Reminders

- Exams are being returned
- Solutions for Exam will be posted later today
- Homework is due Thursday
- Quiz Thursday on Algorithms and Complexity
  - We will discuss what you will need to know

Practice

- Find the runtime and Big-O of these programs
- i := 1
  k := 0
  while i ≤ 1000
  	k := k + 1
  	i := i*2

P vs. NP

- All **tractable** problems are in **P**
- All **intractable** problems are in **NP**
  - Problems in NP are “hard” problems where approximations are generally accepted
  - Tractable if some magic computer (“oracle”) gave us an answer we only have to verify
- It has not been proven that P ≠ NP, but it is widely believed to be true
  - Proving P = NP makes problems like AI easier, but breaks many fundamentals in cryptography

CMSC 203: Lecture 13

Introduction to Induction

What is Induction?

- It's **another proof method**! Yay!
  - Starts with some “base case” that may be true
  - Prove that given any “step” we can reach the next one
- Some constraints
  - Must be sequence (eg: natural numbers, not reals)
  - Must be able to prove k+1 given k
  - Must have a starting case that may be proven true
  - Can not derive new theorems (only proves them)

Concept

- Start with **basis step** to show that P(1) is true
- P(k) as an **inductive hypothesis**
- Create an **inductive step** to show if P(k) then P(k+1)

Details

- Our basis step may also be P(0), or some other “start point”
- P(k) → P(k+1) must work for any arbitrary integer
  - No exceptions or selective k
- Our inductive step shows P(1) proves P(2), which proves P(3), etc.
- It is never assumed that P(k) is true for all integers, just that if we assume P(k), then P(k+1)

Process

1. Express statement as "for all $n \geq b$, P(n)" for some b
2. Write "Basis Step" and show P(b) is true
3. Write "Inductive Step"
4. "Assume P(k) is true for arbitrary k" (hypothesis)
5. "Prove $P(k) \rightarrow P(k+1)$" being explicit about P(k+1)
6. "This completes the inductive step"
7. Conclude the proof with "By mathematical induction, P(n) is true for all $n \geq b$"

Examples

- Prove that if I have a sequence of dominos lined up correctly, I can topple all of them.

- If *n* is a positive integer, then
$$1 + 2 + \ldots + n = n(n+1)/2$$

- Prove $2^n < n!$ for $n \geq 4$

Strong Induction

- Similar to regular mathematical induction
- Sometimes, we can't complete inductive step
- Instead, inductive hypothesis shows:
  - If P(j) is true for all positive integers not exceeding *k*, then P(k+1) is true
  - ie: Assuming P(j) is true j = 1, 2, ..., k

Example

- All positive integers ≥ 2 can be written as product of two primes

- Prove every amount of postage 12 cents or more can be formed using just 4-cent and 5-cent stamps