Illumination and Reflection

Many slides courtesy of Prof. Pat Hanrahan @ Stanford
Many material courtesy of John Hughes @ Brown

Readings (online):

- Textbook chapter 14.4, 14.9-11, 26, 27, 31

- Rendering concepts (a book chapter by Pat Hanrahan) (RenderingConcept.pdf)
- Theory for off-specular reflection from roughened surface, Torrance and Sparrow, Optical Society of American, 57(9), 1105-1114. 1967. (TorranceSparrow.pdf)
- A reflection model for computer graphics, Cook and Torrance, ACM Tran. On Graphics, 1(1), 7-24, 1982. (cookpaper.pdf)

Recap

- Moving from geometry to rendering and appearance
- About 8 lectures
- First couple of lectures
  - Intro to formal illumination, reflection, global illumination
  - Quickly move to new and advanced material

- Remember the Teddy assignment due March 12. Project proposal due March 13. Literature review due Mar. 27.

Light

- Visible electromagnetic radiation
  - Power spectrum

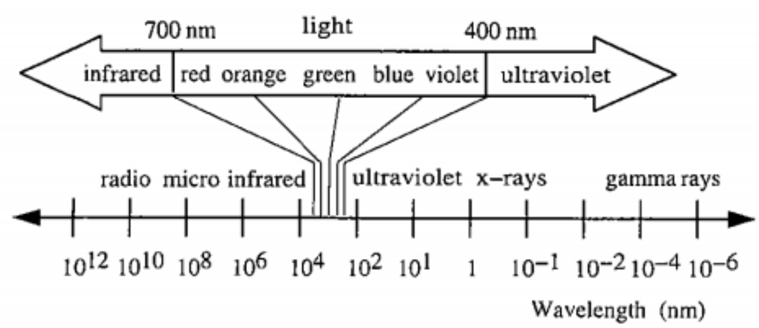


Figure 2.1: *Electromagnetic spectrum.*

- At a macroscopic level, light can be regarded as a kind of energy that flows uninterrupted through empty space along straight lines, but is absorbed into and/or reflected from surfaces that it meets.
  - Photons: light quantized in individual and indivisible packets.
  - a powerful beam of light contains more photons than a weak beam, not more powerful photons.

Light

- Wave-like: our visual system perceives light containing photons of different frequencies as a color somewhere between that created by individual photons.
  - For example, red+green=yellow (similar to yellow photon). We only need three wavelengths on a display.

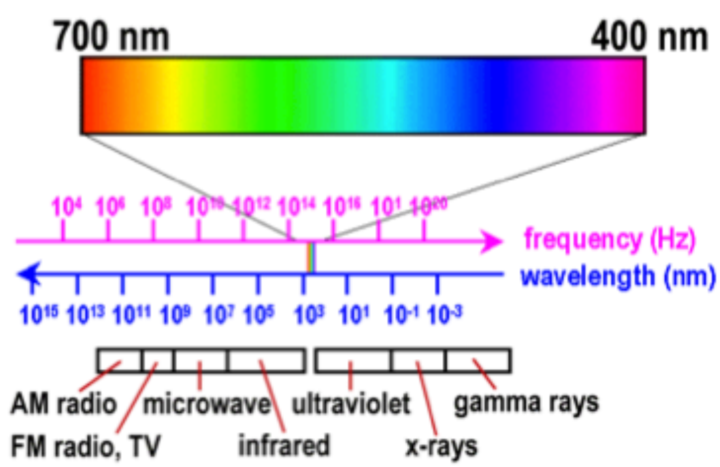


Figure 14.9: The visible spectrum is part of the full electromagnetic spectrum. The color of light that we perceive from an EM wave is determined by its frequency. The relationship between frequency and wavelength is determined by the medium through which the wave is propagating.

Light -> rendering

- Photo-realistic rendering depends on five kinds of models:
  - light
  - light emitter (fluorescent bulbs)
  - light transport
    - the speed of propagation of a photon is determined by a material. The speed of light c ~= 300K kilometers / second. The index of refraction of a material is the ratio of the speed of light in vacuum to the rate of propagation in that material (s):

$$\eta = \frac{c}{s}.$$

For household glasses, $\eta \approx 1.5$

For everyday materials, $s < c$ **so** $\eta \geq 1$

  - matter: geometry
  - sensor (e.g., camera and eyes)

Units

- Photons transport *energy*, which is measured in Joules.
- The *power* of a stream of photons is the rate of energy delivery per unit of time, measured in Watts.
  - Common household lighting solutions today measure the *consumption* in Watts, and convert 4-10 % of the power that they consume into visible light, so a typical 100w light bulb emits at best 10w of visible light.
- *Radiosity* or *irradiance* is the power per unit area *entering* or *leaving* a surface in units of w/m^2.
- *Radiance*: the power per unit area per unit solid angle, measured in w/m^2 sr

