

Description Logics

What Are Description Logics?

- **A family of logic based Knowledge Representation formalisms**
 - **Descendants of semantic networks and KL-ONE**
 - **Describe domain in terms of concepts (classes), roles (relationships) and individuals**
- **Distinguished by:**
 - **Formal semantics (typically model theoretic)**
 - **Decidable fragments of FOL**
 - **Closely related to Propositional Modal & Dynamic Logics**
 - **Provision of inference services**
 - **Sound and complete decision procedures for key problems**
 - **Implemented systems (highly optimized)**

Description Logics

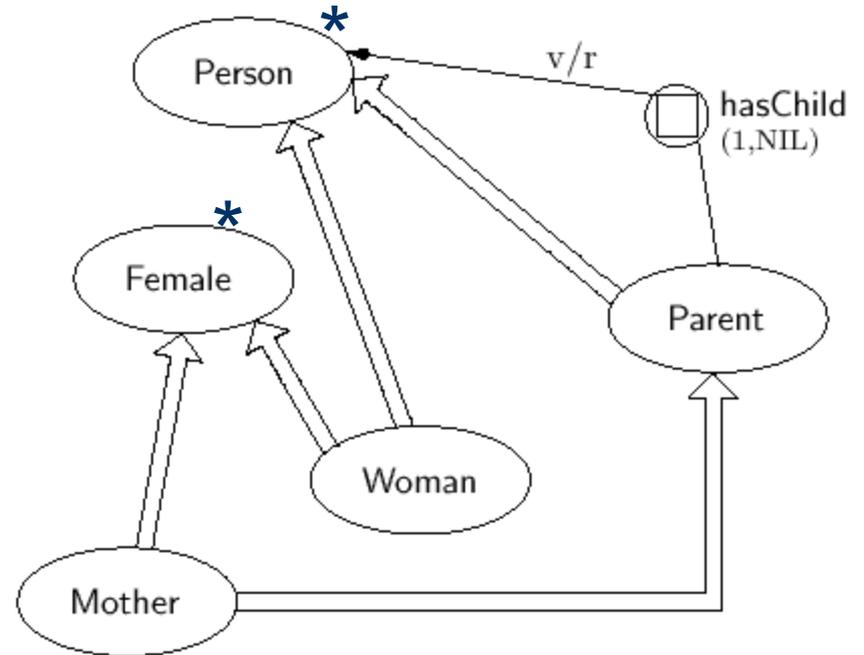
- Major focus of KR research in the 80's
 - Led by Ron Brachman – (AT&T Labs)
 - Grew out of early network-based KR systems like semantic networks and frames.
- Major systems and languages –
 - 80s: KL-ONE, NIKL, KANDOR, BACK, CLASSIC, LOOM
 - 90s: FACT, RACER, ...
 - 00s: DAML+OIL, OWL, Pellet, Jena, FACT++
- Used as the basis for the Semantic web languages DAML+OIL and OWL
- Some commercial systems

Description Logics

- Thought to be well-suited for the representation of and reasoning about
 - ontologies
 - terminological knowledge
 - Configurations and configuration problems
 - database schemata
 - schema design, evolution, and query optimization
 - source integration in heterogeneous databases/data warehouses
 - conceptual modeling of multidimensional aggregation

Example of Network KR

- Person, Female, etc are concepts
- hasChild is a property of Person
 - hasChild relates Parent to Person
 - Nil means infinity. A Parent is a Person with between 1 and infinity children
- Large arrows are “IS-A” links
 - A Mother is a (specialization of a) Parent
- Concepts are either primitive or definitions.
 - Primitive concepts have only necessary properties
 - Defined concepts have necessary and sufficient conditions.



**Graphical notation
introduced by KL-ONE**

DL Paradigm

- A **Description Logic** is mainly characterized by a set of constructors that allow one to build complex descriptions or terms out of **concepts** and **roles** from atomic ones
 - **Concepts** correspond to classes
 - and are interpreted as sets of objects,
 - **Roles** correspond to relations
 - and are interpreted as binary relations on objects
- Set of axioms for asserting **facts** about concepts, roles and **individuals**

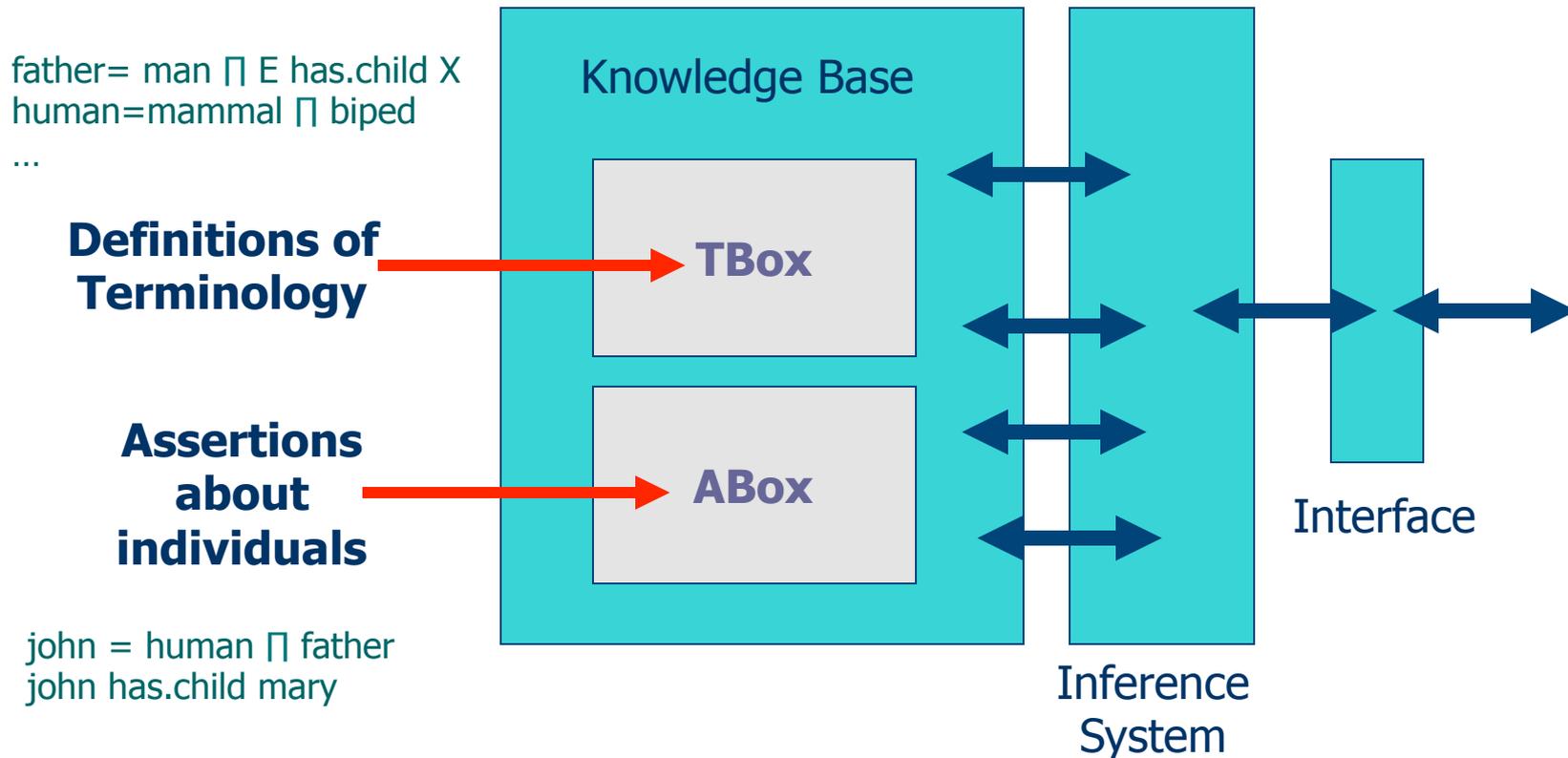
Basic Concepts of a DL

- Individuals are treated exactly the same as constants in FOL
 - *john*
- Concepts are exactly the same as Unary Predicates in FOL
 - *Person(john)*
- Roles are exactly the same as Binary Predicates in FOL
 - *has_mother(john, mary)*

