Homework 4 Rubric

CMSC 203

1. Section 2.1, Exercise 2
   Use set builder notation to give a description of each of these sets.

   a) $\{0, 3, 6, 9, 12\}$
   $\{x \in \mathbb{N} | 0 \leq x \leq 12\}, x mod 3 = 0$
   $\{x \in \mathbb{N} | 0 \leq x \leq 12\}, x = 3k$ for some k

   b) $\{-3, -2, -1, 0, 1, 2, 3\}$
   $\{x \in \mathbb{Z} | -3 \leq x \leq 3\}$

   c) $\{m, n, o, p\}$
   $\{x \in$ alphabet $|$ letter l $< x <$ letter q$\}$ (or some form thereof)

2. Section 2.1, Exercise 9
   Determine whether each of these statements is true or false.

   a) $\emptyset \in \{\emptyset\}$
   True

   b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$
   True

   c) $\{\emptyset\} \in \{\emptyset\}$
   False

   d) $\{\emptyset\} \in \{\{\emptyset\}\}$
   True

   e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
   True

   f) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$
   True

   g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$
   False

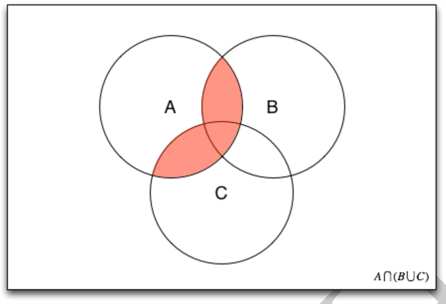
3. Section 2.2, Exercise 12
   Prove the first absorption law from Table 1 by showing that if A and B are sets, then $A \cup (A \cap B) = A$.
   Proving via venn diagram is acceptable. Additionally, you may prove equality by showing $A \cup (A \cap B) \subseteq A$ and $A \subseteq A \cup (A \cap B)$ by proving if some element x is an element in the former then it must exist in the latter for both problems. Below, is the preferred method using set builder notation:

   Let $A \cup (A \cap B) = \{x | x \in A \vee x \in A \cap B\}$
   $= \{x | x \in A \vee (x \in A \wedge x \in B)\}$
   Let $p(x) = x \in A$ and $q(x) = x \in B$
   $= \{x | p(x) \vee (p(x) \wedge q(x))$
   $= \{x | p(x)\}$
   $= \{x | x \in A\}$
   $= A$

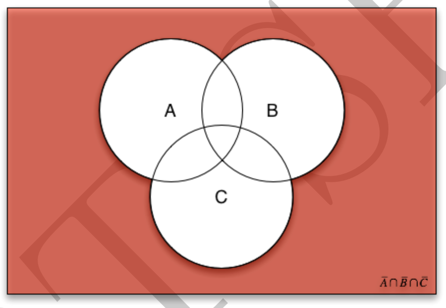
4. Section 2.2, Exercise 14
Find the sets A and B if $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$, $A \cap B = \{3, 6, 9\}$.
A = {1,3,5,6,7,8,9}; B = {2,3,6,9,10}

5. Section 2.2, Exercise 26
Draw the Venn diagrams for each of these combinations of the sets A, B, and C.

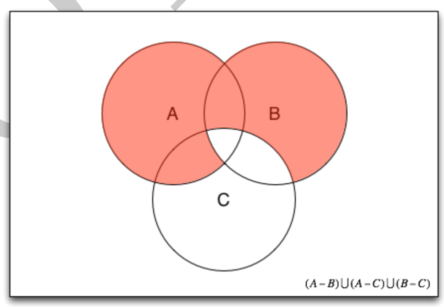
   a) $A \cap (B \cup C)$



   b) $\bar{A} \cap \bar{B} \cap \bar{C}$



   c) $(A - B) \cup (A - C) \cup (B - C)$



6. Section 2.2, Exercise 30
Can you conclude that $A = B$ if A, B, and C are sets such that

   a) $A \cup C = B \cup C$?
   No, $A \subset C$ and $B \subset C$, but A and B may be disjoint
   b) $A \cap C = B \cap C$?
   No, A and B may both be disjoint with C and with each other
   c) $A \cup C = B \cup C$ and $A \cap C = B \cap C$?
   Yes, this can be demonstrated easily with a proof by contradiction given the two prior conditions

7. Section 2.3, Exercise 4
Find the domain and range of these functions. Note that in each case, to find the domain, determine the set of elements assigned values by the function.

a) the function that assigns to each nonnegative integer its last digit

Domain: $\{x \in \mathbb{N} | x \geq 0\}$
Range: $\{x \in \mathbb{N} | x < 10\}$

b) the function that assigns the next largest integer to a positive integer

Domain: $\{x | x \in \mathbb{Z}^+\}$
Range: $\{x \in \mathbb{Z}^+ | x > 1\}$

c) the function that assigns to a bit string the number of one bits in the string

Domain: $\{x | x \text{ is a bit string}\}$
Range: $\{x | x \in \mathbb{N}\}$

d) the function that assigns to a bit string the number of bits in the string

Domain: $\{x | x \text{ is a bit string}\}$
Range: $\{x | x \in \mathbb{Z}^+\}$

8. Section 2.3, Exercise 12
Determine whether each of these functions from $\mathbb{Z}$ to $\mathbb{Z}$ is one-to-one.

a) $f(n) = n - 1$
Yes

b) $f(n) = n^2 + 1$
No (for example, -2 and 2 both map to the same value in $\mathbb{Z}$)

c) $f(n) = n^3$
Yes

d) $f(n) = \lceil n/2 \rceil$
No (for example, 2 and 1 both map to the same value in $\mathbb{Z}$)

9. Section 2.3, Exercise 14
Determine whether $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is onto if

a) $f(m, n) = 2m - n$.
Yes

b) $f(m, n) = m^2 - n^2$.
No (for example, no combination of m and n map to 2)

c) $f(m, n) = m + n + 1$.
Yes

d) $f(m, n) = |m| - |n|$.
Yes

e) $f(m, n) = m^2 - 4$.
No (no m can map to negative values less than -4)

10. Section 2.3, Exercise 22
Determine whether each of these functions is a bijection from $\mathbb{R}$ to $\mathbb{R}$.

a) $f(x) = -3x + 4$
Yes

b) $f(x) = -3x^2 + 7$
No, it is not one-to-one

c) $f(x) = (x + 1)/(x + 2)$
No, it can not map to negative numbers or zero (so it is not onto)

d) $f(x) = x^5 + 1$
Yes