

Reminders

- Homework 6 is Due Thursday
 - We will cover the rest of the material today
- Late Homework 5 will be due Thursday
- Homework 7 will be released Thursday

Merge Sort

- Iteratively split list into two sublists of equal length
 - Or have one sublist with one more element
- Continue until each sublist has one element
- Successively merge pairs of lists
- $O(n \log n)$

Quick Sort

- Take first element (a_1) and form two sublists
 - One contains elements less than a_1
 - Second contains elements greater than a_1
- Place a_1 at the end of the first sublist
- Repeat recursively until all sublists contain 1 item
- Combine sublists of 1 item in order they occur
- $O(n \log n)$ in practice

CMSC 203: Lecture 15

Counting



What is “Counting”

- **Combinatorics** – study of arrangement of objects
- **Enumeration** – counting objects with certain properties
- Useful in a wide variety of situations:
 - Algorithm complexity
 - Availability of telephone numbers / IP addresses
 - Sequencing DNA
 - Probabilities

Product Rule

- **If:** A procedure can be broken into a sequence of two tasks
- There are n_1 ways to do the first task
- Each way to perform task has n_2 ways to do second task
- **Then:** There are $n_1 n_2$ ways to do the procedure

Product Rule Examples

- How many ways to order pasta if there are 3 kinds of noodles and 5 kinds of sauces?
- How many UMBC ID numbers are there if an ID is [letter][letter][digit][digit][digit][digit][digit]?
- How many different binary strings of length 7 are there?

Product Rule Answers

- How many ways to order pasta if there are 3 kinds of noodles and 5 kinds of sauces?
 - $3 \times 5 = 15$ possible pastas
- How many UMBC ID numbers are there if an ID is [letter][letter][digit][digit][digit][digit][digit]?
 - $26 \times 26 \times 10 \times 10 \times 10 \times 10 \times 10 = 26^2 \times 10^5 = 6.76 \times 10^7$
- How many different binary strings of length 7 are there?
 - $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7 = 128$

IPv4 Exhaustion

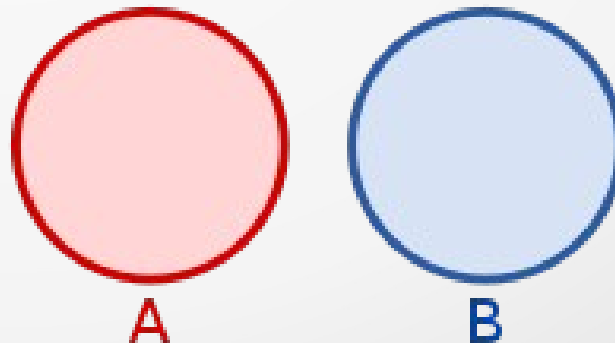
- How many possible IPv4 addresses are there if an IPv4 address can be 4 octets [where an octet is 0-255]?
- Can this handle the estimated 15 billion devices estimated to be connected to the internet in 2015?
- IPv6 contains 8 groups, with each group containing 2 octets. How many IPv6 addresses are possible?

IPv4 Exhaustion Answers

- How many possible IPv4 addresses are there if an IPv4 address can be 4 octets [where an octet is 0-255]?
 - $256 \times 256 \times 256 \times 256 = 256^4 = 4,294,967,296$
- Can this handle the estimated 15 billion devices estimated to be connected to the internet in 2015?
 - Definitely not
- IPv6 contains 8 groups, with each group containing 2 octets. How many IPv6 addresses are possible?
 - $(256 \times 256)^8 = 256^{16} = 3.4 \times 10^{38}$

Sum Rule

- **If:** A task can either be done in either n_1 or n_2 ways
- **But:** None of the n_1 ways are same as any of the n_2 ways
- **Then:** There are $n_1 + n_2$ ways to do task
- Set of ways to perform task one way or the other **MUST** be disjoint



Sum Rule Examples

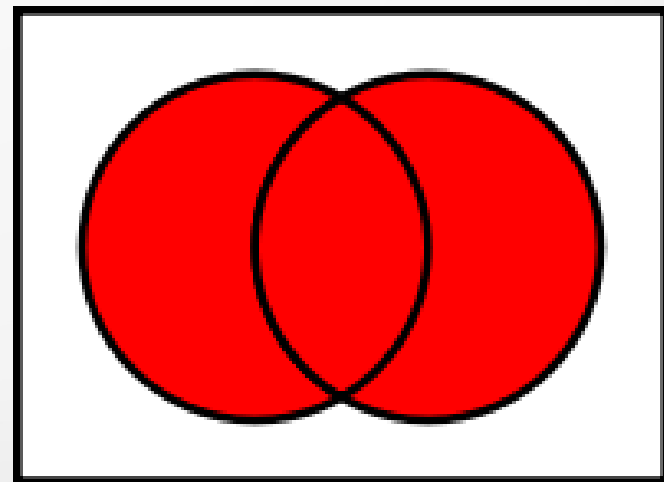
- If I pick one of 8 Freshman, 18 Sophomore, 11 Junior, or 3 Seniors in the class to be class representative. How many options do I have?
- You may pick a project off of one of three lists. The first list has 23 projects, the second has 15, and the last has 19. No project is on more than one list. How many possible ways are there to choose a project?

