Homework 6
Due October 24, 2013

CMSC 203

1. Section 5.1, Exercise 4
   Let $P(n)$ be the statement that $1^3 + 2^3 + \ldots + n^3 = (n(n+1)/2)^2$ for the positive integer n.
   a) What is the statement $P(1)$?
   b) Show that $P(1)$ is true, completing the basis step of the proof.
   c) What is the inductive hypothesis?
   d) What do you need to prove in the inductive step?
   e) Complete the inductive step, identifying where you use the inductive hypothesis.
   f) Explain why these steps show that this formula is true whenever n is a positive integer.

2. Section 5.1, Exercise 20
   Prove that $3^n < n!$ if n is an integer greater than 6.

3. Section 5.1, Exercise 50
   What is wrong with this "proof"?
   "*Theorem*" For every positive integer n, $\sum_{i=1}^{n} i = (n + \frac{1}{2})^2/2$

   Basis Step: The formula is true for $n = 1$.

   Inductive Step: Suppose that $\sum_{i=1}^{n} i = (n+\frac{1}{2})^2/2$. Then $\sum_{i=1}^{n+1} i = (\sum_{i=1}^{n} i)+(n+1)$. By the inductive hypothesis, $\sum_{i=1}^{n+1} i = (n + \frac{1}{2})^2/2 + n + 1 = (n^2 + n + \frac{1}{4})/2 + n + 1 = (n^2 + 3n + \frac{9}{4})/2 = (n + \frac{3}{2})^2/2 = [(n + 1) + \frac{1}{2}]^2/2$, completing the inductive step.

4. Section 5.2, Exercise 12
   Use strong induction to show that every positive integer n can be written as a sum of distinct powers of two, that is, as a sum of a subset of integers $2^0 = 1, 2^1 = 2, 2^2 = 4$, and so on. [*Hint*: For the inductive step, separately consider the case where $k + 1$ is even and where it is odd. When it is even, note that $(k + 1)/2$ is an integer]

5. Section 5.2, Exercise 32
   Find the flaw with the following "proof" that every postage of three cents or more can be formed using just three-cent and four-cent stamps.

   Basis Step: We can form postage of three cents with a single three-cent stamp and we can form postage of four cents using a single four-cent stamp.

   Inductive Step: Assume that we can form postage of j cents for all nonnegative integers j with $j \leq k$ using just three-cent and four-cent stamps. We can then form postage of $k + 1$ cents by replacing one three-cent stamp with a four-cent stamp or by replacing two four-cent stamps by three three-cent stamps.

6. Section 5.4, Exercise 10
   Give a recursive algorithm for finding the maximum of a finite set of integers, making use of the fact that the maximum of n integers is the larger of the last integer in the list and the maximum of the first $n - 1$ integers in the list.

7. Section 5.4, Exercise 22
   Prove that the recursive algorithm you found above is correct.
8. Section 5.4, Exercise 44
   Use a merge sort to sort $4, 3, 2, 5, 1, 8, 7, 6$ into increasing order. Show all steps used by the algorithm.
9. Section 5.4, Exercise 50
   Sort $3, 5, 7, 8, 1, 9, 2, 4, 6$ using the quick sort.