Reminders

- Continue working on Homework #2
  - It is due in class on *Thursday*
  - Check Piazza for answers to questions
- **Extra credit** for stopping by my office (ITE 214)
  - Wednesday @ 10:30 - 11:30
  - Thursday @ 11:30 – 12:30
- Drop date (with a “W”) is *Tomorrow*

Review of Inference

- Use logic to show a proposition is true (or false)
- Basis of all mathematical proofs
- We want to prove a **conclusion** (also called a *conjecture*)
- Start with a set of **premises** (also called *lemmas*)
- Follow valid rules of inference to produce *corollaries*
- A valid series of corollaries to reach the conclusion is a **theorem**

Rules of Inference

p $p \rightarrow q$ $\therefore q$	Modus ponens
$\neg q$ $p \rightarrow q$ $\therefore \neg p$	Modus tollens
$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	Disjunctive syllogism
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	Resolution
p q $\therefore p \wedge q$	Conjunction

Review of Inference

p: It is sunny this afternoon
q: It is colder than yesterday
r: We will go swimming
s: We will take a canoe trip
t: We will be home by sunset

Premises:
1. $\neg p \wedge q$
2. $r \rightarrow p$
3. $\neg r \rightarrow s$
4. $s \rightarrow t$

Conclusion:
t

p p → q ---------- ∴ q	Modus ponens
¬q p → q ---------- ∴ ¬p	Modus tollens
p → q q → r ---------- ∴ p → r	Hypothetical syllogism
p v q ¬p ---------- ∴ q	Disjunctive syllogism
p v q ¬p v r ---------- ∴ q v r	Resolution
p q ---------- ∴ p ∧ q	Conjunction

Review of Inference

1. $\neg p \wedge q$ (premise 1)

p: It is sunny this afternoon
q: It is colder than yesterday
r: We will go swimming
s: We will take a canoe trip
t: We will be home by sunset

Premises:

1. $\neg p \wedge q$
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p v q ¬p ---------- ∴ q	Disjunctive syllogism
p v q ¬p v r ---------- ∴ q v r	Resolution
p q ---------- ∴ p ∧ q	Conjunction

Review of Inference

1. $\neg p \wedge q$ (premise 1)
2. $\neg p$ (conjunction)

p: It is sunny this afternoon
q: It is colder than yesterday
r: We will go swimming
s: We will take a canoe trip
t: We will be home by sunset

Premises:

1. $\neg p \wedge q$
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p v q ¬p v r ---------- ∴ q v r	Resolution
p q ---------- ∴ p ∧ q	Conjunction

Review of Inference

1. $\neg p \wedge q$ (premise 1)
2. $\neg p$ (conjunction)
3. $r \rightarrow p$ (premise 2)

p: It is sunny this afternoon
q: It is colder than yesterday
r: We will go swimming
s: We will take a canoe trip
t: We will be home by sunset

Premises:

1. $\neg p \wedge q$
2. $r \rightarrow p$
3. $\neg r \rightarrow s$
4. $s \rightarrow t$

Conclusion:

t

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p v q ¬p v r ---------- ∴ q v r	Resolution
p q ---------- ∴ p ∧ q	Conjunction

Review of Inference

1. $\neg p \wedge q$ (premise 1)
2. $\neg p$ (conjunction)
3. $r \rightarrow p$ (premise 2)
4. $\neg r$ (modus tollens 2,3)

p: It is sunny this afternoon
q: It is colder than yesterday
r: We will go swimming
s: We will take a canoe trip
t: We will be home by sunset

Premises:

1. $\neg p \wedge q$
2. $r \rightarrow p$
3. $\neg r \rightarrow s$
4. $s \rightarrow t$

Conclusion:
t

p p → q ---------- ∴ q	Modus ponens
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p → q q → r ---------- ∴ p → r	Hypothetical syllogism
p v q ¬p ---------- ∴ q	Disjunctive syllogism
p v q ¬p v r ---------- ∴ q v r	Resolution
p q ---------- ∴ p ∧ q	Conjunction

Review of Inference

1. $\neg p \wedge q$ (premise 1)
2. $\neg p$ (conjunction)
3. $r \rightarrow p$ (premise 2)
4. $\neg r$ (modus tollens 2,3)
5. $\neg r \rightarrow s$ (premise 3)

p: It is sunny this afternoon
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Premises:

1. $\neg p \wedge q$
2. $r \rightarrow p$
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Conclusion:
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p q ---------- ∴ p ∧ q	Conjunction

