Quiz 1 Solutions

1. List all the steps (including any comparisons) used to search for 9 in the sequence $1, 3, 4, 5, 6, 8, 9, 11$ using

   a) a linear search

      i. compare 9 to 1, $i = 0$
      ii. compare 9 to 3, $i = 1$
      iii. compare 9 to 4, $i = 2$
      iv. compare 9 to 5, $i = 3$
      v. compare 9 to 6, $i = 4$
      vi. compare 9 to 8, $i = 5$
      vii. compare 9 to 9, $i = 6$
      viii. return i (6)

   b) a binary search

      i. find the middle element of $\{1, 3, 4, 5, 6, 8, 9, 11\}$, rounding down (5), $i = 3$
      ii. compare 9 to 5 (greater)
      iii. find the middle element of $\{6, 8, 9, 11\}$, rounding down (8), $i = 5$
      iv. compare 9 to 8 (greater)
      v. find the middle element of $\{9, 11\}$, rounding down (9), $i = 6$
      vi. compare 9 to 9
      vii. return this i (6)

2. Use the insertion sort to sort the list in $3, 1, 5, 7, 4$, showing the lists obtained at each step.

   a) $3|1, 5, 7, 4$
   b) $1, 3|5, 7, 4$
   c) $1, 3, 5|7, 4$
   d) $1, 3, 4, 5|7$
   e) $1, 3, 4, 5, 7$

3. Give a big-O estimate for each of these functions. For the function g in your estimate $f(x)$ is $O(g(x))$, use a simple function g of smallest order.

   a) $n \log(n^2 + 1) + n^2 \log n$
   $O(n^2 \log n)$
   b) $(n \log n + 1)^2 + (\log n + 1)(n^2 + 1)$
   $O(n^2 (\log n)^2)$
   c) $n^{2^n} + n^{n^2}$
   $O(n^{2^n})$

4. Arrange the following functions in a list so that each function is big-O of the next function:
   $$\log n, n^n, n \log n, n^{10}, n, 10^n, 1, n!$$
   $1, \log n, n, n \log n, n^{10}, 10^n, n!, n^n$

5. Give a big-O estimate for the number of operations, where an operation is an addition or a multiplication, used in this segment of an algorithm.

   $m := 0$
   for $i := 1 \to n$ **do**
   for $j := i + 1 \to n$ **do**
   $m := \max(a_i a_j, m)$

   $O(n^2)$