CMSC 203: Lecture 2

Introduction to Logic
Propositional Logic

(Or: How I Learn to Stop Assuming and Love the Logic)

Reminders

- Don't forget your homework!
  - Sign up for Piazza
  - Follow directions in HW1 thread
  - Due Thursday
- Slides are up
- Office hours are tomorrow and Thursday
- Get the book, start reading it

What is Logic? (and why care?)

- Everything can be represented as logic, as a set of rules
- If we can translate a problem into logic, solving is trivial
  - Using reasoning and understanding
- Logic is how we assert correctness
  - Basis of all *mathematical* and *automated* reasoning

What is a Proof? (and why care?)

- Correct mathematical argument (using logic)
- Once it is proven true, it is a **theorem**
- Collections of theorems are what we *know* about a topic
- Knowing the theorems means knowing the topic
  - Also allows easily modifying for new situations
- http://www.math.sc.edu/~cooper/proofs.pdf

Propositions

- Building block of logic
- Declarative sentence (a fact)
  - True or false (not both)
  - Can be defined in English
- Letters to denote **propositional variables**
  - Similar to letters in algebra
  - Usually *p, q, r, s, …*

Facts about Propositions

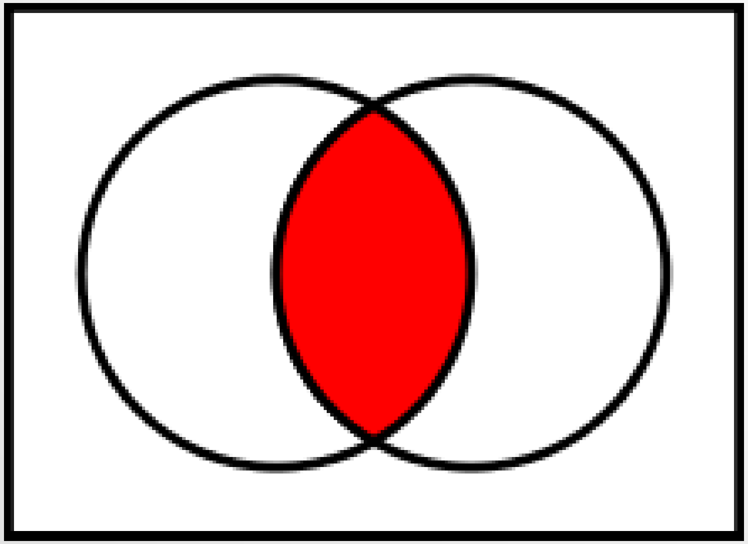
- **Truth value** of a proposition is either T or F
- Area of logic that deals with propositions
  - **Propositional Calculus / Logic**
- You can produce new propositions from ones you have
- Mathematical statements can combine propositions
  - Called **compound propositions**
  - Use *logical operators*
  - Example: “not *p*” is defined as $\neg p$

