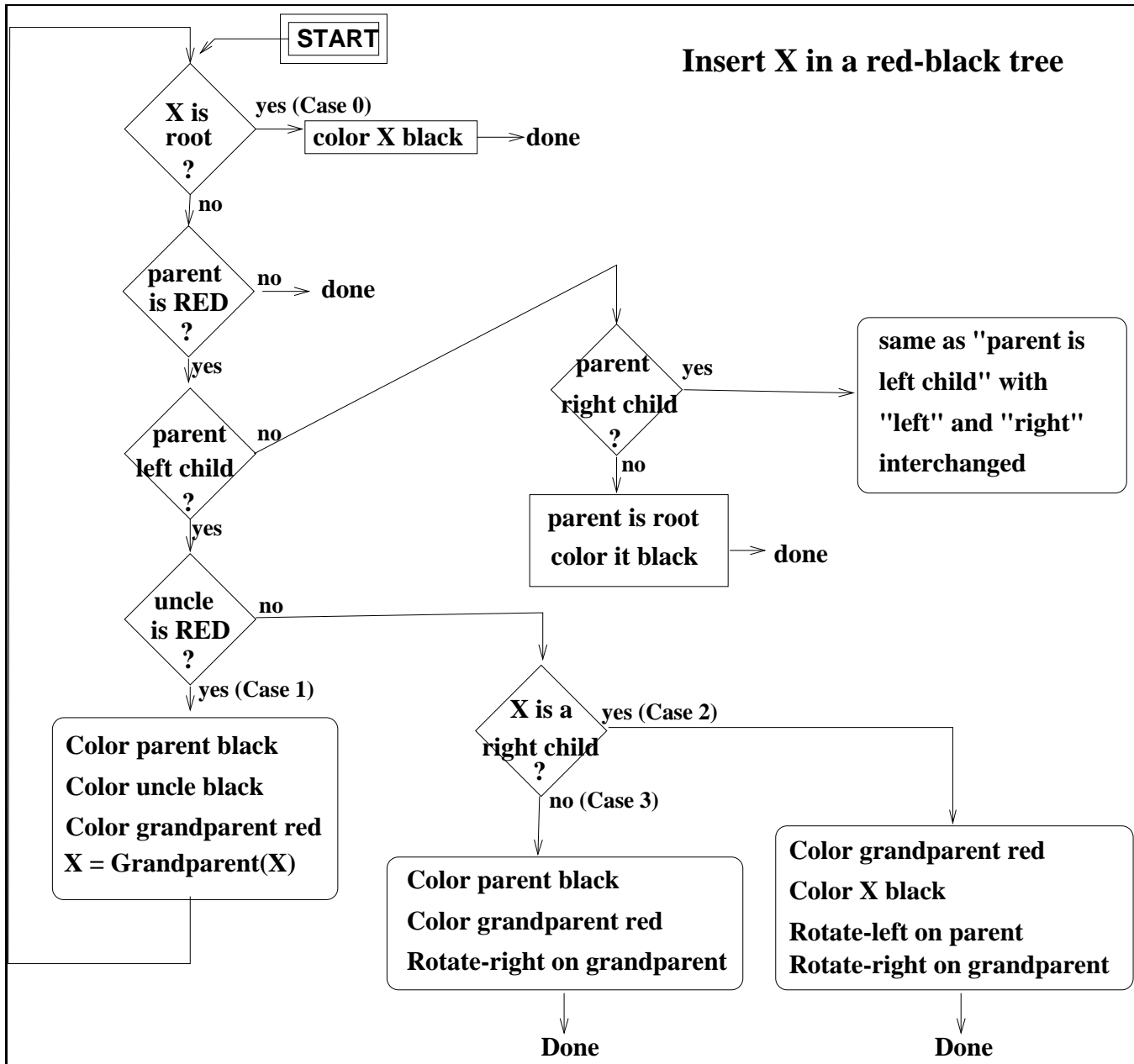
1.5 What property of Red-Black trees is most significant in explaining their “Big-Oh” behavior for the operations **find**, **insert**, and **remove**.

1.6 Prove: Any red-black tree, with root  $x$ , has at least  $n = 2^{\text{bh}(x)} - 1$  internal nodes, where  $\text{bh}(x)$  is the black-height of node  $x$ .

