# Homework 2
# Due September 12, 2013

CMSC 203

*Please write all answers on separate sheets of paper. Show your work!*

## Propositional Logic

1. Section 1.1, exercise 12
   Let $p$, $q$, and $r$ be the propositions
   $p$: You have the flu.
   $q$: You miss the final examination.
   $r$: You pass the course.
   Express each of these propositions as an English sentence.

   a) $p \rightarrow q$

   b) $\neg q \leftrightarrow r$

   c) $q \rightarrow \neg r$

   d) $p \vee q \vee r$

   e) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$

   f) $(p \wedge q) \vee (\neg q \wedge r)$

   g) $p \wedge (q \vee r)$

2. Section 1.1, exercise 14
   Let $p$, $q$, and $r$ be the propositions
   $p$: You get an A on the final exam.
   $q$: You do every exercise in this book.
   $r$: You get an A in this class.
   Write these propositions using $p$, $q$, and $r$ and logical connectives (including negations)

   a) You get an A in this class, but you do not do every exercise in this book.

   b) You get an A on the final, you do every exercise in this book, and you get an A in this class.

   c) To get an A in this class, it is necessary for you to get an A on the final.

   d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.

   e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

   f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

3. Section 1.3, exercise 8
   Use De Morgan's laws to find the negation of each of the following statements.

   a) Johnny will take a job in industry or go to graduate school.

   b) Eric knows Java and calculus.

c) James is young and strong.

d) Marge will move to Oregon or Washington.

4. Section 1.3, exercise 10
   Show that each of these conditional statements is a tautology by using truth tables.

   a) $[\neg p \wedge (p \vee q)] \rightarrow q$

   b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

   c) $[p \wedge (p \rightarrow q)] \rightarrow q$

   d) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

# Predicate Logic

5. Section 1.4, Exercise 14
   Determine the truth value of each of these statements if the domain consists of all real numbers.

   a) $\exists x(x^3 = -1)$

   b) $\exists x(x^4 < x^2)$

   c) $\forall x((-x)^2 = x^2)$

   d) $\forall x(2x > x)$

6. Section 1.4, Exercise 62 (a-d)
   Let $P(x)$, $Q(x)$, $R(x)$, and $S(x)$ be the statements "$x$ is a duck," "$x$ is one of my poultry," "$x$ is an officer," and "$x$ is willing to waltz," respectively. Express each of these statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, $R(x)$, and $S(x)$.

   a) No ducks are willing to waltz.

   b) No officers ever decline to waltz.

   c) All my poultry are ducks.

   d) My poultry are not officers.

7. Section 1.5, Exercise 24
   Translate each of these nested quantifications into an English statement that expresses a mathematical fact. The domain in each case consists of all real numbers.

   a) $\exists x \forall y(x + y = y)$

   b) $\forall x \forall y(((x \geq 0) \wedge (y < 0)) \rightarrow (x - y > 0))$

   c) $\exists x \exists y(((x \leq 0) \wedge (y \leq 0)) \wedge (x - y > 0))$

   d) $\forall x \forall y((x \neq 0) \wedge (y \neq 0) \leftrightarrow (xy \neq 0))$

8. Section 1.5, Exercise 26
   Let $Q(x, y)$ be the statement "$x + y = x - y$." If the domain for both variables consists of all integers, what are the truth values?

   a) $Q(1, 1)$

   b) $Q(2, 0)$

   c) $\forall y Q(1, y)$

   d) $\exists x Q(x, 2)$

   e) $\exists x \exists y Q(x, y)$

   f) $\forall x \exists y Q(x, y)$

   g) $\exists y \forall x Q(x, y)$

   h) $\forall y \exists x Q(x, y)$

   i) $\forall x \forall y Q(x, y)$