CMSC 678 Machine Learning - Fall 2023 Homework Assignment 4 Due by 11:59pm on November 7th

1. (Support Vector Machines) Consider the dataset with instances in \mathbb{R}^2 below:

x_1	x_2	class
-2	-2	1
-2	-1	1
1	2	1
2	1	1
-2	2	-1
0	2	-1
0	-1	-1
2	-1	-1

Consider the non-linear mapping from the input space to a two-dimensional feature space given by the following:

 $(x_1, x_2) \to (x_1^2, x_1 x_2)$

- (a) Plot on graph paper the training instances in the input space and label them according to the class to which they belong. State whether the patterns from the two classes are linearly separable in this space.
- (b) Create a new plot of the training instances in the feature space and label them according to the class to which they belong.
- (c) Find the maximum margin classifier in the feature space. More specifically, find the equations of the classification boundary and of the two margin boundaries. Plot these three boundaries on the same graph that was used in step (b). Also indicate which are the support vectors in the feature space. Note that the boundaries and the support vectors are easy to find by inspection.
- (d) Find which vectors in the input space correspond to the support vectors found in step (c).
- (e) Plot the classification boundary in input space on the same graph that was used in step (a).
- (f) Plot on the same graph the boundaries of the classification margin zone in the input space. Shade the area between these two boundaries to better visualize the classification margin zone in the input space.

(g) Write the inequality that you would use to classify new input patterns with the SVM classifier developed in steps (a) through (f). Choose an inequality that is as simple as possible.