Homework 4
Due October 1, 2013

CMSC 203

1. Section 2.1, Exercise 2
   Use set builder notation to give a description of each of these sets.

   a) $\{0, 3, 6, 9, 12\}$

   b) $\{-3, -2, -1, 0, 1, 2, 3\}$

   c) $\{m, n, o, p\}$

2. Section 2.1, Exercise 9
   Determine whether each of these statements is true or false.

   a) $\emptyset \in \{\emptyset\}$

   b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$

   c) $\{\emptyset\} \in \{\emptyset\}$

   d) $\{\emptyset\} \in \{\{\emptyset\}\}$

   e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$

   f) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$

   g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$

3. Section 2.2, Exercise 12
   Prove the first absorption law from Table 1 by showing that if A and B are sets, then $A \cup (A \cap B) = A$.

4. Section 2.2, Exercise 14
   Find the sets A and B if $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$, $A \cap B = \{3, 6, 9\}$.

5. Section 2.2, Exercise 26
   Draw the Venn diagrams for each of these combinations of the sets A, B, and C.

   a) $A \cap (B \cup C)$

   b) $\bar{A} \cap \bar{B} \cap \bar{C}$

   c) $(A - B) \cup (A - C) \cup (B - C)$

6. Section 2.2, Exercise 30
   Can you conclude that $A = B$ if A, B, and C are sets such that

   a) $A \cup C = B \cup C$?

   b) $A \cap C = B \cap C$?

   c) $A \cup C = B \cup C$ and $A \cap C = B \cap C$?

   Explain your reasoning

7. Section 2.3, Exercise 4
   Find the domain and range of these functions. Note that in each case, to find the domain, determine the set of elements assigned values by the function.

a) the function that assigns to each nonnegative integer its last digit

b) the function that assigns the next largest integer to a positive integer

c) the function that assigns to a bit string the number of one bits in the string

d) the function that assigns to a bit string the number of bits in the string

8. Section 2.3, Exercise 12
   Determine whether each of these functions from $\mathbb{Z}$ to $\mathbb{Z}$ is one-to-one.

   a) $f(n) = n - 1$

   b) $f(n) = n^2 + 1$

   c) $f(n) = n^3$

   d) $f(n) = \lceil n/2 \rceil$

9. Section 2.3, Exercise 14
   Determine whether $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is onto if

   a) $f(m, n) = 2m - n$.

   b) $f(m, n) = m^2 - n^2$.

   c) $f(m, n) = m + n + 1$.

   d) $f(m, n) = |m| - |n|$.

   e) $f(m, n) = m^2 - 4$.

10. Section 2.3, Exercise 22
    Determine whether each of these functions is a bijection from $\mathbb{R}$ to $\mathbb{R}$.

    a) $f(x) = -3x + 4$

    b) $f(x) = -3x^2 + 7$

    c) $f(x) = (x + 1)/(x + 2)$

    d) $f(x) = x^5 + 1$