Sum Rule Answers

- If I pick one of 8 Freshman, 18 Sophomore, 11 Junior, or 3 Seniors in the class to be class representative. How many options do I have?
 - $8 + 18 + 11 + 3 = 40$
- You may pick a project off of one of three lists. The first list has 23 projects, the second has 15, and the last has 19. No project is on more than one list. How many possible ways are there to choose a project?
 - $23 + 15 + 19 = 57$

Subtraction Rule

- **If:** Task can be done in either n_1 ways or n_2 ways
- **Then:** There are $n_1 + n_2 -$ common ways to do the task
- Similar to sum rule, but we remove common elements
- Remember: $|A \cup B| = |A| + |B| - |A \cap B|$



Subtraction Rule Examples

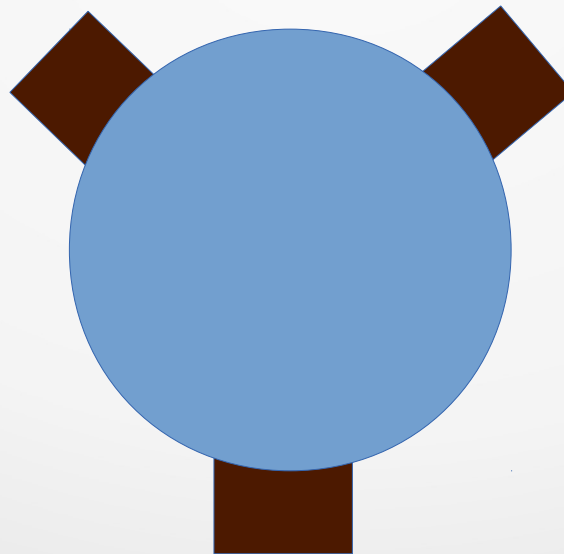
- How many bit strings of length 8 start with a “1” bit or end with two bits “00”?
- A computer company receives 350 applications from computer grads. Suppose that 220 applicants majored in CS, 147 majored in business, and 51 majored in both. How many applicants majored in neither?

Subtraction Rule Examples

- How many bit strings of length 8 start with a “1” bit or end with two bits “00”?
 - $(1 \times 2^7) + (2^6 \times 1 \times 1) - (1 \times 2^5 \times 1 \times 1) = 128 + 64 - 32 = 160$
- A computer company receives 350 applications from computer grads. Suppose that 220 applicants majored in CS, 147 majored in business, and 51 majored in both. How many applicants majored in neither?
 - $350 - (220 + 147 - 51) = 350 - 316 = 34$

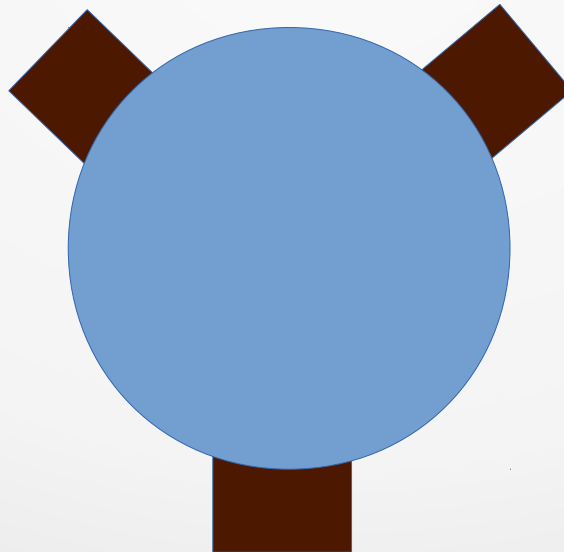
Division Rule

- **If:** A procedure can be done in n ways
- **And:** For every way w , d of n ways correspond to w
- **Then:** There are n / d ways to do the task



Division Rule Example

- How many different ways can you seat Tom, Mary, and Sue in three chairs if the arrangement is the same when each person has the same person on their left and right?



Division Rule Example

- How many different ways can you seat Tom, Mary, and Sue in three chairs if the arrangement is the same when each person has the same person on their left and right?
 - $3 * 2 * 1 = 3! = 6$ possible seatings
 - 3 seatings are the “same”
 - $3! / 3 = 2$ possible ways to seat

Pigeonhole Principle

- If k is a positive integer and $k + 1$ more more objects are placed into k boxes, then there is at least one box containing two or more of the objects
- Can prove this by contraposition
- Generally: If N objects are placed into k boxes, then there is at least one box containing at least $\lceil N/k \rceil$ objects

