CMSC 203
Exam 1
October 3, 2013

If you find yourself spending too long on a problem, skip it and move on to the next one. For all problems, partial credit will be given for incorrect answers only if an explanation is included.

Page	Points	Score
2	5	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	15	
Total:	120	

1 Gimme Points (5 bonus points)

0. Draw a picture of this class (CMSC 203). It can be any picture, including the content we cover, the class itself, your emotions about the class, etc. Be imaginative! (But don't waste too much time on this) (5)

2 Propositional Logic (10 points)

1. Produce a truth table for the following propositional statement: (8)
   $((p \wedge q) \rightarrow r) \iff (p \rightarrow (q \rightarrow r))$

2. Is this either a tautology or a contradiction? (2)

3 Predicate Logic (20 points)

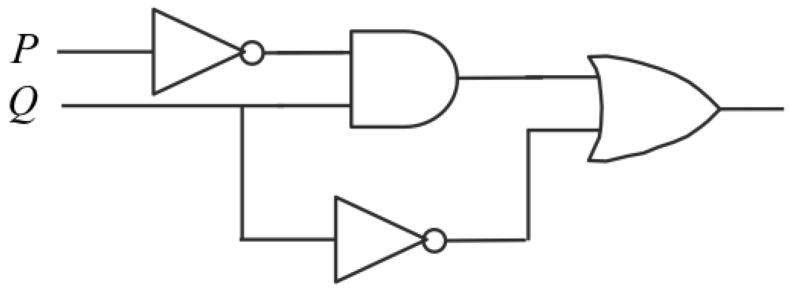
Let P, Q, R, and S be the following predicates:
$P(x) : x$ is a rock
$Q(x) : x$ sinks in water
$R(x) : x$ is very small
$S(x) : x$ can be used to build a bridge

3. Convert the following english sentences to logical sentences in terms of P, Q, R, S, logical connectives, and quantifiers: (10)

   - All rocks can be used to build a bridge.
   - There is nothing very small which sinks in water
   - Anything which is **not** very small cannot be used to build a bridge.
   - The Blarney Stone is a rock

4. Given the sentences above, show that the Blarney Stone floats in water (doesn't sink) using rules of inference and logical equivalences. Show each new sentence in each step of your proof, and apply exactly one rule of inference or equivalence per step. You are welcome to label each rule for each step. (10)

4 Boolean Algebra (10 points)



5. Write the equivalent logic statement represented by the digital logic circuit above. (5)

6. Simplify the logic statement by minimizing the number of connectives used. (5)

5 Proofs (20 points)

7. Prove or disprove that x^3 is irrational, then x is irrational. What kind of proof did you use? (10)

8. Prove that if $n \in \mathbb{Z}$, then $n^2 + 3n + 5$ is an odd integer. *(Hint: n may be even or odd)* (10)

6 Sets (10 points)

9. Prove $\overline{A \cup B \cup C} = \overline{A} \cap \overline{B} \cap \overline{C}$ without using Venn Diagrams. (10)

7 Functions (15 points)

For the following questions, list whether it is **one-to-one**, **onto**, or a **one-to-one correspondence** (bijection).

10. $f(x) = 9x/12$ $f(x) : \mathbb{R} \to \mathbb{R}$ (3)
    One-to-One? Onto? Bijection?

11. $f(x) = \lceil x \rceil * 2$ $f(x) : \mathbb{R} \to \mathbb{Z}$ (3)
    One-to-One? Onto? Bijection?

12. $f(x) = (x^2 + 1)/(x^2 + 2)$ $f(x) : \mathbb{R} \to \mathbb{R}$ (3)
    One-to-One? Onto? Bijection?

13. $f(x) = x^7 + 13$ $f(x) : \mathbb{R} \to \mathbb{R}$ (3)
    One-to-One? Onto? Bijection?

14. $f(x, y) = x^2 - y$ $f(x, y) : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ (3)
    One-to-One? Onto? Bijection?

Cheat Sheet

Logical Equivalences

Identity: $p \wedge \boldsymbol{T} \equiv p$
$p \vee \boldsymbol{F} \equiv p$
Domination: $p \vee \boldsymbol{T} \equiv \boldsymbol{T}$
$p \wedge \boldsymbol{F} \equiv \boldsymbol{F}$
Idempotent: $p \vee p \equiv p$
$p \wedge p \equiv p$
Double negation: $\neg(\neg p) \equiv p$
Commutative: $p \vee q \equiv q \vee p$
$p \wedge q \equiv q \wedge p$
Associative: $(p \vee q) \vee r \equiv p \vee (q \vee r)$
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Distributive: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
De Morgan's: $\neg(p \wedge q) \equiv \neg p \vee \neg q$
$\neg(p \vee q) \equiv \neg p \wedge \neg q$
Absorption: $p \vee (p \wedge q) \equiv p$
$p \wedge (p \vee q) \equiv p$
Negation: $p \vee \neg p \equiv \boldsymbol{T}$
$p \wedge \neg p \equiv \boldsymbol{F}$

Conditional Statements

$p \rightarrow q \equiv \neg p \vee q$
$p \rightarrow q \equiv \neg q \rightarrow \neg p$
$p \vee q \equiv \neg p \rightarrow q$
$p \wedge q \equiv \neg(p \rightarrow \neg q)$
$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$
$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$
$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

Biconditional Statements

$p \iff q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
$p \iff q \equiv \neg p \iff \neg q$
$p \iff q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg(p \iff q) \equiv p \iff \neg q$

Rules of Inference

p $p \rightarrow q$ $\therefore q$	Modus ponens
$\neg q$ $p \rightarrow q$ $\therefore \neg p$	Modus tollens
$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	Disjunctive syllogism
p $\therefore p \vee q$	Addition
$p \wedge q$ $\therefore p$	Simplification
p q $\therefore p \wedge q$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	Resolution

Logic Gates