Descriptions

- As in FOL, we are dealing with (ultimately) sets of individuals and relations between them
- The basic unit of semantic significance is the *Description*.
- “We are **describing** sets of individuals”
- Description logics differ in the operators allowed
- If a “happy father” is a man with both a son and daughter and all of whose children are either rich or happy, then we describe it in DL as
$$\text{HappyFather} = \text{Man} \cap \exists \text{hasChild.Female} \cap \exists \text{hasChild.Male} \cap \forall \text{hasChild.}(\text{Rich} \cup \text{Happy})$$

Typical Architecture



The division into TBox and ABox doesn't have a logical significance, but is made for conceptual and implementation convenience.

A family of languages

- The expressiveness of a description logic is determined by the operators that it uses
- Add or eliminate certain operators (e.g., \neg , \cup), and the statements that can be expressed are increased/reduced in number
- Higher expressiveness implies higher complexity
- *AL* or *Attributive Language* is the base and includes just a few operators
- Other DLs are described by the additional operators they include

AL: Attributive Language

| Constructor | Syntax | Example |
|-------------------------|--------------|--------------------------------|
| atomic concept | C | Human |
| atomic negation | $\sim C$ | \sim Human |
| atomic role | R | hasChild |
| conjunction | $C \wedge D$ | Human \wedge Male |
| value restrict. | $R.C$ | Human \exists hasChild.Blond |
| existential rest. (lim) | $\exists R$ | Human \exists hasChild |
| Top (univ. conc.) | \top | \top |
| bottom (null conc) | \perp | \perp |

for concepts C and D and role R

ALC

ALC is the smallest DL that is propositionally closed (i.e., includes full negation and disjunction) and include booleans (and, or, not) and restrictions on role values

| constructor | Syntax | Example |
|--------------------------|---------------------------------|--|
| atomic concept | C | Human |
| negation | $\sim C$ | $\sim (\text{Human } \vee \text{Ape})$ |
| atomic role | R | hasChild |
| conjunction | $C \wedge D$ | Human \wedge Male |
| disjunction | $C \vee D$ | Nice \vee Rich |
| value restrict. | $\exists R.C$ | Human \exists hasChild.Blond |
| existential rest. | $\exists R.C$ | Human \exists hasChild.Male |
| Top (univ. conc.) | \top | \top |
| bottom (null conc) | \perp | \perp |

Other Constructors

| Constructor | Syntax | Example |
|----------------------|--------------|------------------------------|
| number restriction | $\geq n R$ | ≥ 7 hasChild |
| | $\leq n R$ | ≤ 1 hasmother |
| inverse role | R^- | haschild- |
| Transitive role | R^* | hasChild* |
| Role composition | $R \circ R$ | hasParent \circ hasBrother |
| Qualified # restric. | $\geq n R.C$ | ≥ 2 hasChild.Female |
| Singleton concepts | {<name>} | {Italy} |

\forall and \exists deserve special attention.

- Note that they only can come before a Role:

$\forall \text{HasChild.Girl}$ $\exists \text{isEmployedBy.Farmer}$

- Remember, they describe sets of individuals.

- $\forall \text{HasChild.Girl}$ would be interpreted as:

The set $\{ x \mid \forall(y)(\text{HasChild}(x,y) \rightarrow \text{Girl}(y)) \}$

Note the conditional: Are you in that set?.

- $\exists \text{isEmployedBy.Farmer}$ would be:

The set $\{ x \mid \exists(y)(\text{isEmployedBy}(x,y) \wedge \text{Farmer}(y)) \}$

Special names and combinations

See http://en.wikipedia.org/wiki/Description_logic

- S = ALC + transitive properties
- H = role hierarchy, e.g., `rdfs:subPropertyOf`
- O = nominals, e.g., values constrained by enumerated classes, as in `owl:oneOf` and `owl:hasValue`
- I = inverse properties
- N = cardinality restrictions (`owl:cardinality`, `owl:maxCardonality`)
- ^(D) = use of datatypes properties
- etc.
- **OWL-DL is SHOIN^(D)**
- OWL 2 is SROIQ



Complexity of reasoning in Description Logics

Note: the information here is (always) incomplete and [updated](#) often

Base description logic: *Attributive Language with Complements*

$\mathcal{ALC} ::= \perp \mid A \mid \neg C \mid C \wedge D \mid C \vee D \mid \exists R.C \mid \forall R.C$



Concept constructors:

- \mathcal{F} - functionality²: $(\leq 1 R)$
- \mathcal{N} - (unqualified) number restrictions: $(\geq n R)$, $(\leq n R)$
- \mathcal{Q} - qualified number restrictions: $(\geq n R.C)$, $(\leq n R.C)$
- \mathcal{O} - nominals: $\{a\}$ or $\{a_1, \dots, a_n\}$ ("one-of" constructor)
- μ - least fixpoint operator: $\mu X.C$
- $R \subseteq S$ - role-value-maps
- $f = g$ - agreement of functional role chains ("same-as")

TBox options:

- Empty TBox
- Acyclic TBox ($A \equiv C$, A is a concept name; no cycles)
- General TBox ($C \subseteq D$ for arbitrary concepts C and D)

Role constructors:

- \mathcal{I} - role inverses: R^-
- \cap - role intersection³: $R \cap S$
- \cup - role union: $R \cup S$
- \neg - role complement: $\text{full} \dots$
- \circ - role chain (composition): $R \circ S$
- $*$ - reflexive-transitive closure⁴: R^*
- id - concept identity: $id(C)$
- $\text{Forbid} \dots$ complex roles⁵ in number restrictions⁶

trans reg

Role axioms (RBox):

- \mathcal{S} - Role transitivity: $\text{Trans}(R)$
- \mathcal{H} - Role hierarchy: $R \subseteq S$
- \mathcal{R} - Complex role inclusions: $R \circ S \subseteq R$, $R \circ S \subseteq S$
- s - some additional features

OWL-Lite

OWL-DL

OWL 1.1

Reset

You have selected the Description Logic: \mathcal{ALCIQ}

Complexity of reasoning problems⁷

| Reasoning problem | Complexity ⁸ | Comments and references |
|------------------------|-------------------------|---|
| Concept satisfiability | PSpace-complete | <ul style="list-style-type: none"> • <u>Hardness</u> for \mathcal{ALC}: see [73]. • <u>Upper bound</u> for \mathcal{ALCQI} even with numbers coded in binary: see [77, Theorem 4.29]. |
| ABox consistency | PSpace-complete | <ul style="list-style-type: none"> • <u>Hardness</u> follows from that for concept satisfiability. • <u>Upper bound</u> is obtained in [25, Theorem 4] by a polynomial translation (that preserves satisfiability) into the language without inverse roles. |

Important properties of the description logic

| | | |
|-----------------------|---|--|
| Finite model property | ☺ | |
| Tree model property | ☺ | |



OWL as a DL

- OWL-DL is SHOIN^(D)
- We can think of OWL as having three kinds of statements
- Ways to specify classes
 - the intersection of humans and males
- Ways to state axioms about those classes
 - Humans are a subclass of apes
- Ways to talk about individuals
 - John is a human, john is a male, john has a child mary

