Homework 5
Due October 17, 2013

CMSC 203

1. Section 3.1, Exercise 14
   List all the steps (including any comparisons) used to search for 7 in the sequence $1, 3, 4, 5, 6, 8, 9, 11$ using
   a) a linear search
   b) a binary search
2. Section 3.1, Exercise 24
   Describe an algorithm that determines whether a function from a finite set of integers to another finite set of integers is one-to-one.
3. Section 3.1, Exercise 34
   Use the bubble sort to sort $6, 2, 3, 1, 5, 4$, showing the lists obtained at each step.
4. Section 3.1, Exercise 38
   Use the insertion sort to sort the list in Exercise 34, showing the lists obtained at each step.
5. Section 3.2, Exercise 2
   Determine whether each of these functions is $O(x^2)$.
   a) $f(x) = 17x + 11$
   b) $f(x) = x^2 + 1000$
   c) $f(x) = x \log x$
   d) $f(x) = x^4/2$
   e) $f(x) = 2^x$
   f) $f(x) = \lfloor x \rfloor \cdot \lceil x \rceil$
6. Section 3.2, Exercise 4
   Use the definition of "$f(x)$ is $O(g(x))$" to show that $2^x + 17$ is $O(3^x)$. Include the witness c and k.
7. Section 3.2, Exercise 14abce
   Determine whether x^3 is $O(g(x))$ for each of these functions $g(x)$.
   a) $g(x) = x^2$
   b) $g(x) = x^3$
   c) $g(x) = x^2 + x^3$
   e) $g(x) = 3^x$
8. Section 3.2, Exercise 26
   Give a big-O estimate for each of these functions. For the function g in your estimate $f(x)$ is $O(g(x))$, use a simple function g of smallest order.
   a) $(n^3 + n^2 \log n)(\log n + 1) + (17 \log n + 19)(n^3 + 2)$
   b) $(2^n + n^2)(n^3 + 3^n)$

c) $(n^n + n2^n + 5^n)(n! + 5^n)$

9. Section 3.3, Exercise 12a
   Consider the following algorithm , which takes as input a sequence of n integers $a_1, a_2, \ldots, a_n$ and produces as output a matrix $\mathbf{M} = \{m_{ij}\}$ where m_{ij} is the minimum term in the sequences of integers $a_i, a_{i+1}, \ldots, a_j$ for $j \geq i$ and $m_{ij} = 0$ otherwise.

   initialize **M** so that $m_{ij} = a_i$ if $j \geq i$ and $m_{ij} = 0$ otherwise
   for $i := 1 \to n$ **do**
       for $j := i + 1 \to n$ **do**
           for $k := i + 1 \to j$ **do**
               $m_{ij} := \min(m_{ij}, a_k)$
   return $\mathbf{M} = \{m_{ij}\}\{m_{ij}$ is the minimum term of $a_i, a_{i+1}, \ldots, a_j\}$

   a) Show that this algorithm uses $O(n^3)$ comparisons to compute the matrix **M**.

10. Chapter 3, Supplementary Exercise 26
    Arrange the functions $2^{100n}, 2^{n^2}, 2^{n!}, 2^{2^n}, n^{\log n}, n \log n \log \log n, n^{3/2}, n(\log n)^{3/2},$ and $n^{4/3}(\log n)^2$ in a list so that each function is big-O of the next function. [*Hint:* To determine the relative size of some of these functions, take logarithms.]