Name: ______________________________ November 3, 2013

Quiz 2

1. UMBC is undergoing many changes and wants your help counting. Include a brief *(one sentence)* explanation of what method you would use for each problem, and the variables for this method. *Do not* solve or simplify, but you may write out the equation if you want. Not all methods may be used.
The counting methods include: **Pigeonhole Principle**, **Permutation**, **Combination**, **Permutation with Repetition**, **Combination with Repetition**.

   a) UMBC wants to change the ID numbers to 10 numbers and uppercase letters. How many students can this handle?
   Permutation with Repetition
   $n = 36; r = 10; n^r = 36^{10}$
   There are 36 possible choices (26 uppercase letters and 10 digits) for 10 slots on the card, and the order matters. Each character may be used more than once.

   b) UMBC wants to minimize the amount of parking spaces such that at least one spot needs to fit 5 cars in it.
   Pigeonhole Principle
   $N = \text{cars}; k = \text{parking spaces}; r = 5; \lceil \frac{N}{k} \rceil = 5$
   There are a certain number of cars that need to be fit into parking spaces. We want at least one to have 5, which is our pigeonhole principle.

   c) UMBC wants to know how many possible choices there are to form a 25 person committee from 100 of their favorite faculty.
   Combination
   $n = 100; r = 25; C(n, r) = \frac{n!}{r!(n-r)!} = \frac{100!}{25!75!}$
   A combination is used since we are selecting members, but the order which committee members are chosen does not matter.

   d) UMBC wants to hire 10 new teachers, and assign them to 15 Introduction to Dance sections. How many possible options there are for assignment (assuming some teachers can teach more than one class)
   Combination with Repetition
   $n = 15; r = 10; \frac{(n+r-1)!}{r!(n-1)!} = \frac{24!}{10!14!}$
   A combination is used since we are selecting teachers for the same class, so the order we select them does not matter. We may select teachers more than once, however, so we want to allow for repetition.

   e) UMBC has decided to rename 10 buildings on campus to one of 22 prestigious names. How many possible ways can they rename the buildings?
   Permutation
   $n = 22; r = 10; P(n, r) = \frac{n!}{(n-r)!} = \frac{22!}{12!}$
   We are selecting names to apply to buildings, and order matters, since the ordering in which the names are applied to buildings matters

2. Solve the following counting problems. Fill in the equation, but do not simplify.

   a) (*Pigeonhole Principle*) What is the minimum number of students, each of whom comes from one of the 50 states, who must be enrolled in a university to guarantee there are at least 100 who come from the same state?
   $k = 50, r = 100$
   $N_{\text{min}} = k(r-1)+1$
   $N_{\text{min}} = 50(100-1)+1$

   b) (*Permuation*) How many permutations of $\{a, b, c, d, e, f, g\}$ end with a?
   $n = 6, r = 6$ (we ignore "a" since we know its location at the end)
   $P(n,r) = \frac{n!}{(n-r)!} = \frac{6!}{0!} = 6!$

   c) (*Combination*) A professor writes 40 discrete mathematics true/false questions. Of the statements in the questions, 17 are true. If the questions can be positioned in any order, how many different answer keys are possible?
   $n = 40, r = 17$
   $C(n,r) = \frac{n!}{r!(n-r)!} = \frac{40!}{17!(40-17)!} = \frac{40!}{17!23!}$

   d) (*Repetition*) How many different ways are there for Homer Simpson to choose a dozen donuts from the 21 varieties at a donut shop?
   $n = 21, r = 12$
   $C(n+r-1,r) = \frac{(n+r-1)!}{r!(n-1)!} = \frac{(21+12-1)!}{12!(21-1)!} = \frac{32!}{12!20!}$

3. What is the binomial coefficient of x^4y^5 in the expansion of $(x+y)^9$? Simplify your answer to a single value.

$$\begin{aligned}\binom{9}{5} &= \frac{9!}{5!(9-5)!} \\ &= \frac{9!}{5!4!} \\ &= \frac{9*8*7*6*5}{5*4*3*2*1} \\ &= \frac{3*2*7*3*1}{1*1*1*1*1} \\ &= 126\end{aligned}$$