Homework #1

- Should be turned in by now!
- [Roughly] 25 people say marshmallow, 7 people said Jelly, 3 said either, 2 said both
- Some answers to questions and concerns:
  - The class is about math **and** computer science
    - We will learn math, and apply it to CS
    - You won't need to know a lot of math ahead of time
    - You won't need to know programming either
  - Getting an A and understanding concepts is up to you
    - If you don't understand something, ask questions on Piazza, email, and office hours
  - I will let you know what exams and quizzes will be like
  - There may or may not be donuts

Some Additional Notes

- Homework #2 will be released today
  - Will be due next Thursday
- Piazza now contains slides and homeworks
- **Extra credit**: Come to my office hours during the next week; you don't need a question, just show up
- *Lemma*: “proven statement used as a stepping-stone toward the proof of another statement” – basically a small theorem that is used in another proof

Lightning Review

Lightning Review (Propositions)

Which of these are propositions?

- Do not pass go.
- What time is it?
- 2 + 3 = 5
- 4 + x = 5
- The moon is made of green cheese.
- Answer this question.
- You answered that question.

Lightning Review (Logic → English)

Convert to English, assuming *p* and *q* are propositions

p : I bought a lottery ticket this week

q : I won the million dollar jackpot

(a) $p \rightarrow q$

(b) $p \wedge q$

(c) $\neg p \vee (p \wedge q)$ *(Hint: Order of operations)*

(d) $p \leftrightarrow q$

Lightning Review (English → Logic)

Write using logical connectives, assuming:

p : It is below freezing

q : It is snowing

(a) If it is snowing, then it is freezing

(b) That it is below freezing is necessary and sufficient for it to be snowing

(c) It can be snowing or below freezing (or both)

(d) It is not below freezing and it is not snowing

Lightning Review (Conditionals)

Determine if these propositions are true

- If 1 + 1 = 2, then 2 + 2 = 5
- If 1 + 1 = 3, then 2 + 2 = 5
- 1 + 1 = 2 if and only if 2 + 3 = 4
- 1 + 1 = 3 if and only if monkeys can fly
- If there is a pop quiz, then we will pass

Lightning Review (Truth Tables)

Construct a truth table for:

- $(p \oplus q) \vee (p \oplus \neg q)$
- $(p \rightarrow q) \wedge (\neg p \rightarrow r)$

Label the following as either a *tautology*, *contradiction*, or *contingency*?

$p \wedge \neg p$	$(p \vee q) \rightarrow (p \vee q)$
$p \wedge q$	$(p \vee q) \wedge (\neg p \wedge \neg q)$
$p \vee \neg p$	$p \wedge p$

CMSC 203: Lecture 3

Some Reminders

Reminder of why this matters

- What is the use of logic and proofs in the field of:
  - computer engineering?
  - computer programming?
  - artificial intelligence?
  - mathematics?
  - game development?

Fun Equivalences

- $p \rightarrow q \equiv \neg p \vee q$
- $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
- **Identity Laws**
  - $p \wedge T \equiv p$
  - $p \vee F \equiv p$
- **Domination Laws**
  - $p \vee T \equiv T$
  - $p \wedge F \equiv F$
- **Negation Laws**
  - $p \vee \neg p \equiv T$
  - $p \wedge \neg p \equiv F$
- **Idempotent Laws**
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- **Commutative Laws**
  - $p \vee q \equiv q \wedge p$
  - $p \wedge q \equiv q \wedge p$

More Fun Equivalences

- **Absorption Law**
  - $p \lor (p \land q) \equiv p$
  - $p \land (p \lor q) \equiv p$
- **Distributive Law**
  - $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$
  - $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$
- **De Morgan's Law**
  - $\neg(p \land q) \equiv \neg p \lor \neg q$
  - $\neg(p \lor q) \equiv \neg p \land \neg q$

Why Equivalences?

- How can equivalences be used in the following:
  - in writing a proof?
  - in programming?
  - in writing an artificial intelligence?

Proving Equivalence

Assume our robot knows $\neg p \lor \neg q$ is true

Can it know whether $\neg(p \land (\neg p \lor q))$ is true?

- It can not figure out the values of p or q
- It must prove that $\neg p \lor \neg q \equiv \neg(p \land (\neg p \lor q))$
- If it is equivalent, we don't need the values of p or q
- Equivalence can be proven with a proof table (slow!)
- Or we may attempt to use our laws to reform
  - (Think back to *solve for x* in algebra!)