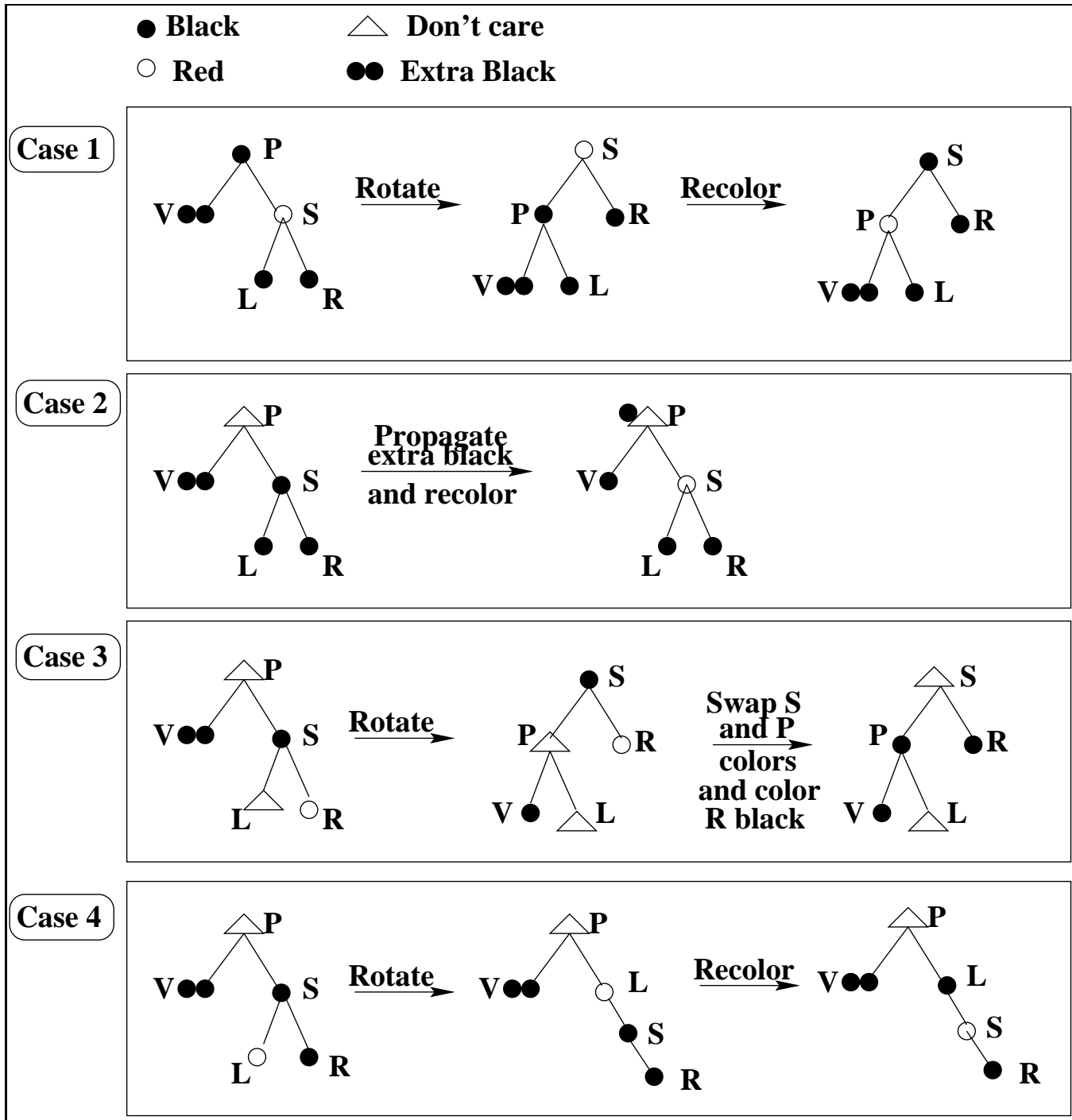
1.7 Prove: In a red-black tree, at least half of the nodes on any path from root to a leaf must be black.

1.8 Prove: In a red-black tree, no path from any node  $N$  to a leaf is more than twice as long as any other path from  $N$  to any other leaf.

# Flow-chart for bottom-up insertion in Red-Black Trees



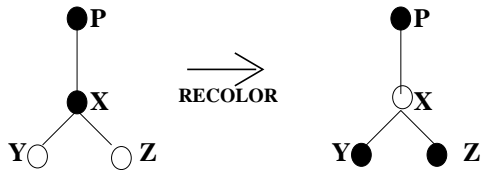
# Cases for Bottom-Up Deletion in Red-Black Trees



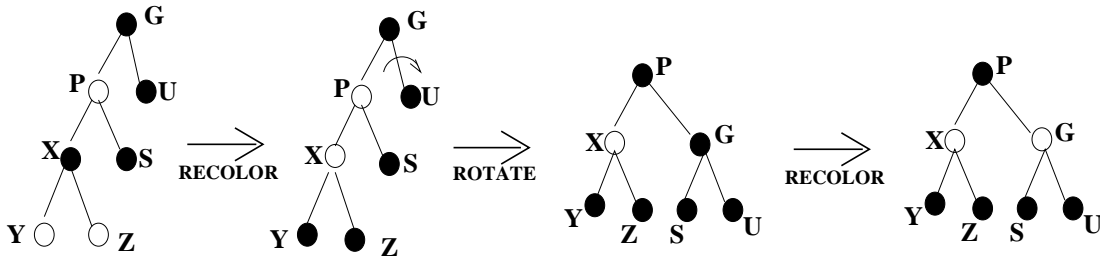
# Cases for top-down insertion in Red-Black Trees

## Top-Down Insertion Cases for Red-Black Trees (Cases for which black X has two red children)

Case 1



Case 2



Case 3