p	$\neg p$
T	F
F	T

Connectives ∧ You

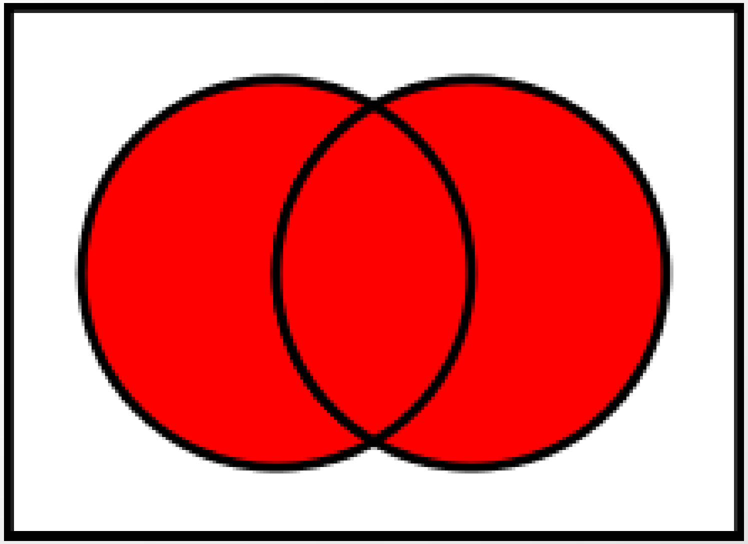
Conjunction

- Denoted by ∧
- Proposition "*p* **and** *q*"
- T if *p* = T and *q* = T



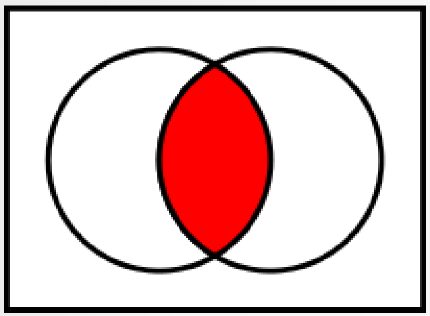
Disjunction

- Denoted by ∨
- Proposition "*p* **or** *q*"
- F if p = F and q = F



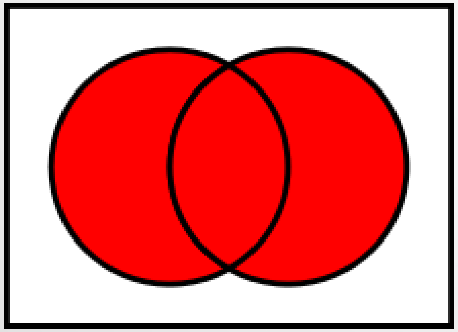
Truth Tables

Conjunction



p	*q*	*p* ∧ *q*
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

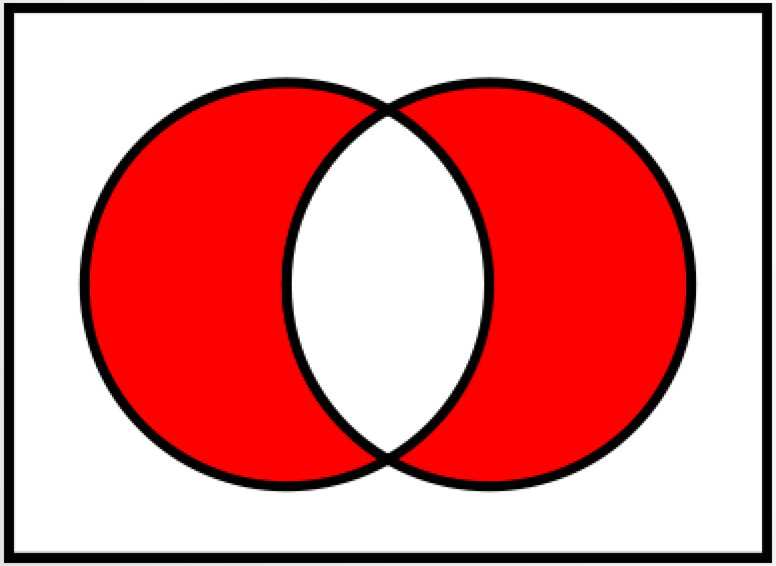


p	*q*	*p* ∨ *q*
T	T	T
T	F	T
F	T	T
F	F	F

The other kind of or...

Exclusive or

- Denoted by ⊕
- Proposition "*p* **or** *q* **but not both**"



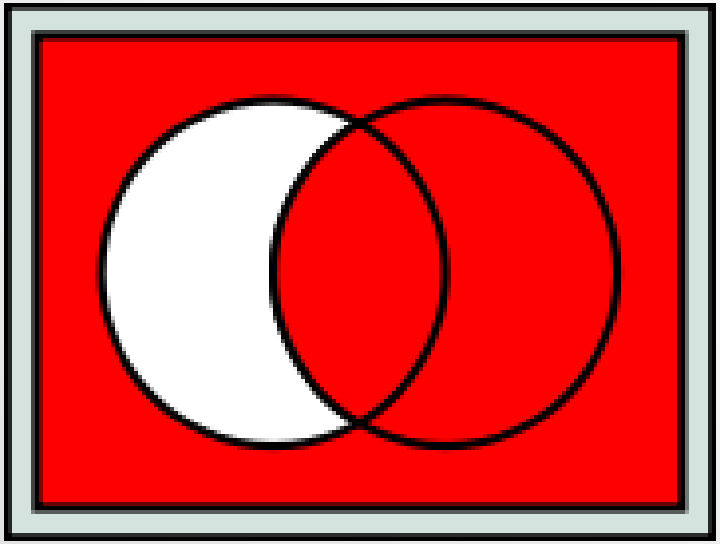
p	***q***	***p* ⊕ *q***
T	T	F
T	F	T
F	T	T
F	F	F