Implementation

```
class Color3 {
public:

    /** Magnitude near 650 THz (``red''), either at a single
        frequency or representing a broad range centered at
        650 THz, depending on the usage context. 650 THz
        photons have a wavelength of about 450 nm in air.*/
    float r;

    /** Near 550 THz (``green''); about 500 nm in air. */
    float g;

    /** Near 450 THz (``blue''); about 650 nm in air. */
    float b;

    Color3() : r(0), g(0), b(0) {}
    Color3(float r, float g, float b) : r(r), g(g), b(b);
    Color3 operator*(float s) const {
      return Color3(s * r, s * g, s * b);
    }
    ...
};
```

Listing 14.1: A general class for recording quantities sampled at three visible frequencies.

```
typedef Color3 Power3;
typedef Color3 Radiosity3;
typedef Color3 Radiance3;
typedef Color3 Biradiance3;
```

Listing 14.2: Aliases of `Color3` *with unit semantics.*

Emitters

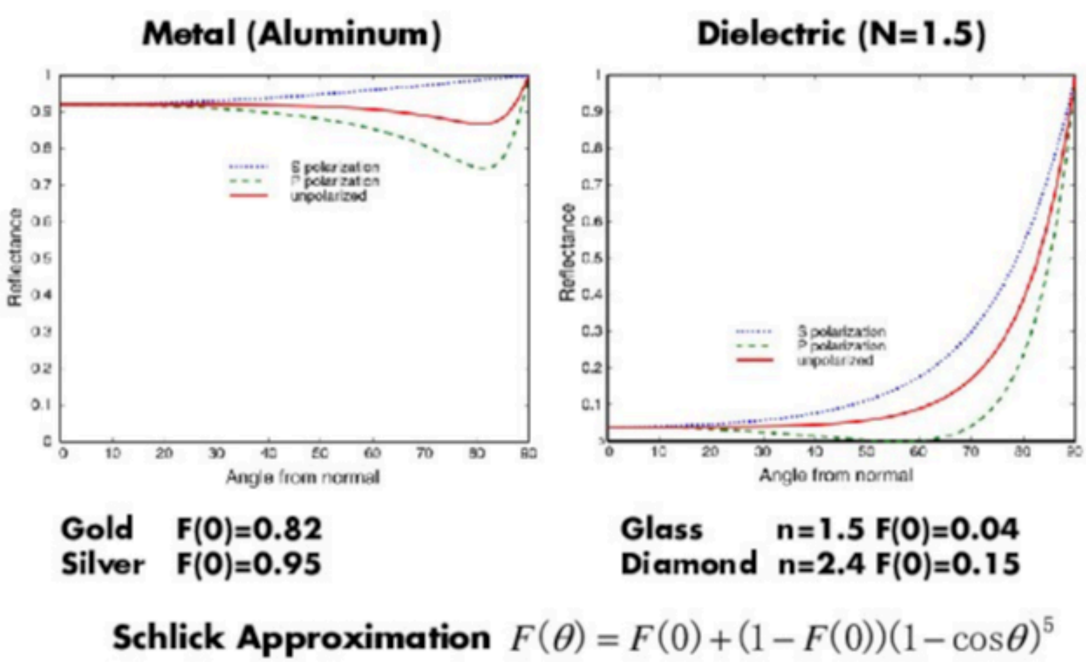
- Create and cast photons into the scene. The photons have locations, propagation directions, and frequencies (i.e., colors), and are emitted at some rate.
- Graphics cannot replicate the physical world of trillions of individual photons, but render about a few million of them, with each represents many real ones.
- Tricks in graphics for real-time rendering: aggregate of photons can be used by the later light transport steps.

Light Transport

- Almost always modeled by ray optics on un-collimated, un-polarized light.
  - Photon no interfere with one another
  - Energy contribution simply sums.

Fresnel reflectance

- Ignoring photon interference and polarization to simplify the representation of light energy is what forces us to complicate our representation of matter.



Fresnel reflectance

- For example, glossy and perfect reflection arises from the interference of nearly-parallel streams of photons. This interference does not arise under ray optics, so we much introduce Fresnel reflectance., such as the Fresnel terms to material to model more realistic scene.

