Homework 7
Due October 31, 2013

CMSC 203

1. Section 6.1, Exercise 4
   A particular brand of shirt comes in 12 colors, has a male version and a female version, and comes in three sizes for each sex. How many different types of this shirt are made?
   $12 * 2 * 3 = 72$

2. Section 6.1, Exercise 16
   How many strings are there of four lowercase letters that have the letter "x" in them?
   $26 * 26 * 26 = 17,576$

3. Section 6.1, Exercise 28
   How many license plates can be made using either three digits followed by three uppercase English letters, or three uppercase English letters followed by three digits?
   $10 * 10 * 10 * 26 * 26 * 26 + 26 * 26 * 26 * 10 * 10 * 10 = 17,576,000 + 17,576,000 = 35,152,000$

4. Section 6.2, Exercise 8
   Show that if f is a function from S to T, where S and T are finite sets with $|S| > |T|$, then there are elements s_1 and s_2 in S such that $f(s_1) = f(s_2)$, or in other words, f is not one-to-one.
   If $|S| > |T|$ then when applying the pigeonhole principle, $N = |S|$ and $k = |T|$ (since we mapping a set of $|S|$ items into a set of $|T|$ items). Since $|S| > |T|$, we know $N > k$, so $\frac{N}{k} > 1$. Therefore, $\lceil \frac{N}{k} \rceil \geq 2$. This means that there must be at least two images that are mapped to by the same pre-image, so it is not one-to-one.

5. Section 6.2, Exercise 14

   a) Show that if seven integers are selected from the first 10 positive integers, there must be at least two pairs of these integers with the sum of 11.
   Possible pairs of numbers that sum to 11 are: $\{(1,10),(2,9),(3,8),(4,7),(5,6)\}$
   Therefore, we have $k = 5$ value pairs to choose from and $N = 7$ values to choose
   We must select one pair, which the pigeonhole principle states will be $\lceil \frac{N}{k} \rceil = \lceil \frac{7}{5} \rceil = \lceil 1.4 \rceil = 2$
   Therefore, at least one set will have two elements (ie: a pair). However, we must have at least two pairs, so we must perform another selection assuming we correctly chose a pair with our first two options. In other words, $k = 4$ for the remaining pairs and $N = 5$ for the remaining selections. Thus, $\lceil \frac{5}{4} \rceil = \lceil 1.25 \rceil = 2$. Therefore, we can pick another pair, so we may find at least two pairs from selecting the first 7 integers.

   b) Is the conclusion in part (a) true if six integers are selected rather than seven? No. Following the previous logic, we first have $k = 5$ and $N = 6$, which is $\lceil \frac{6}{5} \rceil = \lceil 1.2 \rceil = 2$, so we may have at least one pair. We attempt to pick another pair with the remainder, $k = 4$ and $N = 4$ to have $\lceil \frac{4}{4} \rceil = \lceil 1 \rceil = 1$, which does not guarantee we will have another pair.

6. Section 6.2, Exercise 34
   Assuming that no one has more than 1,000,000 hairs on the head of any person and that the population of New York City was 8,008,278 in 2010, show there had to be at least nine people in New York City in 2010 with the same number of hairs on their heads.
   We have $N = 8,008,278$ people to assign different quantities of hairs from $0 \rightarrow 1,000,000$, which is $k = 1,000,001$.
   $\lceil \frac{N}{k} \rceil = \lceil \frac{8008278}{1000001} \rceil \approx \lceil 8.008 \rceil = 9$

7. Section 6.3, Exercise 6
   Find the value of each of these quantities.

a) $C(5,1) = \frac{5!}{1!(5-1)!} = \frac{5}{1*1} = 5$

b) $C(5,3) = \frac{5!}{3!(5-3)!} = \frac{5*4}{2*1} = 10$

c) $C(8,4) = \frac{8!}{4!(8-4)!} = \frac{8*7*6*5}{4*3*2*1} = 2*7*1*5 = 70$

d) $C(8,8) = \frac{8!}{8!(8-8)!} = \frac{1}{1*1} = 1$

e) $C(8,0) = \frac{8!}{0!(8-0)!} = \frac{1!}{1*1} = 1$

f) $C(12,6) = \frac{12!}{6!(12-6)!} = \frac{12*11*10*9*8*7}{6*5*4*3*2*1} = 2*11*2*3*1*7 = 9$

8. Section 6.3, Exercise 16
How many subsets with an odd number of elements does a set with 10 elements have?
Since counting subsets is done using a combination, and the r for each combination is the number of elements in it, we must find all combinations such that $C(10,1) + C(10,3) + C(10,5) + C(10,7) + C(10,9)$. Also, since $C(n,r) = C(n,n-r)$ we know that $C(10,1) = C(10,9)$ and $C(10,3) = C(10,7)$. Therefore, $2*C(10,1) + 2*C(10,3) + C(10,5) = 2*10 + 2*120 + 252 = 512$

9. Section 6.3, Exercise 22
How many permutations of the letters $ABCDEFGH$ contain

a) the string ED? $P(7,7) = 7! = 5040$

b) the string CDE? $P(6,6) = 6! = 720$

c) the strings BA and FGH? $P(5,5) = 5! = 120$

d) the strings AB, DE, and GH? $P(5,5) = 5! = 120$

e) the strings CAB and BED? $P(4,4) = 4! = 24$

f) the strings BCA and ABF? Since there is no possible way to have BCA and ABF, there are 0

10. Section 6.3, Exercise 30
Seven women and nine men are on the faculty in the mathematics department at a school.

a) How many ways are there to select a committee of five members of the department if at least one woman must be on the committee?
$C(7,1)C(9,4)+C(7,2)C(9,3)+C(7,3)C(9,2)+C(7,4)C(9,1)+C(7,5) = 7*126+21*84+35*36+35*9+21 = 882+1764+1260+315+21 = 4242$

b) How many ways are there to select a committee of five members of the department if at least one women and at least one man must be on the committee?
$C(7,1)C(9,4) + C(7,2)C(9,3) + C(7,3)C(9,2) + C(7,4)C(9,1) = 7*126 + 21*84 + 35*36 + 35*9 + 21 = 882 + 1764 + 1260 + 315 = 4221$