Review of Inference

1. $\neg p \wedge q$ (premise 1)
2. $\neg p$ (conjunction)
3. $r \rightarrow p$ (premise 2)
4. $\neg r$ (modus tollens 2,3)
5. $\neg r \rightarrow s$ (premise 3)
6. s (modus ponens 4,5)

p: It is sunny this afternoon
q: It is colder than yesterday
r: We will go swimming
s: We will take a canoe trip
t: We will be home by sunset

Premises:

1. $\neg p \wedge q$
2. $r \rightarrow p$
3. $\neg r \rightarrow s$
4. $s \rightarrow t$

Conclusion:
t

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p q ---------- ∴ p ∧ q	Conjunction

Review of Inference

1. $\neg p \wedge q$	(premise 1)
2. $\neg p$	(conjunction)
3. $r \rightarrow p$	(premise 2)
4. $\neg r$	(modus tollens 2,3)
5. $\neg r \rightarrow s$	(premise 3)
6. s	(modus ponens 4,5)
7. $s \rightarrow t$	(premise 4)

p: It is sunny this afternoon
q: It is colder than yesterday
r: We will go swimming
s: We will take a canoe trip
t: We will be home by sunset

Premises:

1. $\neg p \wedge q$
2. $r \rightarrow p$
3. $\neg r \rightarrow s$
4. $s \rightarrow t$

Conclusion:

t

p p → q ---------- ∴ q	Modus ponens
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p v q ¬p v r ---------- ∴ q v r	Resolution
p q ---------- ∴ p ∧ q	Conjunction

Review of Inference

1. $\neg p \wedge q$	(premise 1)
2. $\neg p$	(conjunction)
3. $r \rightarrow p$	(premise 2)
4. $\neg r$	(modus tollens 2,3)
5. $\neg r \rightarrow s$	(premise 3)
6. s	(modus ponens 4,5)
7. $s \rightarrow t$	(premise 4)
8. t	(modus ponens 6,7)

p: It is sunny this afternoon
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r: We will go swimming
s: We will take a canoe trip
t: We will be home by sunset

Premises:

1. $\neg p \wedge q$
2. $r \rightarrow p$
3. $\neg r \rightarrow s$
4. $s \rightarrow t$

Conclusion:

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p p → q ---------- ∴ q	Modus ponens
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p v q ¬p v r ---------- ∴ q v r	Resolution
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Fallacies

- Affirming the consequent (*abductive reasoning*)
  - If q and $p \rightarrow$ q, then p
  - Eg: If Bill Gates owns Fort Knox, then he is rich. Bill Gates is rich. Therefore, he owns Fort Knox.
- Denying the atecedent (*inverse error*)
  - If $\neg p$ and $p \rightarrow q$, then $\neg q$
  - If Queen Elizabeth is an American citizen, then she is a human being. Queen Elizabeth is not an American citizen. Therefore, Queen Elizabeth is not a human being.
- Circular reasoning
  - Prove something is true by assuming it is true

CMSC 203: Lecture 4

Boolean Algebra and Circuits

Background

- George Boole, *The Laws of Thought* (1854)
  - Claude Shannon adapted to circuit design (1938)
  - These rules are the basis of Boolean Algebra
- Values contain either 1 (T, voltage) or 0 (F, no voltage)
- Apply logic to create Boolean functions
- **Circuit**: Boolean function that specifies value of output for each set of inputs
  - Building block of computers

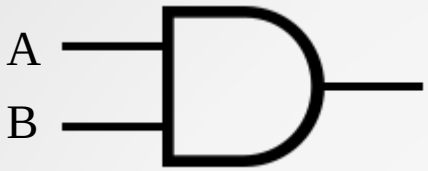
Boolean Logic

- **Complement**
  - Logical “NOT” : Denoted by bar, or by ¬ (or ~)
  - $\overline{0} = 1$, $\overline{1} = 0$
- **Sum**
  - Logical “OR” : Denoted by +, or by ∨
  - 1 + 1 = 1; 1 + 0 = 1; 0 + 1 = 1; 0 + 0 = 0
- **Product**
  - Logical “AND” : Denoted by •, or by ∧
  - 1 • 1 = 1; 1 • 0 = 0; 0 • 1 = 0; 0 • 0 = 0

Logic Gates

- **Gates**: basic elements of circuits
- Implementats a Boolean operation
- Gates we look at have no “memory”
  - Thus, input → output
- Multiple inputs may be fed, but only one output