OWL Class Constructors

| Constructor | DL Syntax | Example | (Modal Syntax) |
|----------------|-------------------------------|---------------------------|-------------------------------|
| intersectionOf | $C_1 \sqcap \dots \sqcap C_n$ | Human \sqcap Male | $C_1 \wedge \dots \wedge C_n$ |
| unionOf | $C_1 \sqcup \dots \sqcup C_n$ | Doctor \sqcup Lawyer | $C_1 \vee \dots \vee C_n$ |
| complementOf | $\neg C$ | \neg Male | $\neg C$ |
| oneOf | $\{x_1 \dots x_n\}$ | {john, mary} | $x_1 \vee \dots \vee x_n$ |
| allValuesFrom | $\forall P.C$ | \forall hasChild.Doctor | $[P]C$ |
| someValuesFrom | $\exists P.C$ | \exists hasChild.Lawyer | $\langle P \rangle C$ |
| maxCardinality | $\leq nP$ | ≤ 1 hasChild | $[P]_{n+1}$ |
| minCardinality | $\geq nP$ | ≥ 2 hasChild | $\langle P \rangle_n$ |

- ➡ XMLS **datatypes** as well as classes in $\forall P.C$ and $\exists P.C$
 - E.g., \exists hasAge.nonNegativeInteger
- ➡ Arbitrarily complex **nesting** of constructors
 - E.g., Person \sqcap \forall hasChild.(Doctor \sqcup \exists hasChild.Doctor)

OWL Axioms

| Axiom | DL Syntax | Example |
|---------------------------|-----------------------------------|---|
| subClassOf | $C_1 \sqsubseteq C_2$ | Human \sqsubseteq Animal \sqcap Biped |
| equivalentClass | $C_1 \equiv C_2$ | Man \equiv Human \sqcap Male |
| disjointWith | $C_1 \sqsubseteq \neg C_2$ | Male $\sqsubseteq \neg$ Female |
| sameIndividualAs | $\{x_1\} \equiv \{x_2\}$ | {President_Bush} \equiv {G_W_B} |
| differentFrom | $\{x_1\} \sqsubseteq \neg\{x_2\}$ | {john} $\sqsubseteq \neg$ {peter} |
| subPropertyOf | $P_1 \sqsubseteq P_2$ | hasDaughter \sqsubseteq hasChild |
| equivalentProperty | $P_1 \equiv P_2$ | cost \equiv price |
| inverseOf | $P_1 \equiv P_2^-$ | hasChild \equiv hasParent ⁻ |
| transitiveProperty | $P^+ \sqsubseteq P$ | ancestor ⁺ \sqsubseteq ancestor |
| functionalProperty | $\top \sqsubseteq \leq 1P$ | $\top \sqsubseteq \leq 1$ hasMother |
| inverseFunctionalProperty | $\top \sqsubseteq \leq 1P^-$ | $\top \sqsubseteq \leq 1$ hasSSN ⁻ |

☞ \mathcal{I} **satisfies** $C_1 \sqsubseteq C_2$ iff $C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$; satisfies $P_1 \sqsubseteq P_2$ iff $P_1^{\mathcal{I}} \subseteq P_2^{\mathcal{I}}$

☞ \mathcal{I} satisfies ontology \mathcal{O} (is a **model** of \mathcal{O}) iff satisfies every axiom in \mathcal{O}

Subsumption: $D \subseteq C$?

- Concept C subsumes D iff on every interpretation I
 - $I(D) \subseteq I(C)$
- This means the same as (for complex statements D and C) the assertion:
 - $\forall(x)(D(x) \rightarrow C(x))$
- Determining whether one concept *logically* contains another is called the *subsumption problem*.
- Subsumption is undecidable for reasonably expressive languages,
- and non-polynomial for fairly restricted ones.

Other reasoning problems

- These problems can be reduced to subsumption (for languages with negation)
- Can be reduced to the satisfiability problem, as well

Satisfiability of Concept or KB $\{C, \neg C\}$

Instance Checking $\text{Father}(\text{john})?$

Equivalence $\text{CreatureWithHeart} \equiv \text{CreatureWithKidney}$

Disjointness $C \sqcap D$

Retrieval $\text{Father}(X)? \quad X = \{\text{john}, \text{robert}\}$

Realization $X(\text{john})? \quad X = \{\text{Father}\}$

Definitions

- A **definition** is a description of a concept or a relationship.
- It is used to assign a meaning to a term.
- In description logics, definitions use a specialized logical language.
- Description logics are able to do limited reasoning about concepts expressed in their logic.
- One important inference is classification (computation of subsumption).

Necessary versus Sufficient

- *Necessary* properties of an object are properties common to all objects of that type.
 - Being a man is a necessary condition for being a father.
- *Sufficient* properties are properties that allow one to identify an object as belonging to a type. They need be common to all members of the type.
 - Speeding is a sufficient reason for being stopped by the police.
- Definitions are often necessary and sufficient

Subsumption

- Meaning of Subsumption

- *A more general concept or description is said to subsume a more specific one. Members of a subsumed concept are necessarily members of a subsuming concept*

