Reminders

- Homework #3 is out
  - Should cover proofs (including some from today)
  - Due next Tuesday (September 24)
- Office hours Wednesday and Thursday

Is this Proof Correct?

If n is odd, then $3n + 2$ is odd.
p: n is odd, **q**: $3n + 2$ is odd

1. Start with $\neg p$: "n is even" (contradiction)
2. $n = 2k$, given definition of even numbers
3. Substitute n into $3n + 2$
4. $3(2k) + 2$
5. $6k + 2$
6. $2(3k + 1)$
7. $2(k\prime)$
8. $3n + 2$ is even
9. q is false if p is false
10. Therefore, the inverse is true:
    if n is odd, then $3n + 2$ is odd

Is this Proof Correct?

If n^2 is even, then n is even.
p: n^2 is even, **q**: n is even

1. Start with q: "n is even"
2. $n = 2k$ given definition of even numbers
3. Substitute n into n^2
4. $(2k)^2$
5. $4k^2$
6. $2(2k^2)$
7. $2k\prime$
8. Therefore, n^2 is even

Is this Proof Correct?

If x is a real number, then x^2 is a positive real number.
p: x is a real number, **q**: x^2 is a positive real number
p_1: x is positive, p_2: x is negative

1. Given p_1, a positive times a positive is still positive
2. Therefore, p_1 is true
3. Given p_2, a negative times a negative is still positive
4. Therefore, p_2 is true
5. Since $p_1 \rightarrow q$ is T and $p_2 \rightarrow q$ is T, p is T

CMSC 203: Lecture 6

Proof Methods and Stragies

Existence Proof

- Useful if theorem is of form $\exists x P(x)$
- **Constructive:** Prove an element *a* exists as a witness
  - You are producing a solution
  - *Example from book*: “There exists a positive integer that can be written as the sum of cubes of positive integers in two different ways.”
  - $1729 = 10^3 + 9^3 = 12^3 + 1^3$

Existence Proof

- **Nonconstructive**: Do not produce a witness
  - You are proving using another proof methd
  - Must show that some solution *must* exist
  - Proof by contradiction, for example
  - *Example from book*: “Show that there exist irrational numbers *x* and *y* such that x^y is rational”

Uniqueness Proof

- Useful if theorem asserts unique element with property
  - Exactly **one** element in a set with property
- Two steps
  - **Existence**: Show an element x exists with property
  - **Uniqueness**: Show that if $y \neq x$, y does not have property (and if y does have property, $y = x$)
- *Example from book*: If a and b are real numbers and $a \neq 0$, then there is a unique real number r such that $ar + b = 0$

Summary of Proofs

- **Direct Proof** – Assume P(x) and and show Q(x) using rules of inference (such as basic mathematical axioms)
- **Contrapositive** – Assume Q(x) is false, and use rules of inference to show P(x) is also false
- **Contradiction** – Assume P(x) is false [or Q(x) in conditionals], and use rules of inference to show any contradiction
- **Exhaustive Proof** – Show every solution possible
- **Proof by Cases** – Break into multiple, provable “cases”
- **Existence** and **Uniqueness** are special problems

Proof Strategies

1. Write down your hypotheses [p and q]
2. Write down all premises (assumed truths)
3. Use existing proof methods, and adapt
4. If conditional ($p \rightarrow q$), attempt to follow rules of inference from p to reach q (**direct proof**)

    Fall-back on indirect proofs (like proof by contradiction)

    Know what proof you are trying to use!
5. Attempt to find a counterexample, and try to prove it

    (This may not solve the problem, but will be helpful)

Some Tips

- Write down every True statement, and every statement you are assuming True (like in proof by contradiction)
- Know your start point (hypothesis) and end point (hypothesis, or contradiction)
- Only follow known axioms, definitions, or rules of inference
- Never start with q and try to prove p (this is *begging the question*)
- Never assume $\neg p$ and try to show $\neg q$, even with a proof by contradiction (this is *denying the antecedent*)

Practice

- Prove the sum of two odd integers is even.
- Prove every odd integer is the difference of two squares.
- Prove if *n* is an integer and n^3+5 is odd, then *n* is even.
- Prove $\max(x, y) + \min(x, y) = x + y$
- Prove or disprove there is a rational number *x* and an irrational number *y* such that x^y is irrational
- Prove given real number *x* there exist unique numbers *n* and *e* such that $x = n - e$, *n* is an integer, and $0 <= e < 1$