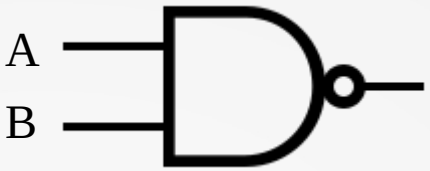
Logic Symbols



AB

$A \wedge B$

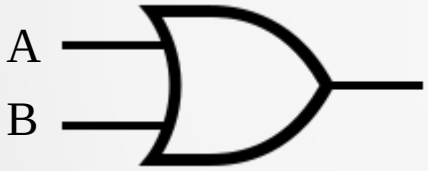
AND



$\overline{AB}$

$\neg(A \wedge B)$

NAND



$A + B$

$A \vee B$

OR

$\overline{A + B}$

$\neg(A \vee B)$

NOR

$\overline{A}$

$\neg A$

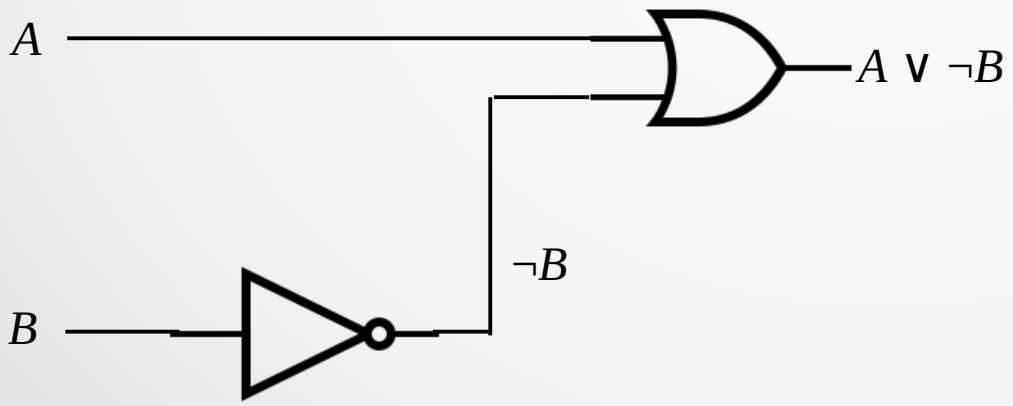
NOT

$A \oplus B$

XOR

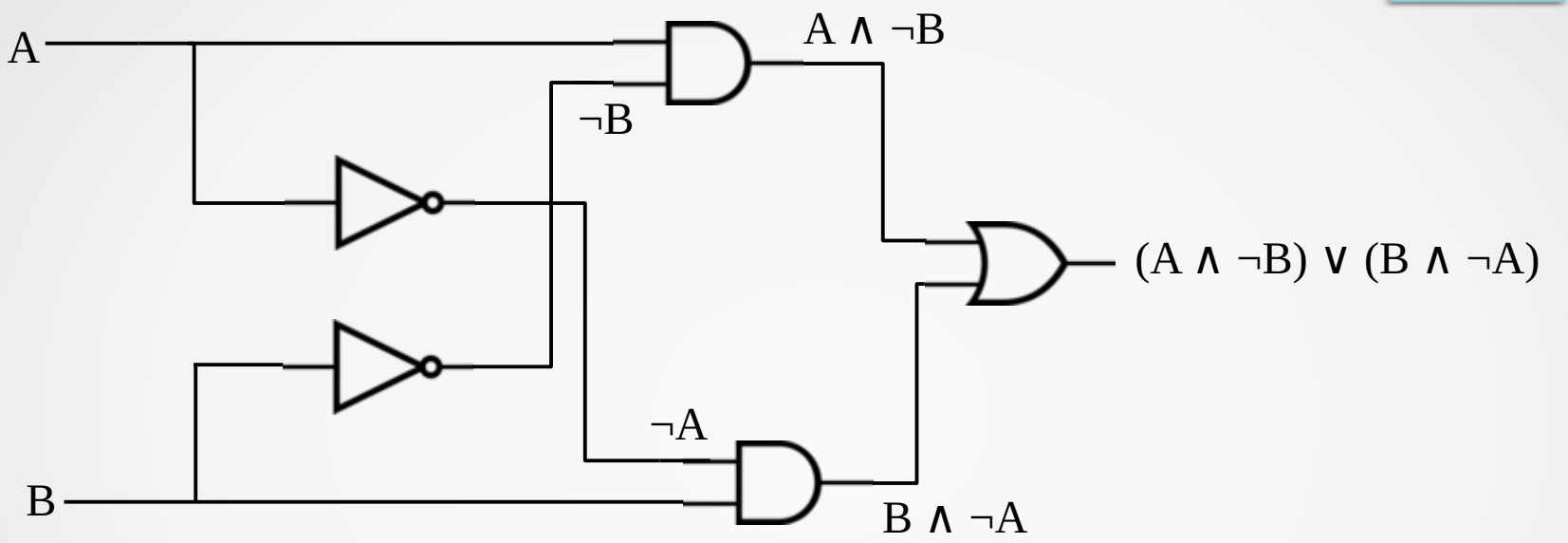
Combining Gates

- We can chain logic gates into logic circuits
  - Similar to combining logical operators into logic statements



