Homework 3
Due September 19, 2013

CMSC 203

1. Section 1.6, Exercise 4 *(modified)*
   What rule of inference is used in each of these arguments?

   a) Kangaroos live in Australia. Kangaroos are marsupials. Therefore, kangaroos are Australian and marsupials.
      Conjunction

   b) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.
      Disjunctive syllogism

   c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.
      Modus ponens

   d) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.
      Hypothetical syllogism

2. Section 1.7, Exercise 2
   Use a direct proof to show that the sum of two even integers is even. Assume even number n
   Using the axiom of even numbers, we know $n = 2k$, where k is some value
   $n + n = 2k + 2k\prime$
   $n + n = 2(k + k\prime)$ by factoring out 2
   $n + n = 2k\prime\prime$ by substituting $k + k\prime = k\prime\prime$
   $\therefore n + n$ is an even number

3. Section 1.7, Exercise 18
   Prove that if n is an integer and $3n + 2$ is even, then n is even using

   a) a proof by contraposition.
      Assume n is odd
      Using axioms of odd numbers, we know $n = 2k + 1$ where k is some value
      $3(2k + 1) + 2$ by substituting n into $3n + 2$
      $6k + 3 + 2$
      $6k + 5$
      $2(3k) + 5$
      $2k\prime + 5$, which is an even number plus an odd number
      Using axioms of math, we know that an even number plus an odd number is odd
      This shows $2k\prime + 5$ is odd, which means that our proposition "$3n + 2$ is even" is false
      $\therefore$ our proposition is true, since $\neg q \to \neg p$

   b) a proof by contradiction. Assume $3n + 2$ is even and n is odd
      Because n is odd, and the product of two odd numbers is odd, $3n$ must be odd
      $3n + 2 = 3n + 2$ is true
      $2 = (3n + 2) - 3n$ is an even number ($3n + 2$, which is given above) minus an odd number ($3n$)

An even number minus an odd number is odd, which is contradictory since 2 is not odd
$\therefore$ our original supposition that $3n + 2$ is even and n is odd is false

4. Section 1.7, Exercise 22
Show that if you pick three socks from a drawer containing just blue socks and black socks, you must get either a pair of blue socks or a pair of black socks.
Exhaustive Proof:
Pick socks one at a time, and assume that picking 2 of the same color produces a pair:

Pick 1	Pick 2	Pick 3	Pair?
Blue	Blue	Blue	Yes (blue)
Blue	Blue	Black	Yes (blue)
Blue	Black	Blue	Yes (blue)
Blue	Black	Black	Yes (black)
Black	Blue	Blue	Yes (blue)
Black	Blue	Black	Yes (black)
Black	Black	Blue	Yes (black)
Black	Black	Black	Yes (black)

Proof by Contradiction:
Assume we picked three socks, but do not have a pair of either color
To not have a pair of either color, we must have no two socks that are the same color
If no two socks are the same color, each sock must be a different color
Since we picked three socks, we must have 3 different colors
However, this is contradictory to our statement that the drawer just contains blue and black socks
$\therefore$ our original supposition that we do not have a pair must be false

5. Section 1.8, Exercise 4
Use a proof by cases to show that $\min(a, \min(b, c)) = \min(\min(a, b), c)$ whenever a, b, and c are real numbers.

$$\begin{aligned}
\text{if } a \le b \le c:\ & \min(a, \min(b, c)) = \min(a, b) = a \\
& \min(\min(a, b), c) = \min(a, c) = a \\
\text{if } a \le c \le b:\ & \min(a, \min(b, c)) = \min(a, c) = a \\
& \min(\min(a, b), c) = \min(a, c) = a \\
\text{if } b \le a \le c:\ & \min(a, \min(b, c)) = \min(a, b) = b \\
& \min(\min(a, b), c) = \min(b, c) = b \\
\text{if } b \le c \le a:\ & \min(a, \min(b, c)) = \min(a, b) = b \\
& \min(\min(a, b), c) = \min(b, c) = b \\
\text{if } c \le a \le b:\ & \min(a, \min(b, c)) = \min(a, c) = c \\
& \min(\min(a, b), c) = \min(a, c) = c \\
\text{if } c \le b \le a:\ & \min(a, \min(b, c)) = \min(a, c) = c \\
& \min(\min(a, b), c) = \min(b, c) = c
\end{aligned}$$

this covers all possible cases relating a, b, and c
$\therefore$ in all cases, $\min(a, \min(b, c)) = \min(\min(a, b), c)$

6. Section 1.8, Exercise 8
Prove that there is a positive integer that equals the sum of the positive integers not exceeding it. Is your proof constructive or nonconstructive?
Note: "Not exceeding" means all the positive numbers less or equal to that number
Depending on interpretation, $n = 1$ or $n = 3$

If "not exceeding" means $\le$, then $n = 1$:
the set of positive integers $\le 1 = \{1\}$
the sum of the elements in this set is trivially $1 = n$

If "not exceeding" means $<$, then $n = 3$:
the set of positive integers $\leq 3 = \{1, 2\}$
the sum of the elements in this set is $1 + 2 = 3 = n$
in either case, we have shown that there exists such an n (students need not show both cases, either is sufficient)
$\therefore$ there is a positive integer that equals the sum of the positive integers not exceeding it

This is a constructive proof

7. Section 1.8 Exercise 16
Show that if a, b, and c are real numbers and $a \neq 0$ then there is a unique solution of the equation $ax + b = c$.
Existence:
    $ax + b = c$
    $ax = c - b$
    $x = (c - b)/a$
    Thus, some value x does exist
Uniqueness:
    Presume a real number s
    $as + b = ax + b$ where $x = (c - b)/a$
    $as = ax$ by subtracting b
    $s = x$ by dividing by a
$\therefore$ s is equal to x, so x is a unique solution