Reflections from a shiny floor

From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

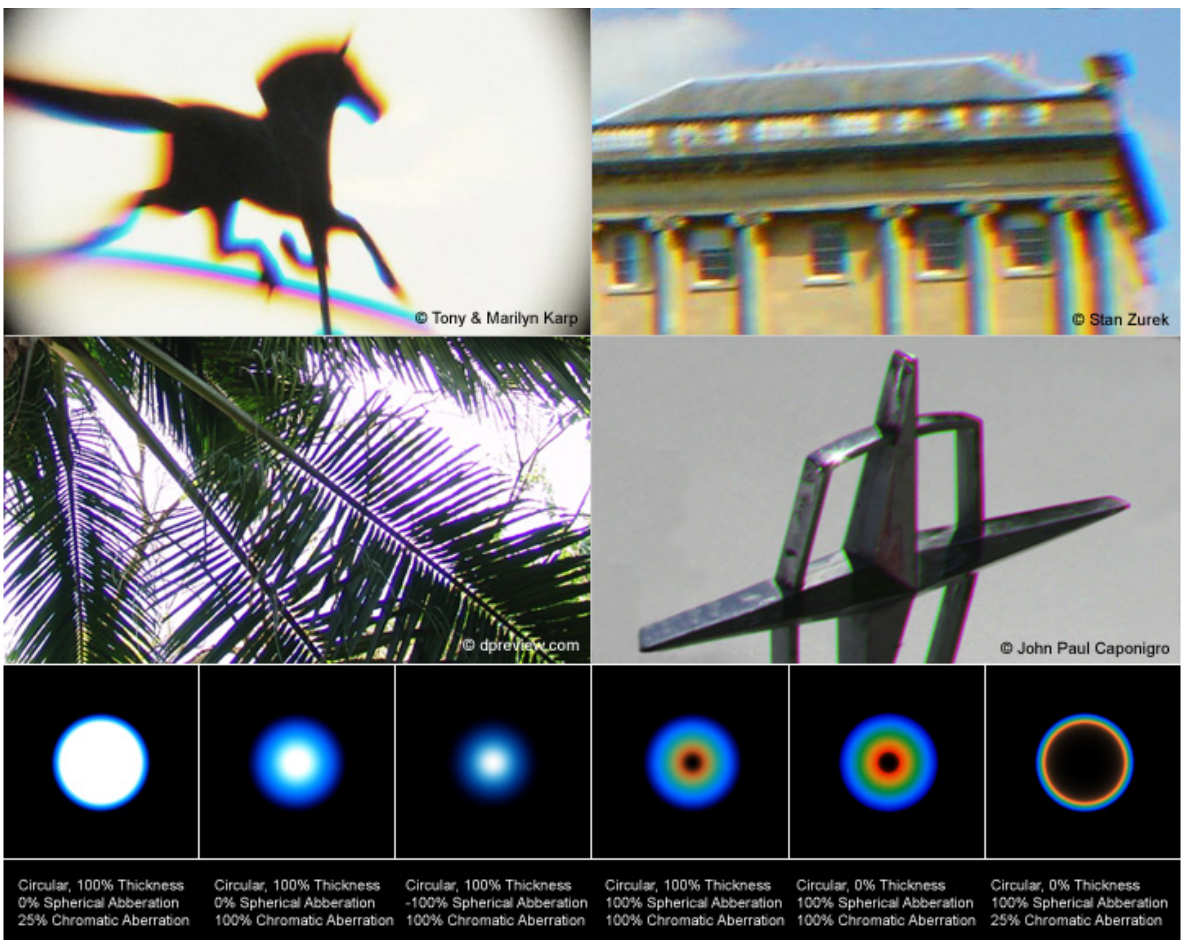
Matter

- Geometry that scatters light
- A simplified model: the surface scattering model only counts the surfaces of opaque objects to reduces the complexity of a scene substantially,
  - Good: a computer graphics house might be only a façade.
  - Poor: skin and fog

- Complex models: represent surfaces at greater details. But often, this depends on the distance to the eyes: tree-> cones if too far away.

Camera

- Lenses (eyes) and sensors (cameras) are complicated.
  - The rainbow-like edges on the objects in the photograph are caused by chromatic aberration in the camera's lens.



Material models

- It is common to distinguish at least five artistically and perceptually significant phenomena:
  - *Sharp specular* (mirror) reflection (e.g., glass)
  - Glossy highlights and reflections (e.g., waxed apple)
  - Shallow subsurface scattering producing matte Lambertian shading that is independent of the viewer's orientation (e.g., "flat" wall paint)

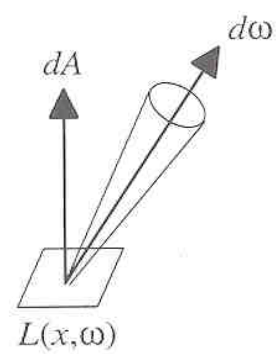
  Lambert's law: most flat, rough surfaces reflect light energy proportional to the cosine of the angle between their surface normal and the direction of the incoming light.
  - Deep subsurface scattering, where light diffuses beneath the surface (e.g., skin, marble to appear soft)
  - Transmission, for translucent material (e.g., water, fog) to produce slighted diffused effects)
- Described by a scattering function

