Hello

- Only one more exam!
  - Half cumulative, half new material
- Email me for your grade
  - Homework 1-5, Quiz 1, Exam 1 have been graded
- Homework due dates have been pushed back
  - Sorry! :(
- Exams will be graded when they are graded

CMSC 203: Lecture 19

Probability

Why Probability?

- Started off with gambling odds
- Now used for many cases
  - Average case complexity
  - Probabilistic algorithm
  - Showing objects with properties exist
  - Probability theory for uncertainty (eg: spam blocking)

Finite Probability

- **Experiment**: procedure that yields one of a given set of possible outcomes
- **Sample space**: Set of all possible outcomes
- **Event**: Subset of the sample space
- If *S* is a finite nonempty sample space of *equally likely* outcomes, and *E* is an event, the **probability** of *E* is:

$$p(E) = \frac{|E|}{|S|}$$

Finite Probability (cont)

- Probability of an event is between 0 and 1
- Examples:
  - What is the probability of drawing a blue ball from a box with 4 blue and 5 red balls?
  - What is the probability of rolling two dice and getting the sum of 7?
  - What is the probability of winning a Pick 4 lottery?
    - What about getting ¾ of the Pick 4 correct?

Practice with Cards

- Probability that a hand of five cards is a four-of-a-kind
  - 4 different suites; same rank
- Probability poker hand is full house
  - 3 of one rank; 2 of another

Practice with Cards

- Probability that a hand of five cards is a four-of-a-kind
  - $\frac{C(13,1)C(4,4)C(48,1)}{C(52,5)} = \frac{13 * 1 * 48}{2598960} \approx 0.00024$
- Probability poker hand is full house
  - $\frac{C(13,2)C(4,3)C(4,2)}{C(52,5)} = \frac{13 * 12 * 4 * 6}{2598960} \approx 0.0014$

Complements and Unions

- $p(\bar{E}) = 1 - p(E)$
- $p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$
- Examples:
  - What is the probability 10 random bits contains a 0?
  - What is the probability a random integer ≤ 100 is divisible by ether 2 or 5?
  - **Monty Hall Problem**

General Probabilities

- For sample space *S* with countable outcomes, probability *p(s)* for each outcome *s* meets conditions:

  1) $0 \leq p(s) \leq 1 \text{ for each } s \in S$

  2) $\sum_{s \in S} p(s) = 1$

- Function *p* is the **probability distribution**
- *p(s)* should equal limit of the times *s* occurs divided by number of times experiment is performed (as experiment count grows without bound)

Probability Distributions

- If *S* is a set with *n* elements, a **uniform distribution** assigns probability of 1/*n* for each element of *S*
- $$p(E) = \sum_{s \in E} p(s)$$
- Selecting element from sample space with uniform distribution is selecting an element **at random**
- *Example*: What is probability of rolling an odd number on a dice if the dice is loaded so 3 comes up twice as often as each other number?

Conditional Probability

- **Conditional probability**: Probability *E* will occur given *F*, where *E* and *F* are events with p(*F*) > 0

$$p(E|F) = \frac{p(E \cap F)}{p(F)}$$

- Examples:
  - Bit string of length 4 is generated at random. What is the probability it contains two consecutive 0s given that the first bit is 0?
  - What is the probability a family will have two boys, given they already have one boy?

Independence

- When two events are **independent**, the occurrence of one of the events gives does not affect the other
- Two events are independent $\iff p(E \cap F) = p(E)P(F)$
- Example: *E* is an the event that a randomly generated bit string of length 4 begins with a 1, and *F* is the event that this bit string contains an even number of 1s. Are *E* and *F* independent?