Conditional Statements - Implies

Implication

- Designated by →
- Proposition "*p* **implies** *q*"
- Asserts q = T if p = T

- *p* is sufficient for *q*
- *q* is necessary for *p*

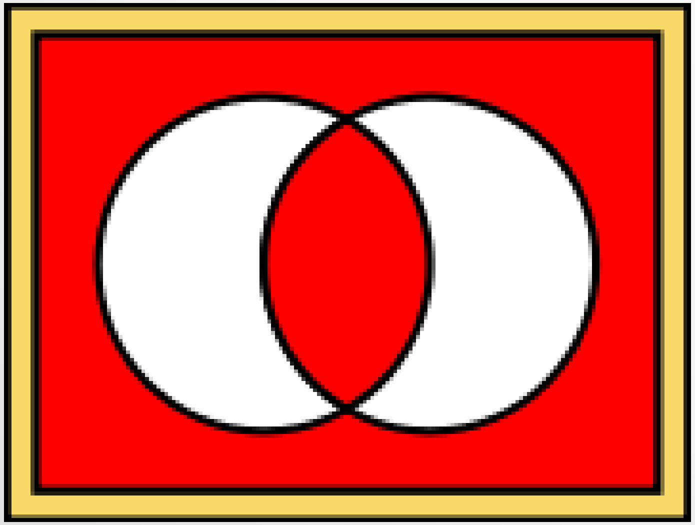


p	***q***	***p* → *q***
T	T	T
T	F	F
F	T	T
F	F	T

This slide implies another conditional

Biconditional

- Designated by ↔
- Proposition "*p* **if and only if (iff)** *q*"
- Also defined as "p → q ∧ q → p"



p	***q***	***p* → *q***
T	T	T
T	F	F
F	T	F
F	F	T

Putting them together (pt. 1)

p	q	$\neg q$	$p \land \neg q$	$p \land q$	$(p \land \neg q) \rightarrow (p \land q)$
T	T				
T	F				
F	T				
F	F				

Putting them together (pt. 2)

p	q	$\neg q$	$p \wedge \neg q$	$p \wedge q$	$(p \wedge \neg q) \rightarrow (p \wedge q)$
T	T	F			
T	F	T			
F	T	F			
F	F	T			

Putting them together (pt. 3)

p	**q**	**¬*q***	**p ∧ ¬*q***	**p ∧ *q***	**(p ∧ ¬*q*) → (*p* ∧ *q*)**
T	T	F	T	T	
T	F	T	T	F	
F	T	F	F	F	
F	F	T	T	F	

Putting them together (pt. 4)

p	q	$\neg q$	$p \land \neg q$	$p \land q$	$(p \land \neg q) \rightarrow (p \land q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Bitwise operations

- Can perform bitwise operations, like **OR, AND, XOR**
- VERY useful in Boolean Algebra (more on that later)
- Treat 1s as T and 0s as F
- We will be dealing with this later, and you will see it a lot in the future

Equivalences

- Propositions p and q are equivalent if truth values are always the same
- Designated as $p \equiv q$ (not a connective)
  - Defines p iff q as a tautology
- Can judge by the truth table
- If p is always T, it is a **tautology**
- If p is always F, it is a **contradiction**

Important Laws (of logic)

- **Absorption Law**
  - $p \lor (p \land q) \equiv p$
  - $p \land (p \lor q) \equiv p$
- **Distributive Law**
  - $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$
  - $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$
- **De Morgan's Law**
  - $\neg(p \land q) \equiv \neg p \lor \neg q$
  - $\neg(p \lor q) \equiv \neg p \land \neg q$
- **More in book** (pg. 27) – Check them out!