- Formalization of Meaning

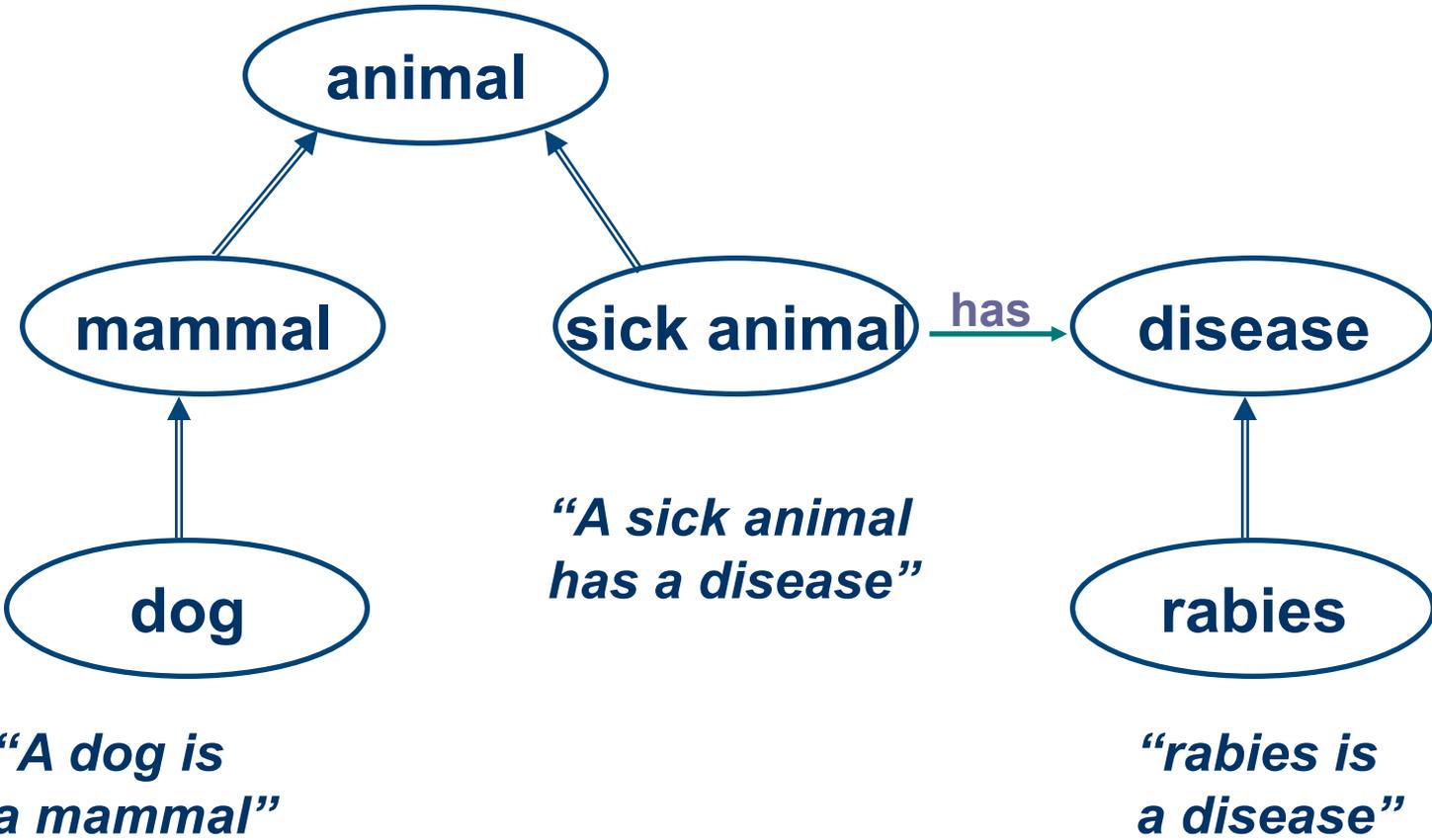
- Logic

- Satisfying a subsumed concept implies that the subsuming concept is satisfied also.
- E.g., if john is a person, he is also an animal

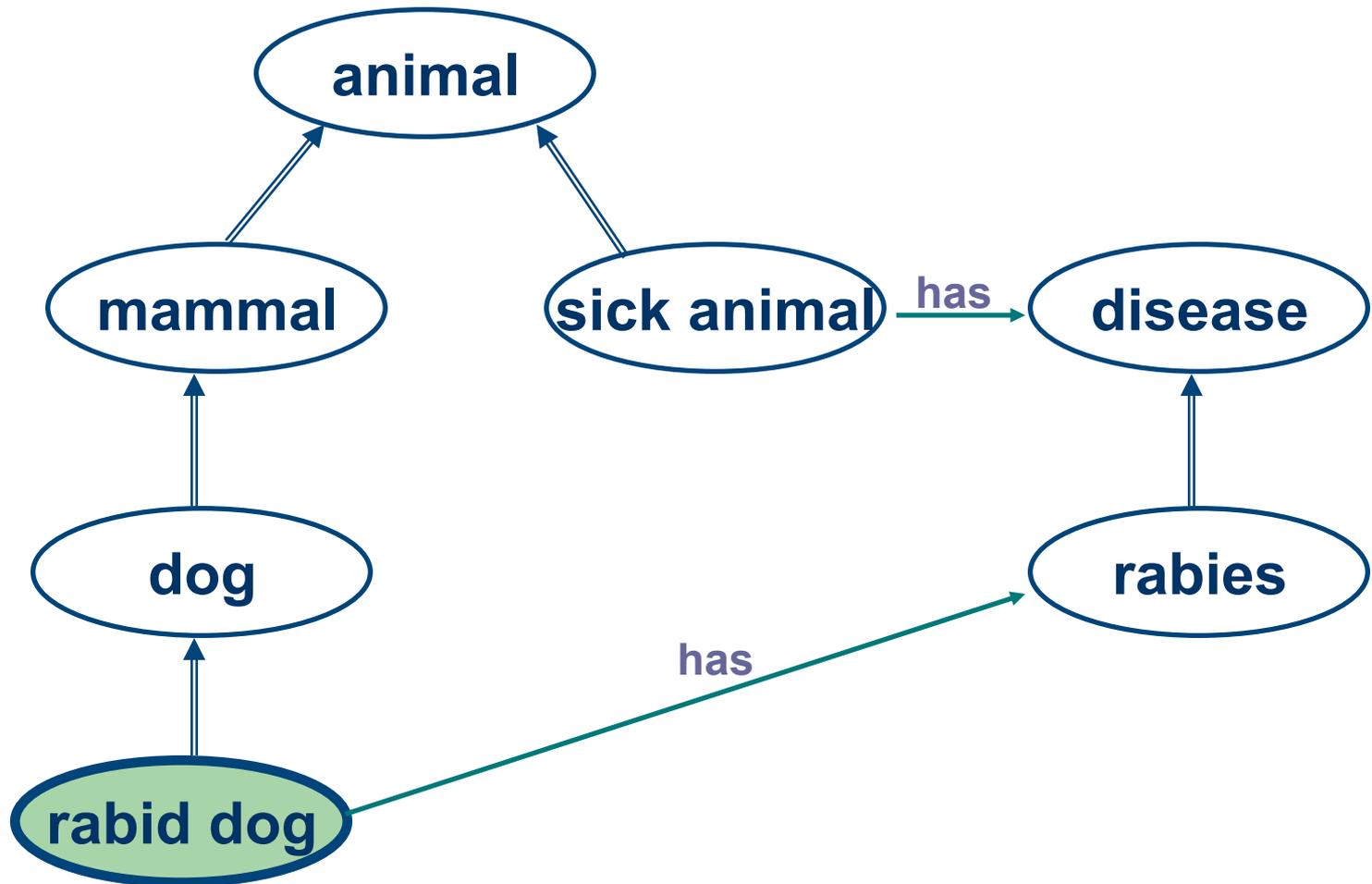
- Sets

- The instances of subsumed concept are necessarily a subset of the subsuming concept's instances.
- E.g., the set of all persons is a subset of all animals

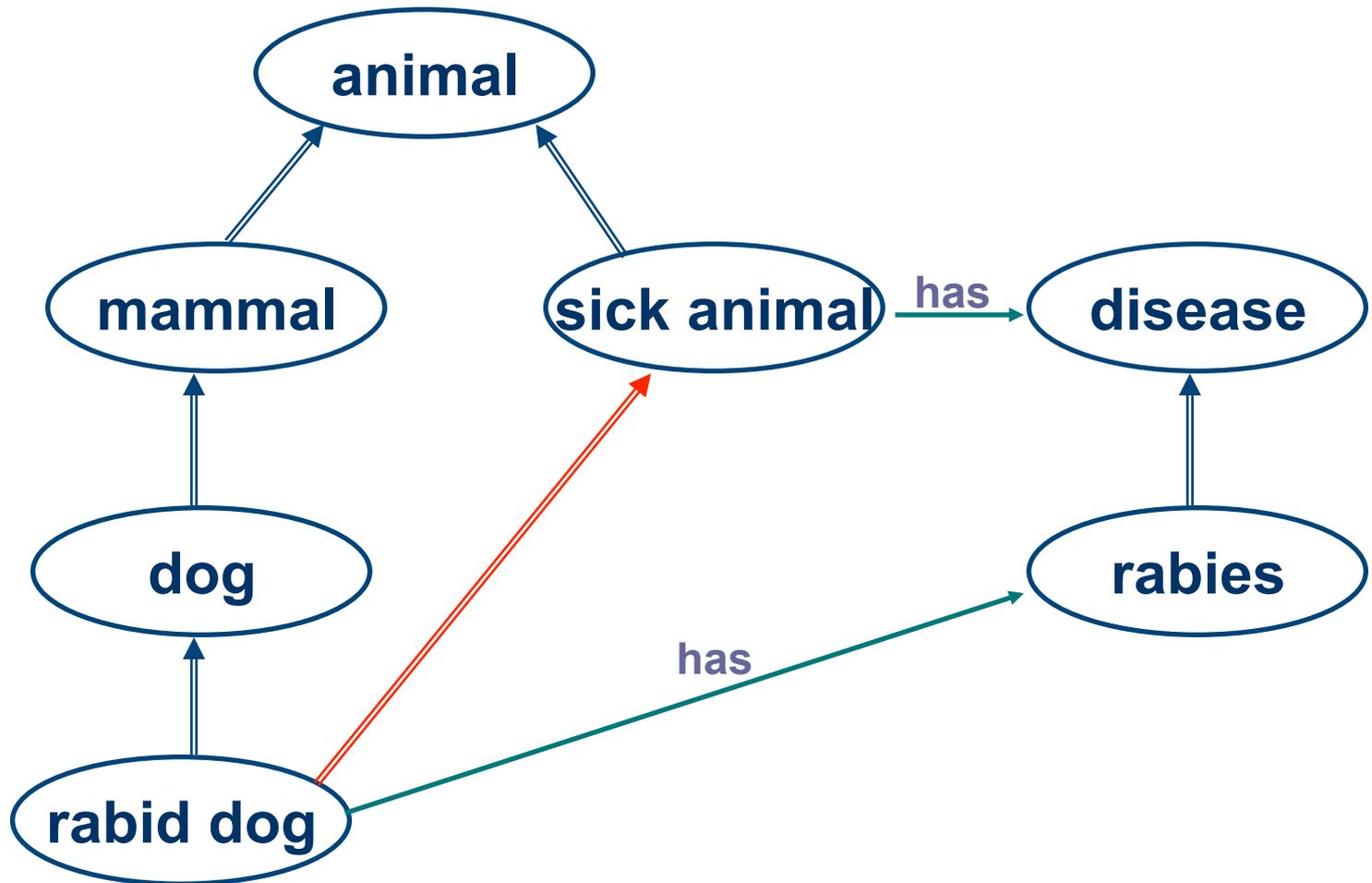
How Does Classification Work?



