Reminders

- Homework #3 is due Tuesday
- Office Hours can be scheduled for Monday
- Should have your Homework 2 grades back on Tuesday

Practice

- Prove the sum of two odd integers is even.
- Prove every odd integer is the difference of two squares.
- Prove if n is an integer and n^3+5 is odd, then n is even.
- Prove $\max(x, y) + \min(x, y) = x + y$
- Prove or disprove there is a rational number x and an irrational number y such that x^y is irrational
- Prove given real number x there exist unique numbers n and e such that $x = n - e$, n is an integer, and $0 <= e < 1$

CMSC 203: Lecture 7

Sets and Functions

What is a set?

- Discrete structure (the first one we look at)
- Used to group objects together (**contain**)
- Usually, **elements** have similar properties
- Unordered
- Given element *a*, we say:
  - $a \in A$ (a is an element in set A)
  - $a \notin A$ (a is not an element in set A)

Defining a Set

- List all elements using the **roster method**
  - {a, b, c, d}
  - {1, 2, 3, …, 100}
- Use **set builder** notation
  - $O = \{x \mid x \text{ is an odd positive integer less than } 10\}$
  - $O = \{x \in \mathbf{Z}^+ \mid x \text{ is odd and } x < 10\}$
  - $O = \{x \in \mathbf{R} \mid x = p/q,\ p \in \mathbf{Z}^+,\ q \in \mathbf{Z}^+\}$

Common Sets

- **N** = {0, 1, 2, 3, …}, set of **natural numbers**
- **Z** = {…, -2, -1, 0, 1, 2, …}, set of **integers**
- $\mathbf{Z}^+$ = {1, 2, 3, …}, set of **positive integers**
- **Q** = {p/q | p ∈ **Z**, q ∈ **Z**, q ≠ 0}, set of **rational numbers**
- **R**, the set of **real numbers**
- $\mathbf{R}^+$, the set of **positive real numbers**
- **C**, the set of complex numbers

Equality

- Two sets are equal *iff* they have the same elements
- $\forall x(x \in A \leftrightarrow x \in B)$
- Denoted by $A = B$
- Examples:
  - $\{1, 3, 5\} \stackrel{?}{=} \{3, 5, 1\}$
  - $\{1, 3, 3, 3, 5, 5, 5\} \stackrel{?}{=} \{1, 3, 5\}$