A	B	¬B	A ∨ ¬B
1	1	0	1
1	0	1	1
0	1	0	0
0	0	1	1

Circuit Example



A	B	¬A	¬B	A ∧ ¬B	B ∧ ¬A	(A ∧ ¬B) ∨ (B ∧ ¬A)
1	1	0	0	0	0	0
1	0	0	1	1	0	1
0	1	1	0	0	1	1
0	0	1	1	0	0	0

Equivalent Circuits

We may have **equivalences** of circuits, as with logical functions

A	B	¬A	¬B	A ∧ ¬B	B ∧ ¬A	(A ∧ ¬B) ∨ (B ∧ ¬A)
1	1	0	0	0	0	0
1	0	0	1	1	0	1
0	1	1	0	0	1	1
0	0	1	1	0	0	0

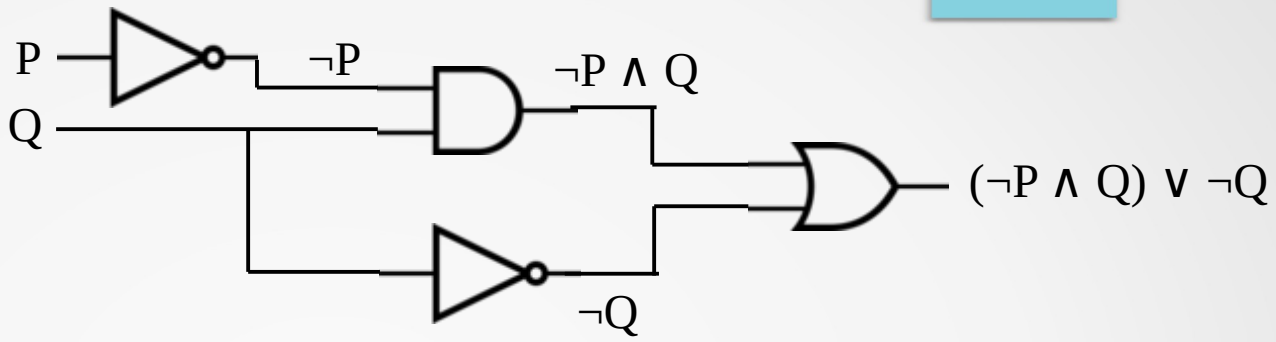
Logic → Circuit

$$(\neg p \wedge q) \vee \neg q$$

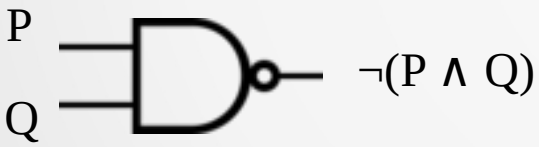
- Create a digital circuit from the statement
- Simplify the circuit by finding logical equivalences
- Draw the new simplified digital circuit
- Create a truth table for the statement

Logic → Circuits

$(\neg p \wedge q) \vee \neg q$



$(\neg p \wedge q) \vee \neg q \equiv (\neg p \vee \neg q) \wedge (q \vee \neg q) \equiv \neg p \vee \neg q \equiv \neg(p \wedge q)$



P	Q	¬P	¬Q	¬(P ∧ Q) ∨ ¬ Q	¬(P ∧ Q)
1	1	0	0	0	0
1	0	0	1	1	1
0	1	1	0	1	1
0	0	1	1	1	1

Truth Table → Logic / Circuit

- Creating logic / circuits from truth tables is hard
- One system:
  1. Find rows with output = 1
  2. Write row as conjunction
  3. Write disjunction of conjunctions
- This is **disjunctive normal** form
- It is always correct
  - But not always simplest

	P	*Q*	*R*	*S*
1	1	1	1	1
2	1	1	0	0
3	1	0	1	1
4	1	0	0	1
5	0	1	1	0
6	0	1	0	0
7	0	0	1	0
8	0	0	0	0

Simplifying Logic

- Simplying logic is **NP-Hard**
- There are methods such as Karnaugh maps and Quine-McCluskey algorithm
- There are tradeoffs in hardware
  - Sometimes preferable to build larger designs
- CMPE will see more of this than CMSC
- You won't be tested on this ;)

Review

- Boolean Algebra is another form of logical algebra
  - 1 ≡ True and 0 ≡ False
- There are several common circuits
  - OR, AND, NOT, NOR, NAND, XOR
- These blocks are basics of computers and digital design
- We can convert logic to circuits and vice versa
- You will see these more in CMPE 212 and CMSC 411