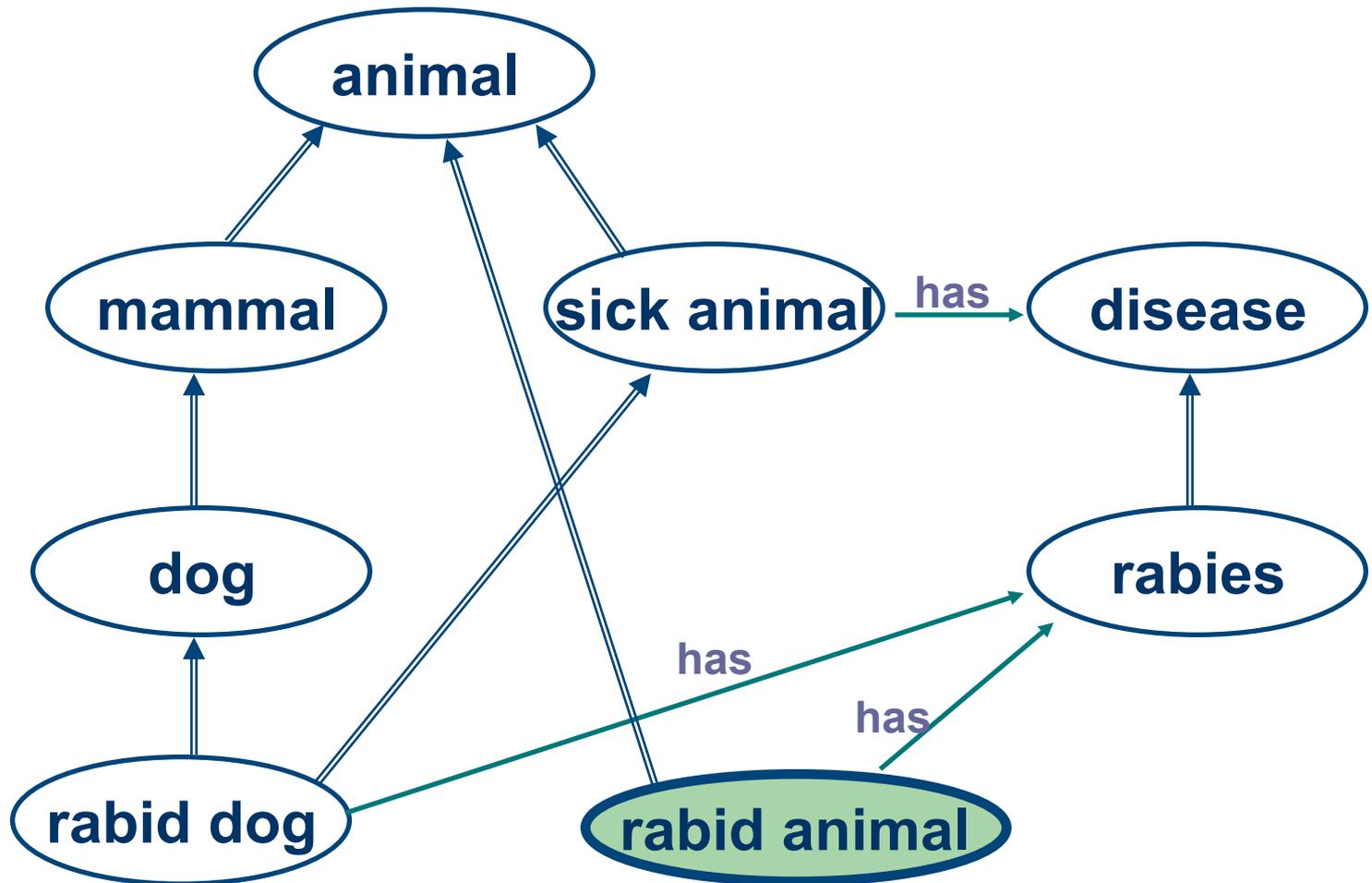
Defining a “rabid dog”