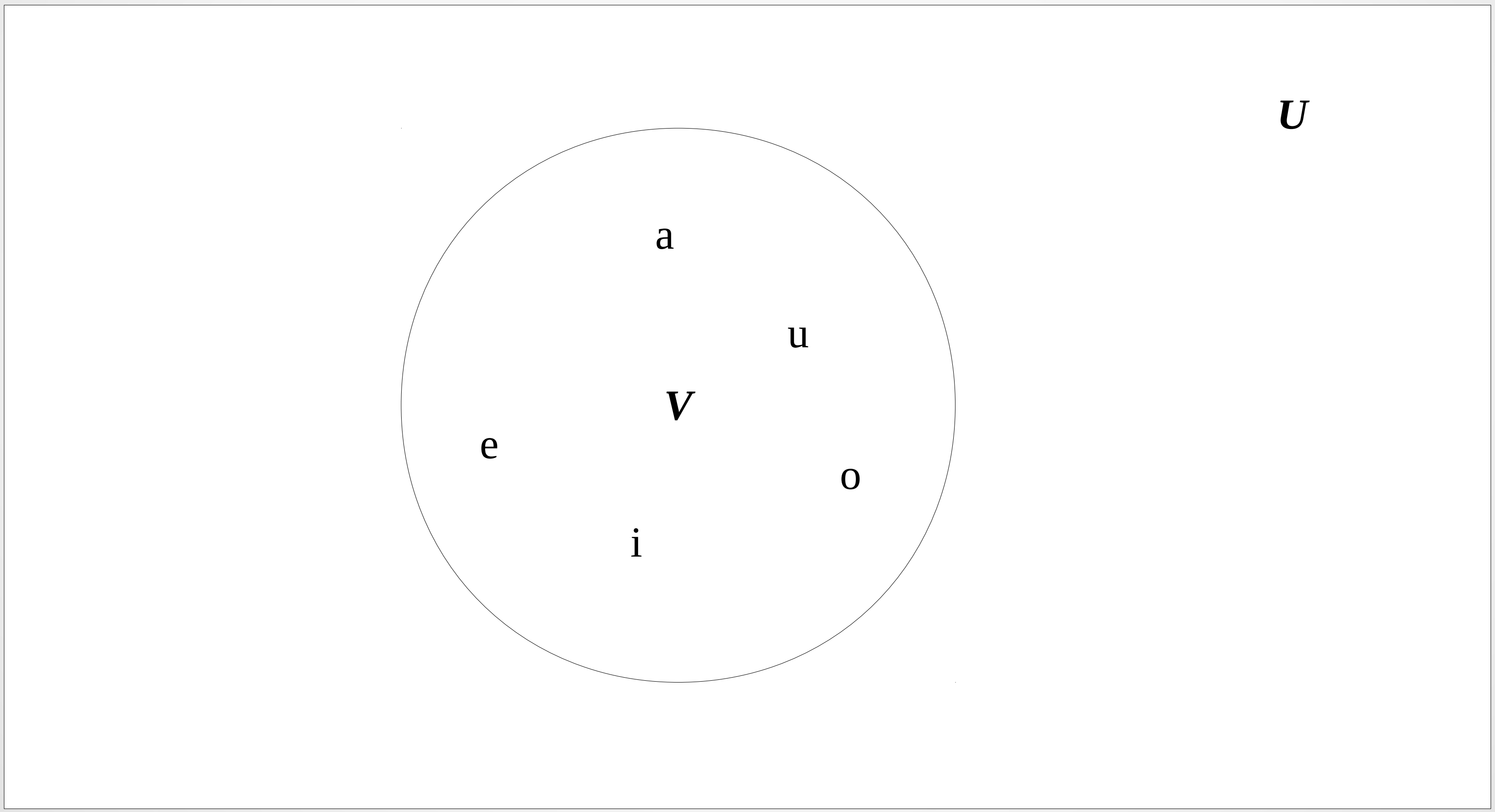
Special Sets

- Set with no elements
  - Called either **empty set** or **null set**
  - Designated by $\varnothing$ or { }
  - Example: $\{x \in \mathbf{Z}^+ \mid x > x^2\} = \varnothing$
- Set with one element
  - Called a **singleton set**
  - $\varnothing \neq \{\varnothing\}$
- Set of all elements in (considered) universe
  - Called **universal set**
  - Designated by **U**

Naive Set Theory

- When Georg Cantor “invented” the notion of sets (in 1895) he defined it as a collection of objects
- Leads to notion that any property contains a set of exactly the properties
  - This leads to paradoxes (logical inconsistencies)
  - $S = \{x \mid x \notin x\}$ leads to $S \in S \longleftrightarrow S \notin S$
- We will still use Naive Set Theory because it's easy and works in many cases

Venn Diagrams



Subsets

- Set *A* is a subset of *B* if and only if every element of A is an element of B
  - $\forall x(x \in A \rightarrow x \in B)$
  - Denoted by $A \subseteq B$
- Two ways to prove
  - Show that if *x* belongs to *A* then *x* also belongs to *B*
    - This proves $A \subseteq B$
  - Find a single $x \in A$ such that $x \notin B$
    - This proves $A \nsubseteq B$

Examples

- $\{x \mid x \text{ is odd and } x < 10\} \subseteq \{0, 1, 2, \ldots, 9\}$
- $\mathbf{Z} \subseteq \mathbf{R}$
- $\mathbf{Z}^{+} \subseteq \mathbf{Z}$
- $\mathbf{N} \subseteq \mathbf{Z}^{+}$
- $\varnothing \subseteq S$ where S is an arbitrary set
- $S \subseteq S$

Size of Sets

- If n distinct elements in set S, S is a **finite** set with **cardinality** n
  - Cardinality is denoted by |S|
  - Cardinality is the "size" if there are no duplicates
- If a set is not finite, it is **infinite**

Set Operations

- **Union:** $A \cup B$
  - set of all elements in A **or** B
- **Intersection:** $A \cap B$
  - set all all elements in A **and** B
  - **Disjoint** if $A \cap B = \varnothing$
- **Difference:** $A - B$
  - set of elements in A but not in B
- **Complement**: $\overline{A}$
  - set of all elements in U but not in A

Sample Sets

- Let *A* be the set of students who live within one mile of school and let *B* be the set of students who walk to classes. Describe the students in the following sets:
  - $A \cap B$
  - $A \cup B$
  - $A - B$
  - $B - A$