Homework 2
Rubric

CMSC 203

These are the answers to the homework problems

Propositional Logic

1. Section 1.1, exercise 12
   Let p, q, and r be the propositions
   p: You have the flu.
   q: You miss the final examination.
   r: You pass the course.
   Express each of these propositions as an English sentence.

   a) $p \rightarrow q$
   If you have the flu, then you will miss the final examination.

   b) $\neg q \leftrightarrow r$
   Attending the final exam is necessary and sufficient for passing the course.
   or: **You don't miss the final exam if and only if you pass the course.**

   c) $q \rightarrow \neg r$
   If you miss the final exam, then you will not pass the course.

   d) $p \vee q \vee r$
   You have the flu, or miss the final exam, or pass the course.

   e) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$
   If you have the flu, you will not pass the course; or if you miss the final exam, you will not pass the course.
   or: **If you have the flu and miss the final exam, you will not pass the course.**

   f) $(p \wedge q) \vee (\neg q \wedge r)$
   You have the flu and miss the final exam, or you do not miss the final exam and pass the course.

   g) $p \wedge (q \vee r)$
   You miss the final exam or pass the course (or both), and have the flu.
   or: **You have flu and miss the exam, or you have the flu and pass the course (or both).**

2. Section 1.1, exercise 14
   Let p, q, and r be the propositions
   p: You get an A on the final exam.
   q: You do every exercise in this book.
   r: You get an A in this class.
   Write these propositions using p, q, and r and logical connectives (including negations)

   a) You get an A in this class, but you do not do every exercise in this book.
   Answer: $r \wedge \neg q$

b) You get an A on the final, you do every exercise in this book, and you get an A in this class.
Answer: $p \wedge q \wedge r$

c) To get an A in this class, it is necessary for you to get an A on the final.
Answer: $p \rightarrow r$

d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.
Answer: $r(p \wedge \neg q)$

e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
Answer: $(p \wedge q) \rightarrow r$

f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.
Answer: $r \leftrightarrow (q \vee p)$

3. Section 1.3, exercise 8
Use De Morgan's laws to find the negation of each of the following statements.

a) Johnny will take a job in industry or go to graduate school.
Johnny will not take a job in industry and he will not go to graduate school

b) Eric knows Java and calculus.
Eric does not know Java or he does not know calculus.
Note: It is possible for Eric to know Java but not calculus, and vice versa

c) James is young and strong.
James is not young or he is not strong.
Note: It is possible for James to be not young but strong, and vice versa. Also, ¬young does not mean old and ¬strong does not mean weak

d) Marge will move to Oregon or Washington.
Marge will not move to Oregon and will not move to Washington.
or: **Marge will neither move to Oregon nor Washington.**

4. Section 1.3, exercise 10
Show that each of these conditional statements is a tautology by using truth tables.

a) $[\neg p \wedge (p \vee q)] \rightarrow q$

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \rightarrow q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$p \rightarrow r$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

c) $[p \wedge (p \to q)] \to q$

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

d) $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$(p \vee q) \wedge (p \to r) \wedge (q \to r)$	$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	T

Predicate Logic

5. Section 1.4, Exercise 14
   Determine the truth value of each of these statements if the domain consists of all real numbers.

   a) $\exists x(x^3 = -1)$
   True

   b) $\exists x(x^4 < x^2)$
   True

   c) $\forall x((-x)^2 = x^2)$
   True

   d) $\forall x(2x > x)$
   False

6. Section 1.4, Exercise 62 (a-d)
   Let $P(x)$, $Q(x)$, $R(x)$, and $S(x)$ be the statements "x is a duck," "x is one of my poultry," "x is an officer," and "x is willing to waltz," respectively. Express each of these statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, $R(x)$, and $S(x)$.

   a) No ducks are willing to waltz.
   Answer 1: $\forall x \neg(P(x) \wedge S(x))$
   Answer 2: $\forall x \neg P(x) \vee \neg S(x)$
   Answer 3: $\neg(\exists x P(x) \wedge S(x))$
   Answer 4: $\forall x P(x) \to \neg S(x)$
   Note: These are all equivalent statements

   b) No officers ever decline to waltz.
   Answer 1: $\forall x \neg(R(x) \wedge \neg S(x))$
   Answer 2: $\forall x \neg R(x) \vee S(x))$
   Answer 3: $\neg(\exists x R(x) \wedge \neg S(x))$
   Note: These are all equivalent statements

   c) All my poultry are ducks.
   Answer: $\forall x Q(x) \to P(x)$

d) My poultry are not officers.
Answer: $\forall x Q(x) \rightarrow \neg R(x)$

7. Section 1.5, Exercise 24
Translate each of these nested quantifications into an English statement that expresses a mathematical fact. The domain in each case consists of all real numbers.

a) $\exists x \forall y(x + y = y)$
There is a number that, when added to any other number, will equal that other number
or: **There exists an x such that when x is added to any number y, y does not change.**

b) $\forall x \forall y(((x \geq 0) \wedge (y < 0)) \rightarrow (x - y > 0))$
For all values of x and y, if x is greater than or equal to zero (non-negative) and y is negative, then $x - y$ is greater than zero (positive)
or: **Any non-negative number minus any negative number is a positive number**

c) $\exists x \exists y(((x \leq 0) \wedge (y \leq 0)) \wedge (x - y > 0))$
There is an x and y value where x is less than or equal to zero (non-positive) and y is less than or equal to zero (non-positive) and $x - y$ is greater than zero (positive)
or: **There exists two non-positive numbers that, $x - y$ is positive**

d) $\forall x \forall y((x \neq 0) \wedge (y \neq 0) \leftrightarrow (xy \neq 0))$
For all values of x and y, x and y are not 0 if and only if x times y is not zero
or: **For all values x and y, it is necessary and sufficient for neither x nor y to be zero if their product is non-zero**
or: **The only way for xy to be nonzero is if x and y are both nonzero.**

8. Section 1.5, Exercise 26
Let $Q(x, y)$ be the statement "$x + y = x - y$." If the domain for both variables consists of all integers, what are the truth values?

a) $Q(1, 1)$
False

b) $Q(2, 0)$
True

c) $\forall y Q(1, y)$
False

d) $\exists x Q(x, 2)$
False

e) $\exists x \exists y Q(x, y)$
True

f) $\forall x \exists y Q(x, y)$
True

g) $\exists y \forall x Q(x, y)$
True

h) $\forall y \exists x Q(x, y)$
False

i) $\forall x \forall y Q(x, y)$
False