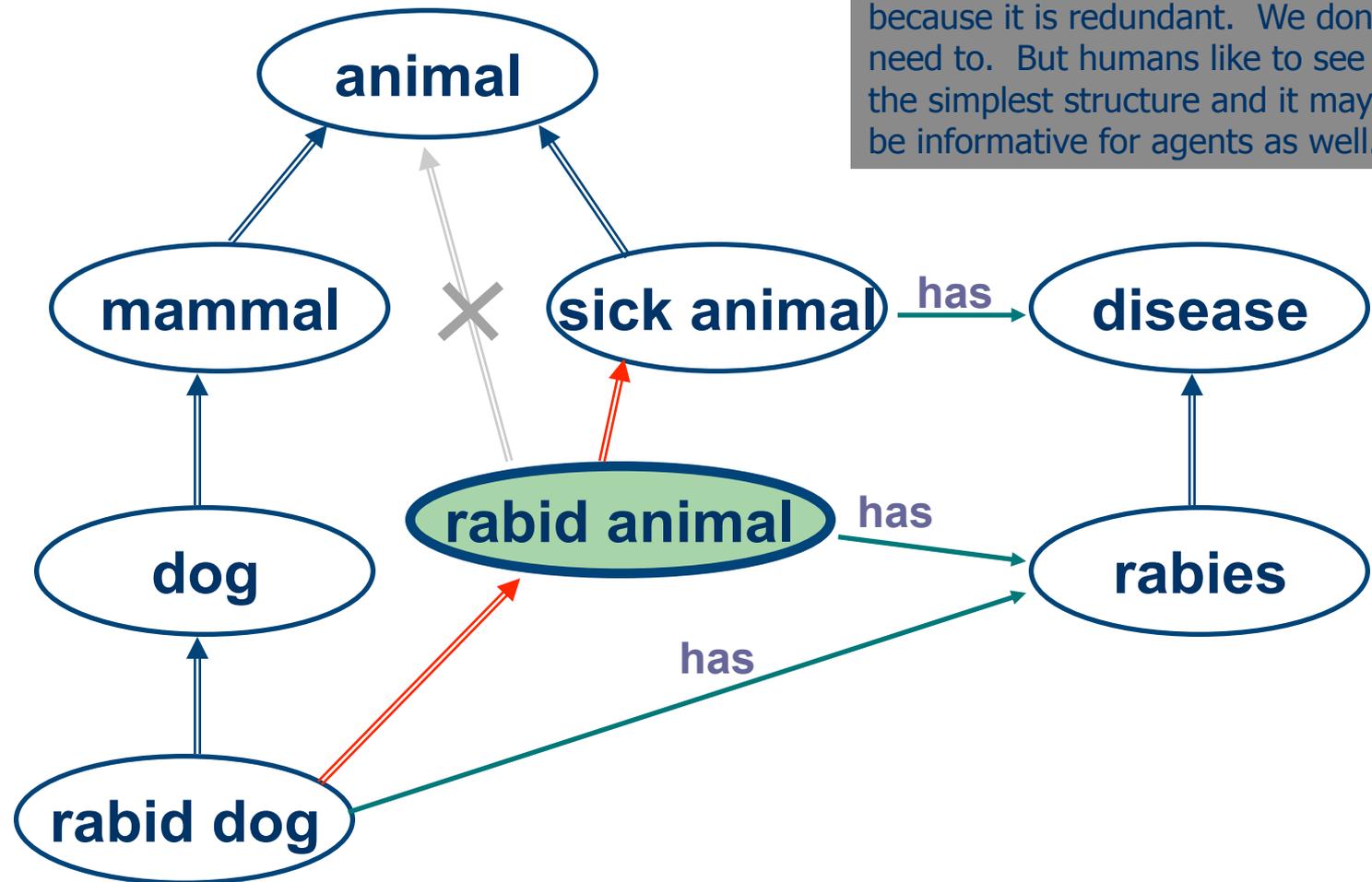
Classification as a “sick animal”



Defining “rabid animal”



Loom Places Concept in Hierarchy



Note: we can remove the subclass link from rabad animal to animal because it is redundant. We don't need to. But humans like to see the simplest structure and it may be informative for agents as well.

Primitive versus Structured (Defined)

- Description logics reason with definitions
 - They prefer to have *complete* descriptions
 - A complete definition includes both necessary conditions and sufficient conditions
- This is often impractical or impossible, especially with natural kinds.
- A “primitive” definition is an incomplete definition
 - This limits the amount of classification that can be done automatically
- Example:
 - Primitive: A Person
 - Defined: Parent = Person with at least 1 child

Intentional versus Extensional Semantics

- **Extensional Semantics** are a model-theoretic idea. They define the meaning of a description by enumerating the set of objects that satisfy the description.
- **Intensional Semantics** defines the meaning of a description based on the intent or use of the description.
- Example:
 - Morning-Star Evening-Star
 - Extensional: Same object, namely Venus
 - Intensional: Different objects, one meaning Venus seen in the morning and one in the evening.

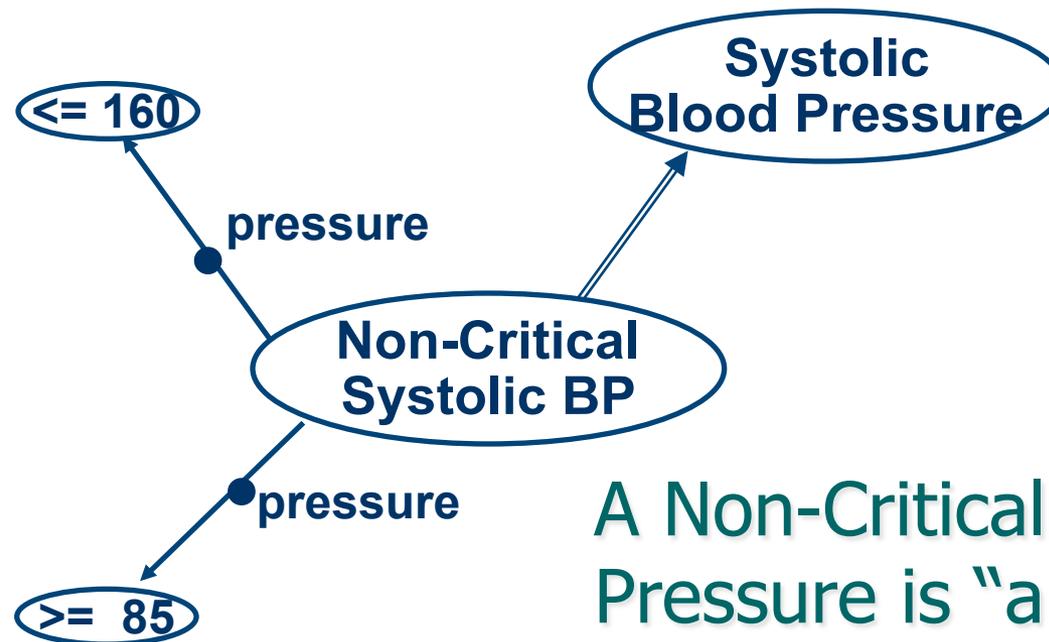
Definition versus Assertion

- A **definition** is used to describe *intrinsic* properties of an object. The parts of a description have meaning as a part of a composite description of an object
- An **assertion** is used to describe an *incidental* property of an object. Asserted facts have meaning on their own.
- Example: “a black telephone”
Could be either a description or an assertion, depending on the meaning and import of “blackness” on the concept telephone.