CMSC 203: Lecture 3

Predicate Logic

Predicate Logic

- Problems with Propositional Logic
  - We can not express meaning of all statements in mathematics and natural language
- Example:
  - Assume every computer connected to the school network is functioning properly
  - Determine if MATH3 is functioning properly
- Predicate logic is more powerful than propositional logic

What is in Predicate Logic?

"Computer x is functioning properly"

- **Variables**
  - *x* is the variable (a la computer programs / algebra)
  - Subject of the statement
- **Predicate**
  - *"is functioning properly"* is the predicate
  - Properties the subject can have
  - Think of them as **functions**
    - You may even express them as functions

Predicates

- Predicates can be expressed as *P(x)*
  - Similar to functions, *x* is some input
  - Once *x* is assigned a value, *P(x)* becomes a proposition *(recall what a proposition is)*
- Example:
  - *P(x) denotes "x > 3"*
    - What is the value of *P(4)* and *P(2)* ?
  - R(x, y, z) denotes *"x hates y, but y loves z"*
    - What is the truth value of *R(Tybalt, Romeo, Juliet)*

Quantification

- **Quantifiers**
  - Designate a *range* in which a predicate is true
  - Two kinds we will look at:
    - **Universal –** Predicate is true under every element
      - Eg: “*Everybody* loves Raymond”
    - **Existential –** Predicate is true for *at least* one
      - Eg: “*Somebody* is President”

Universal Quantifier

- Asserts predicate is true for all variables in **domain**
- $\forall x P(x)$ denotes the universal quantification for P(x)

  Read: "for all/every *x P(x)*"

- ***True*** when P(x) is true for *every* x
- ***False*** when there is an x for which P(x) is false

Existential Quantifier

- Asserts an element exists in **domain** for which predicate is true
- $\exists x P(x)$ denotes the existential quantification for P(x)

  Read: “There exists some *x* such that *P(x)*”

- ***True*** when there is an x in which P(x) is true
- ***False*** when P(x) is false for every x

Quantifier Practice

- $\exists x$ 203Student(x) ∧ 202Student(x)
- $\forall x$ 203Student(x) → Sophomore(x)
- $\forall x$ Senior(x) ∧ Tall(x)
- Every gardener likes the sun.
- All purple mushrooms are poisonous.
- No purple mushroom is poisonous.

De Morgan's Laws (again!)

- $\neg\exists\, x\, P(x) \equiv \forall x\, \neg P(x)$
- $\neg\forall x\, P(x) \equiv \exists x\, \neg P(x)$
- There does not exist an element that... ≡ For all elements, it is not true that...

Nested Quantifiers

- Quantifiers can have *scope*
- We can stack / nest our quantifiers for more flexibility
- For example: $\forall x \exists y \equiv$ "For all x, there exists a y…"
- **Order is important!**
- Think of them as nested "loops"

Nested Quantifier Examples

- $\exists x \exists y$ Montague(x) $\wedge$ Capulet(y) $\rightarrow$ Enemies(x, y)
- $\forall x \forall y \; (x + y = y + x)$
- $\forall x \exists y$ Father(x,y) $\wedge$ Male(y)
- $\exists x \forall y$ (Student(x) $\wedge$ Hard(y)) $\rightarrow$ Failed(x, y)

Inference

- Special case of a mathematical proof
- Show some sentence *p* is true, given a set of **premises**
  - *p* is called the **conclusion**
- Our **argument** is **valid** if the truth of the conclusion follows the premises
- For example, given *premises* p and $p \rightarrow q$, we can reach the *valid conclusion* q

Rules of Inference

p $p \rightarrow q$ ———————— $\therefore q$	Modus ponens
$\neg q$ $p \rightarrow q$ ———————— $\therefore \neg p$	Modus tollens

$p \rightarrow q$ $q \rightarrow r$ ———————— $\therefore p \rightarrow r$	Hypothetical syllogism
$p \vee q$ $\neg p$ ———————— $\therefore q$	Disjunctive syllogism

$p \vee q$ $\neg p \vee r$ ———————— $\therefore q \vee r$	Resolution
p q ———————— $\therefore p \wedge q$	Conjunction

Inference Example

- Given:
  - It is humid
  - If it is humid, then it is hot
  - If it is hot and humid, then it is raining
- Show:
  - It is raining

Review

- Today we:
  - Finished looking at equivalences
  - Discovered predicate logic is, and why it's used
    - Learned what quantifiers are, and how to use them
      - Including how to nest them properly
  - Looked at inference and how to do it
    - This is our first major introduction to doing proofs!