

Quiz 3

1. What is the probability that a card selected at random from a standard deck of 52 cards is an ace or a heart? [points](#)

$$p(\text{Heart}) = \frac{13}{52}$$

$$p(\text{Ace}) = \frac{4}{52}$$

$$p(\text{Heart} \cap \text{Ace}) = \frac{1}{52}$$

$$p(\text{Heart} \cup \text{Ace}) = p(\text{Heart}) + p(\text{Ace}) - p(\text{Heart} \cap \text{Ace}) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13} \approx 0.308$$

2. What is the conditional probability that exactly four heads appear when a fair coin is flipped five times, given that the first flip came up heads? [points](#)

E: Four heads appear when coin is flipped five times

F: First flip is heads

$$p(E) = \frac{5}{2^5} = \frac{5}{32}$$

$$p(F) = \frac{1}{2}$$

$$p(E \cap F) = \frac{4}{32} = \frac{1}{8}$$

$$p(E|F) = \frac{p(E \cap F)}{p(F)} = \frac{1/8}{1/2} = \frac{1}{4}$$

3. Are the events in problem 2 independent or dependent? Why? [points](#)

$$p(E \cap F) = \frac{1}{8}$$

$$p(E)p(F) = \frac{5}{32} \cdot \frac{1}{2} = \frac{5}{64}$$

$p(E \cap F) \neq p(E)p(F)$, so events are dependent

4. An electronics company is planning to introduce a new gaming console. The company commissions a marketing report for each new product that predicts either the success or the failure of the product. Of new products introduced by the company, 60% have been successes. Furthermore, 70% of their successful products were predicted to be successes, while 40% of failed products were predicted to be successes. Find the probability that this new console will be successful if its success has been predicted. [points](#)

E: Successful product

F: Success predicted

$$p(E) = 0.6$$

$$p(\overline{E}) = 1 - p(E) = 0.4$$

$$p(F|E) = 0.7$$

$$p(F|\overline{E}) = 0.4$$

$$p(E|F) = \frac{p(F|E)p(E)}{p(F|E)p(E) + p(F|\overline{E})p(\overline{E})} = \frac{0.7 \cdot 0.6}{0.7 \cdot 0.6 + 0.4 \cdot 0.4} \approx 0.724$$

5. What is the expected value when a \$1 lottery ticket is bought in which the purchaser wins exactly \$10 million if the ticket contains the six winning numbers chosen from the set $\{1, 2, 3, \dots, 50\}$ and the purchaser wins nothing otherwise.

[points](#)

$$p(\text{win}) = 1/C(50, 6) = 1/15890700$$

$$V(\text{win}) = p(\text{win}) \cdot 10000000 = .629$$

$$V(\text{lose}) = (1 - p(\text{win})) \cdot 0 = 0$$

$$V(\text{play}) = V(\text{win}) + V(\text{lose}) - 1 = -.371$$