Definition versus Assertion

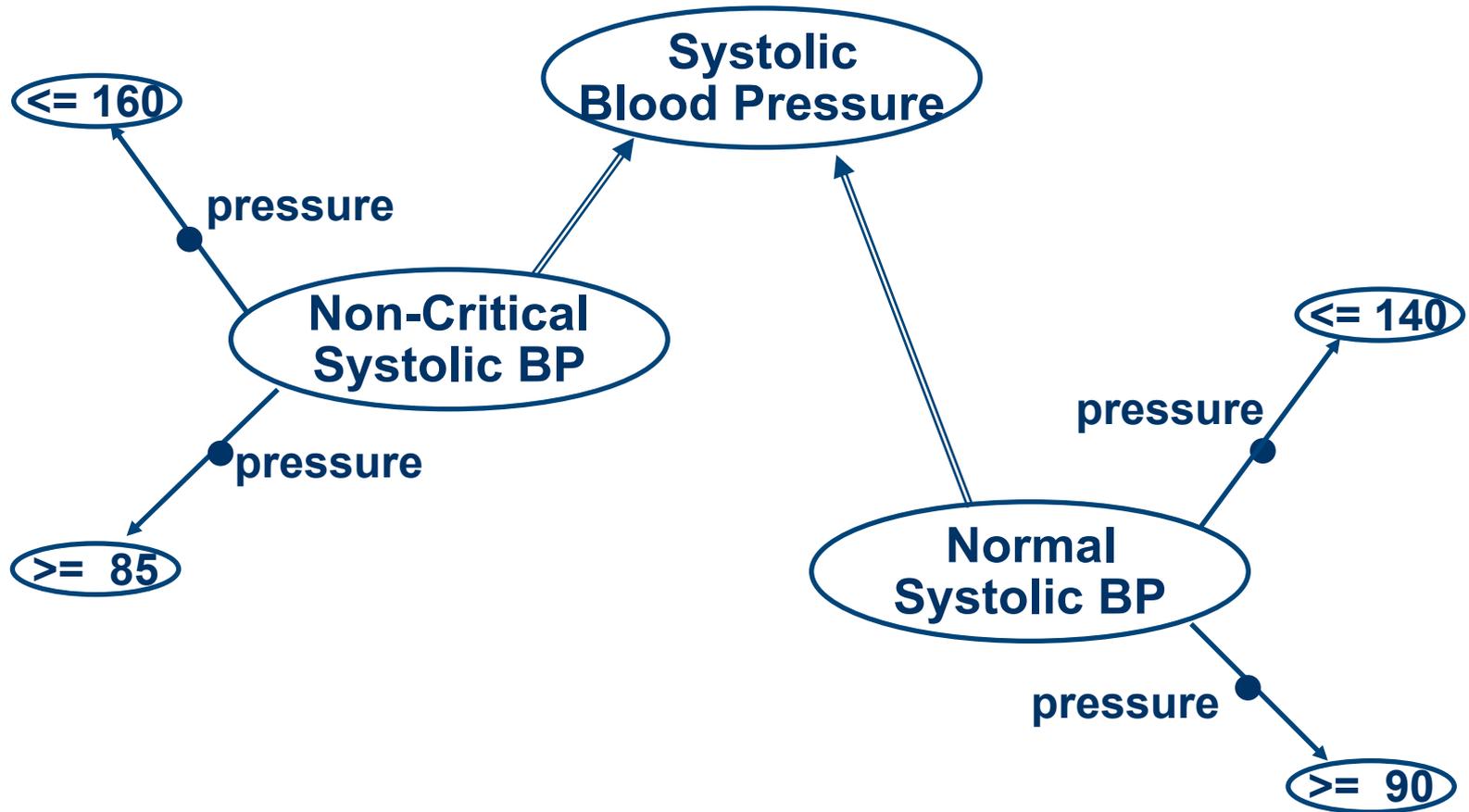
- In English, “a black telephone” is ambiguous
 - (1) A black telephone is a common sight in an office
 - (2) A black telephone is on the corner of my desk
- KR languages should not be ambiguous so typically distinguish between descriptions of classes and descriptions of individuals
- KR languages often also allow additional assertions to be made that are not part of the definition (often called annotation properties)

Example: Blood Pressure



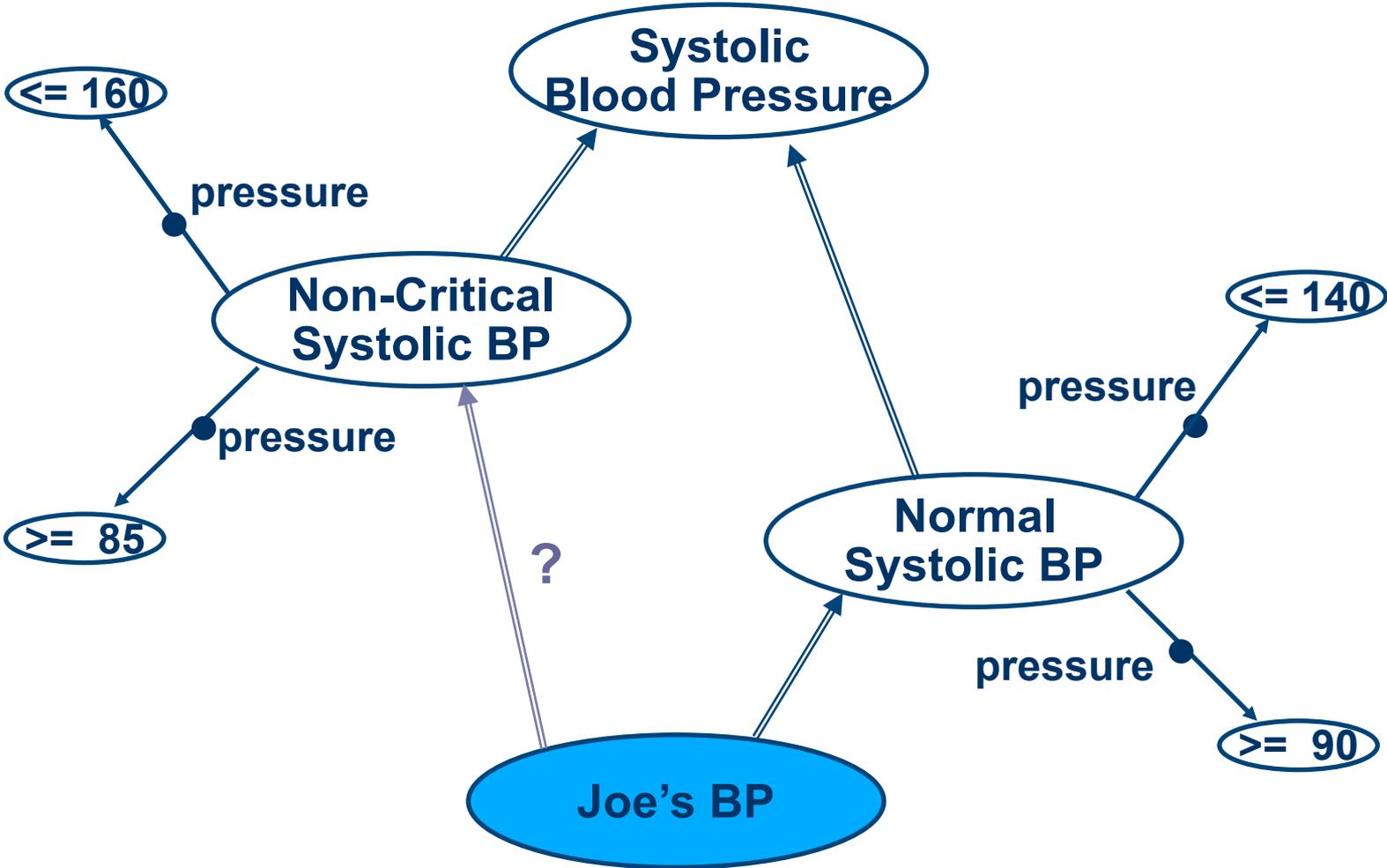
A Non-Critical Blood Pressure is "a Systolic B.P. between 85 and 160."

