Reminders

- Homework #2 is being returned
  - The rubric is online (Piazza and Website)
- Homework #3 is due today
  - If you are not turning it in, email me to use your late
- Homework #4 is released
  - It is due on October 1st
- Final exam is next week

CMSC 203: Lecture 8

Sets and Functions (continued!)

Refresher

- Given element *a*, we say:
  - $a \in A$ (a is an element in set A)
  - $a \notin A$ (a is not an element in set *A*)
- Can define sets using **set builder notation** or **roster method**
- Given two sets, we can say:
  - $A \subseteq B$ (A is a subset of B)
  - $A \subset B$ (A is a proper subset of B)

Common Sets

- **N** = {0, 1, 2, 3, …}, set of **natural numbers**
- **Z** = {…, -2, -1, 0, 1, 2, …}, set of **integers**
- **Z+** = {1, 2, 3, …}, set of **positive integers**
- **Q** = {p/q | p ∈ **Z**, q ∈ **Z**, q ≠ 0}, set of **rational numbers**
- **R**, the set of **real numbers**
- **R+**, the set of **positive real numbers**
- **C**, the set of complex numbers

Size of Sets

- If n distinct elements in set S, S is a **finite** set with **cardinality** n
  - Cardinality is denoted by $|S|$
  - Cardinality is the "size" if there are no duplicates
- If a set is not finite, it is **infinite**

Set Operations

- **Union:** $A \cup B$
  - set of all elements in A **or** B
- **Intersection:** $A \cap B$
  - set of all elements in A **and** B
  - **Disjoint** if $A \cap B = \emptyset$
- **Difference:** $A - B$
  - set of elements in A but not in B
- **Complement**: $\overline{A}$
  - set of all elements in U but not in A

Practice Set Operations

- Let *A* be the set of students who live within one mile of school and let *B* be the set of students who walk to classes. Describe the students in the following sets:
  - $A \cap B$
  - $A \cup B$
  - $A - B$
  - $B - A$

Power Sets

- The **power set** of *S* is the set of all subsets of *S*
  - Denoted by $\mathcal{P}(S)$
  - Set of all combinations of elements
  - Example:

    $\wp(\{0,1,2\}) = \{\emptyset, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, \{0,1,2\}\}$

Cartesian Product

- The **cartesian product** of A and B is set of all ordered pairs (a, b) where $a \in A$ and $b \in B$
  - Denoted by A x B
  - Set of all ordered pairs
  - Example:

    $A = \{1, 2\}$ and $B = \{a, b, c\}$

    $AxB = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c)\}$

    $BxA = \{(a,1), (a,2), (b,1), (b,2), (c,1), (c,2)\}$

    $AxB \neq BxA$

Set Equality

- $|A \cup B| = |A| + |B| - |A \cap B|$
- Common identities are similar to our identities with logic
- We can prove sets are equal
  - Showing both sets are a subset of the other
  - Using set builder notation and definitions
  - Using **membership tables** (similar to truth tables)

Functions

- Assigning elements of one set to another set (possibly the same)
- *Example*: Assigning set of students to set of grades
- This assignment is a **function** (also called **mapping** or **transformation**)
- Functions are extremely important to computer science
  - Definition of discrete structures
  - Runtime analysis
  - Calculating values (input → output)
  - Understanding recursive functions

Defining Functions

- Function *f* from A to B is assignment of **exactly one** element of B to **each** element of A
- Written as *f(a) = b*
  - Can be done as f(x) = x + 1
  - Can also be done by f(Apple) = Tree
- *b* is a unique element, assigned by *f* to element *a*
- If *f* is a function from A to B, write “f : A → B”

Definitions

If f is a function from A to B, and f(a) = b

- **Domain**: A is the domain of f
- **Codomain**: B is the codomain of f
- **Image**: b is the image of a
- **Preimage**: a is the preimage of b
- **Range**: Set of all images of elements of A
  - *Note:* This is **not** to be confused with codomain
- **Maps**: f maps A to B

Special Function Types

- **One-to-One / Injunctive**: f(a) = f(b) implies that a = b for all a and b in the domain
  - I.e., each image only has one preimage
  - $\forall a \forall b(f(a) = f(b) \rightarrow a = b)$, or $\forall a \forall b(a \neq b \rightarrow f(a) \neq f(b))$
- **Onto / Surjective**: Every element $b \in B$ has element $a \in A$ with f(a) = b
  - I.e., every element in the codomain is an image
  - $\forall y \exists x(f(x) = y)$
- **One-to-One correspondence / Bijective**: Function *f* is both, one-to-one and onto

Function Operations

- **Inverse Function**
  - Denoted as f^{-1}
  - If f is a bijection, then $f^{-1}(b) = a$ whenever $f(a) = b$
  - i.e., f^{-1} assigns to b the element a such that $f(a) = b$
- **Composition**
  - Let f and g be functions from B to C and A to B (respectively)
  - $f(g(a)) = (f \circ g)(a)$ is the composition of f and g
  - Range of g must be a subset of the domain of f