

T minus 5 classes

- Quiz on Probability
- Homework 8 due Thursday
- Homework 9 released Thursday
 - Will be due Tuesday after Thanksgiving
- Next quiz won't be until 12/10
- Exams will be graded by Thursday

CMSC 203: Lecture 22

Relations

What is Relations?

- **Relation:** A structure that represents the relationships between elements of sets
- Subset of the Cartesian product of the sets
- Examples:
 - Pairs of cities linked by airline flights?
 - Phases of a project in a viable order?
 - Storing information in a database?

Expressing Relations

- Most direct way to express relationship between elements in two sets is *ordered pairs*
- Sets of ordered pairs are **binary relations**
 - Binary relation from A to B is a subset of $A \times B$
 - aRb denotes $(a, b) \in R$
- *Example* : Let A be the set of students at UMBC and B be the set of courses. Let R be the relation that consist of pairs (a, b) where a is a student enrolled in course b .

Functions as Relations

- Recall function f from A to B maps elements exactly one element in B to every element in A
- Graph of f is a set of ordered pairs (a, b) where $b = f(a)$
- Graph of f is a subset of $A \times B$, so it is a relation
- Relations are a generalizations of graphs of functions

Relations on a Set

- A **relation on a set A** is a relation from A to A
- *Example s:*
 - A is the set $\{1, 2, 3, 4\}$. Which ordered pairs are in the relation $R = \{(a, b) \mid a \text{ divides } b\}$?
 - Which of these relations contain the pair (1, 2):
 - $R_1 = \{(a, b) \mid a \leq b\}$
 - $R_2 = \{(a, b) \mid a = b + 1\}$
 - $R_3 = \{(a, b) \mid a + b \leq 3\}$

Properties of Relations

- **Reflexive**

- $\forall a((a, a) \in R)$

- **Symmetric**

- $\forall a \forall b((a, b) \in R \rightarrow (b, a) \in R)$

- **Antisymmetric**

- $\forall a \forall b(((a, b) \in R \wedge (b, a) \in R) \rightarrow (a = b))$

- **Transitive**

- $\forall a \forall b \forall c(((a, b) \in R \wedge (b, c) \in R) \rightarrow (a, c) \in R)$

Combining Relations

- Two relations from A to B can be combined in any way two sets can be combined
 - Due to relations being subsets of $A \times B$
- We can perform operations such as
 - $R_1 \cup R_2$
 - $R_1 \cap R_2$
 - $R_1 - R_2$
 - $R_2 - R_1$
 - $R_1 \oplus R_2$

n-ary Relations

- Subset of $A_1 \times A_2 \times \dots \times A_n$
 - A_k is a **domain** of the relation
 - n is the **degree**
- Used for computer databases
 - One method is a **relational data model**

Relational Data Model

- A **database** consists of **records** which are n-tuples made up of **fields**
 - Example fields: name, number, major, GPA
 - This would be a 4-tuple
- Relations are also called **tables**
- There may be a **primary key** – a domain that unique identifies an n-tuple
- Current collection of n-tuples is **extension** of relation
- The permanent attributes of relation is **intension**