Example: Blood Pressure

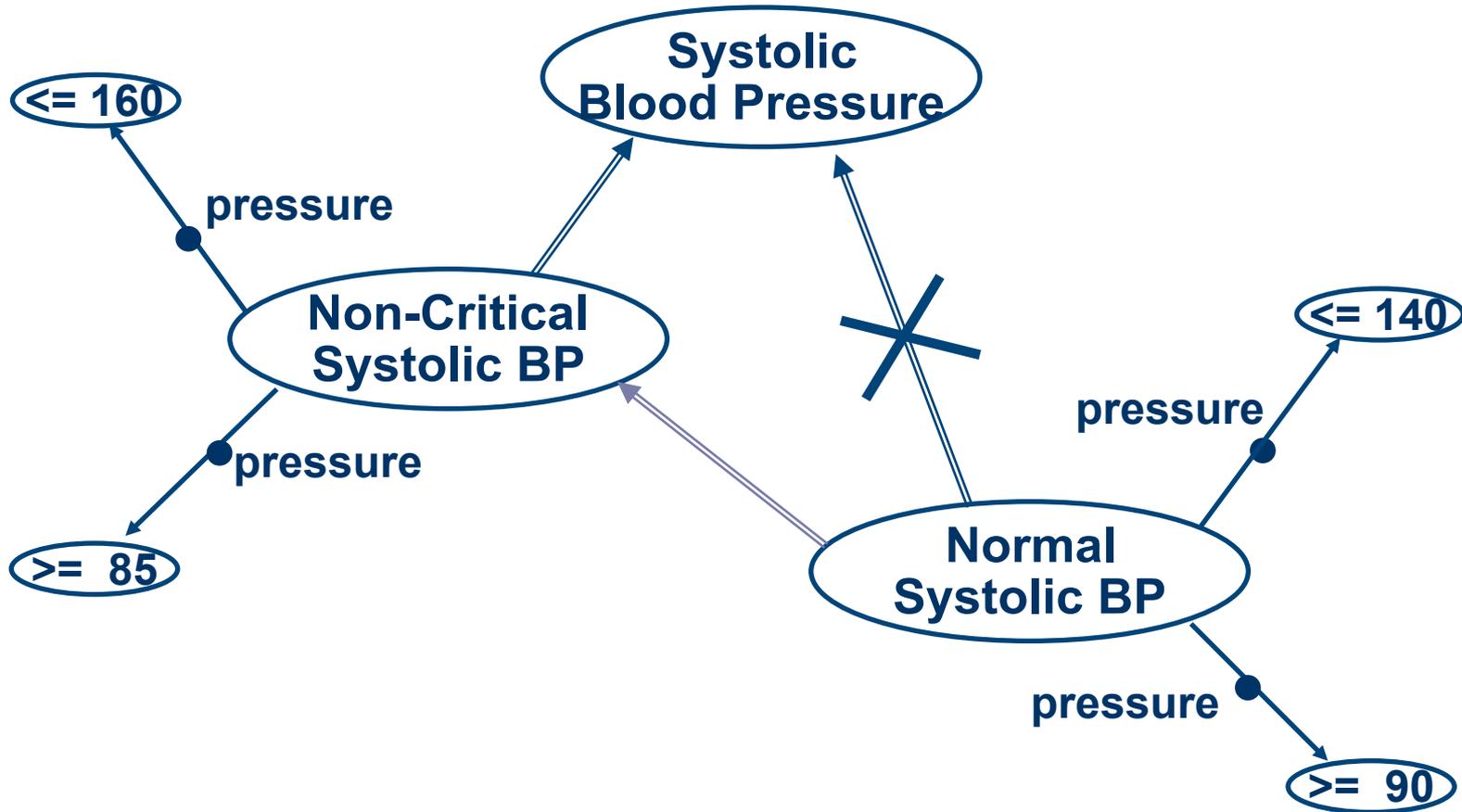


Normal Systolic B.P. is “a Systolic B.P. between 90 and 140.

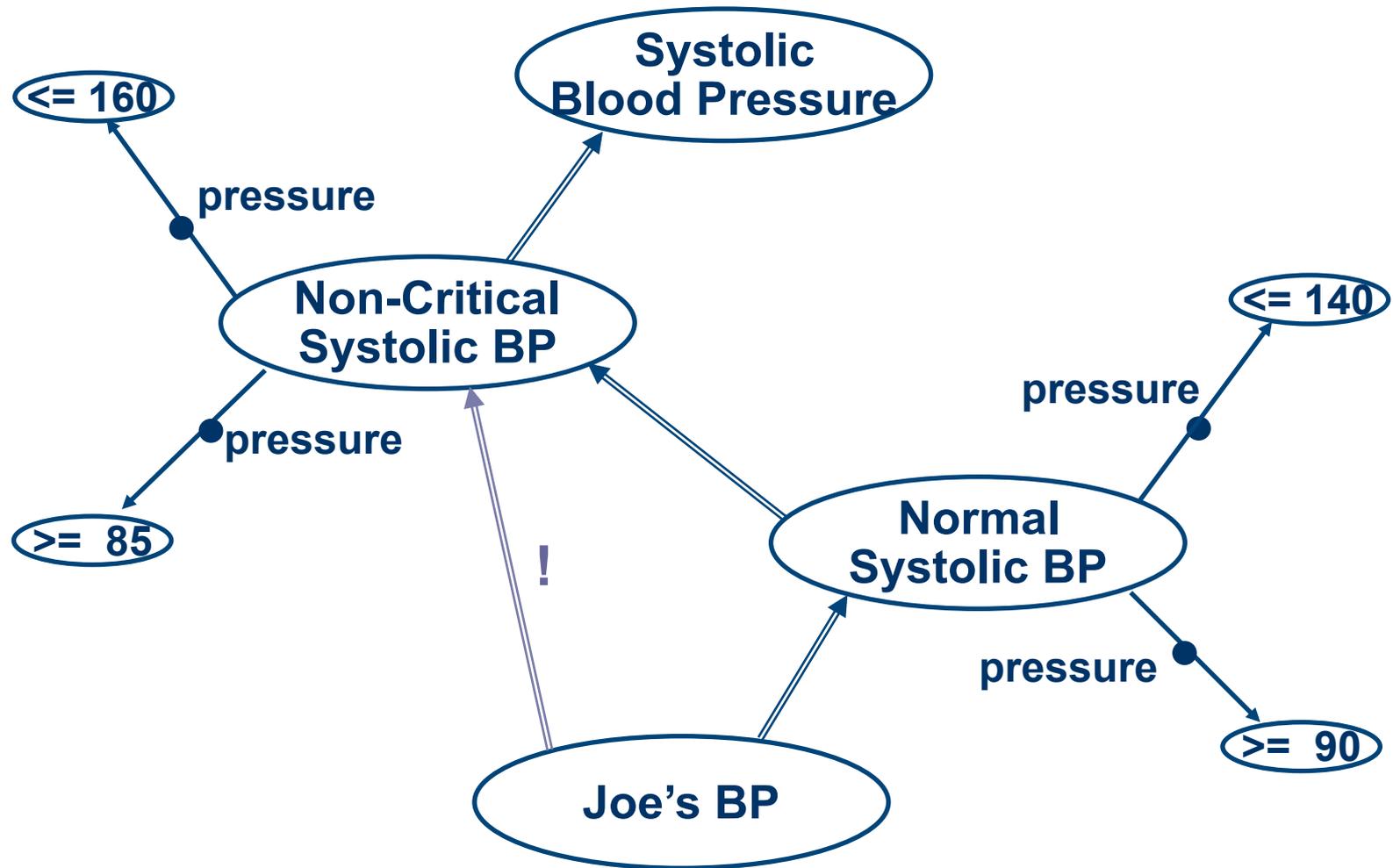
If Joe's BP is Normal is it also Non-Critical?



Concept Classification Infers Normal BP is Subsumed by Non-Critical BP



With Classified Concepts the Answer is Easy to Compute



Classification is very useful

- Classification is a powerful kind of reasoning that is very useful
- Many expert systems can be usefully thought of as doing “heuristic classification”
- Logical classification over structured descriptions and individuals is also quite useful.
- But... can classification ever deduce something about an individual other than what classes it belongs to?
- And what does *that* tell us?

Incidental properties

- If we allow incidental properties (e.g., ones that don't participate in the description mechanism) then these can be deduced via classification.

Some DL reasoners

- See http://en.wikipedia.org/wiki/Description_logic
 - [CEL](#), free (for non-commercial use), LISP
 - [Cerebra Engine](#), commercial, C++
 - [FaCT++](#), free, open-source, C++
 - [KAON2](#) free (for non-commercial usage), Java
 - [MSPASS](#) free, open-source, C
 - [Pellet](#) free, open-source, Java
 - [RacerPro](#) commercial, LISP
- DIG is a standard XML based interface to a DL reasoner
- Protégé uses DIG and can thus use any of several DL reasoners that have a DIG interface

Dig API: <http://dig.sourceforge.net/>

