

Reminders

- Late Homework 5 is due
- Homework 6 is due
- Homework 7 will be released today
- Quiz on Counting next Thursday
- Review for Exam 2 next Thursday

CMSC 203: Lecture 16

Counting 2



Last Class

- We discussed rules of counting:
 - Product Rule
 - Sum Rule
 - Difference Rule
 - Division Rule
- We discussed Pigeonhole Principle

More on Pigeonhole Principle

What is the minimum number of objects such that at least r objects must be in one of the k boxes (or categories) provided?

- $\lceil \frac{N}{k} \rceil \geq r$

$$\frac{N}{k} > r - 1$$

$$N_{\min} = k(r - 1) + 1$$

Pigeonhole Examples

- I have a drawer of socks in unmatched pairs. There are 5 pairs of black, 5 pairs of blue and 5 pairs of dark grey. How many socks do I need to draw from the box to assure that I get one pair of any color?
- How many people are needed in a room to guarantee we have two people with the same birthday?

Permutations

- An **ordered** arrangement of elements in a set of objects
- Ordered arrangement of r elements is an **r-permutation** (where r is some number)
- Denoted by $P(n, r)$

Solving Permutations

- Uses product rule to solve
- Selecting one element for a slot reduces the size of available selections for future slots by one
- Eg: Number of ways to order 3 students from a group of 5 to stand in line for a picture? What about all 5?

Solving Permutations

- Uses product rule to solve
- Selecting one element for a slot reduces the size of available selections for future slots by one
- Eg: Number of ways to order 3 students from a group of 5 to stand in line for a picture? What about all 5?
- $P(n, r) = n(n - 1)(n - 2) \dots (n - r + 1) =$

$$P(n, r) = \frac{n!}{(n - r)!}$$

Permutation Examples

- How many ways are there to select a first-, second-, and third-prize winner from 100 people in a contest?
- A salesperson has to visit 8 cities. They are assigned the first city, but can visit the other cities in any order. How many possible orders can the person visit these cities?
- How many permutations of the letters ABCDEFGH contain the string ABC?

Permutation Examples

- How many ways are there to select a first-, second-, and third-prize winner from 100 people in a contest?
 - $P(100, 3) = 100 * 99 * 98 = 970,200$
- A salesperson has to visit 8 cities. They are assigned the first city, but can visit the other cities in any order. How many possible orders can the person visit these cities?
 - $7! = 7 * 6 * 5 * 4 * 3 * 2 * 1 = 5040$
- How many permutations of the letters ABCDEFGH contain the string ABC?
 - Permutations of ABC, D, E, F, G, and H: $6! = 720$

Combinations

- An **unordered** selection of objects from a set
- Unordered selection of r elements is an **r-combination**
- Denoted by $C(n,r)$ or $\binom{n}{r}$

Solving Combinations

- Finding number of subsets of a particular size
- We may use division rule on $P(n,r)$ and dividing by $P(r,r)$

$$C(n, r) = \frac{n!}{r!(n - r)!}$$

- May have to cancel out terms in denominator by numerator using factors

Equivalence in Combinations

- $C(n, r) = C(n, n - r)$
- Eg: How many poker hands of 5 cards can be dealt? How many ways are there to select 47 cards from the deck?

Combination Practice

- How many ways can you select 6 crew members to go to Mars from 30 trained astronauts?
- How many bit strings of length n contain exactly r 1s?
- Suppose there are 9 faculty members in Math, and 11 in CS. How many ways can you select a committee that consists of 3 math faculty and 4 CS faculty?

Combination Practice

- How many ways can you select 6 crew members to go to Mars from 30 trained astronauts?

$$- \frac{30!}{6!24!} = \frac{30 * 29 * 28 * 27 * 26 * 25}{6 * 5 * 4 * 3 * 2 * 1} = 593,775$$

- How many bit strings of length n contain exactly r 1s?

$$- C(n, r)$$

- Suppose there are 9 faculty members in Math, and 11 in CS. How many ways can you select a committee that consists of 3 math faculty and 4 CS faculty?

$$- C(9, 3) * C(11, 4) = \frac{9!}{3!6!} * \frac{11!}{4!7!} = 84 * 330 = 27,720$$

More on Mars

- The group of 30 consists of 22 men and 8 women. How many ways are there to select a crew of 6 that must have at least 2 men and at least 2 women?