Radiometry

- Physical measurement of electromagnetic energy
  - Integrals of radiance
  - The study of the measurement of radiation is called radiometry
- *Measure spatial (and angular) properties of light*
  - *Radiance, Irradiance*
  - Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF
  - Reflection Equation
  - Simple BRDF models
- Environment Maps

The radiance function

- The amount of light incident at X from direction ω.
  - If there is a photon passing through the point x, traveling in direction ω, then L != 0.
- Mathematically, it is the power per unit projected area perpendicular to the ray per unit solid angle in the direction of the ray



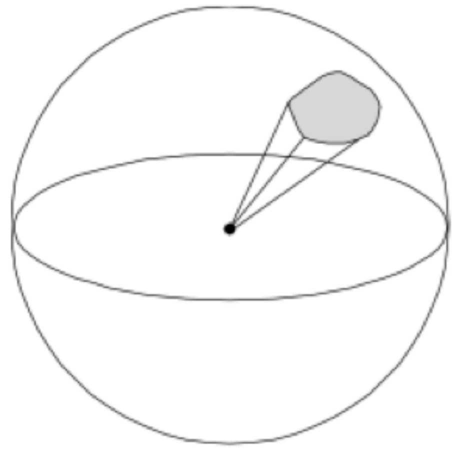
- Symbol:
  - $L(x,\omega)$ (W/m^2 sr)
  - Flux given by $d\Phi = L(x,\omega) \cos\theta \, d\omega \, dA$

Angles and solid angles

Surface areas are measured in units of square meters**, called angle.** Steradian are the spherical analogy of angular measure**, called solid angle.**

- **Angle** $\theta = \frac{l}{r}$

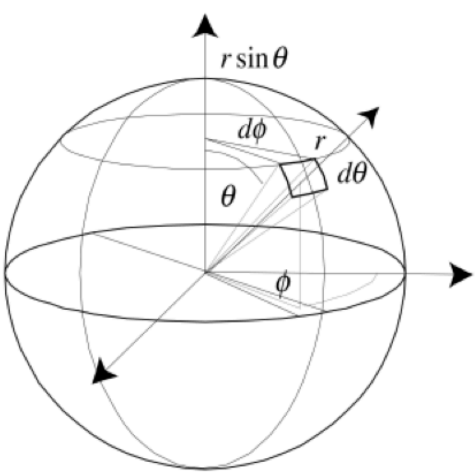
  $\Rightarrow$ circle has 2π radians



- **Solid angle** $\Omega = \frac{A}{R^2}$

  $\Rightarrow$ sphere has 4π steradians

Differential solid angles



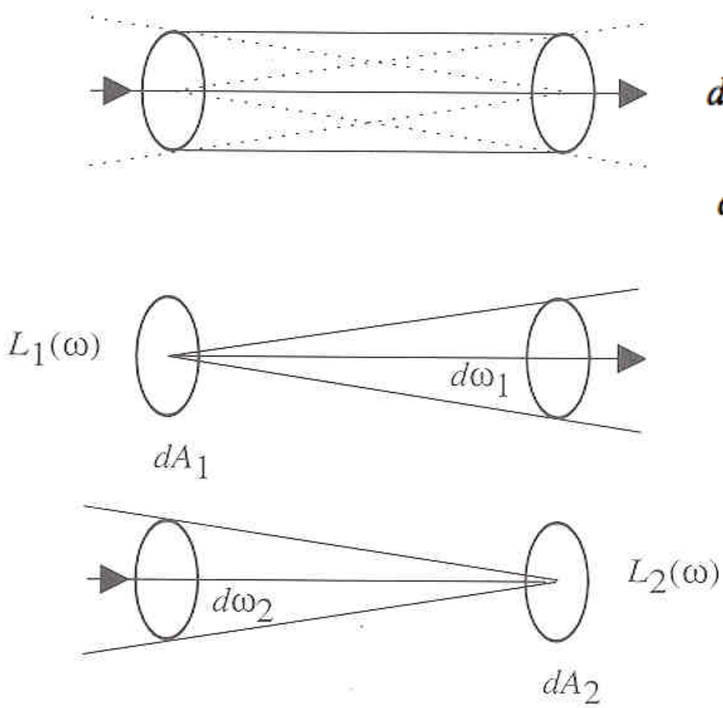
$$dA = (r\,d\theta)(r\sin\theta\,d\phi) = r^2 \sin\theta\,d\theta\,d\phi$$

$$d\omega = \sin\theta\,d\theta\,d\phi$$

$$\Omega = \int_{S^2} d\omega = \int_0^{\pi}\int_0^{2\pi} \sin\theta\,d\theta\,d\phi = \int_{-1}^{1}\int_0^{2\pi} d\cos\theta\;d\phi = 4\pi$$

Radiance properties

- Radiance constant as propagates along ray
  - Derived from conservation of energy
  - Fundamental in light transport



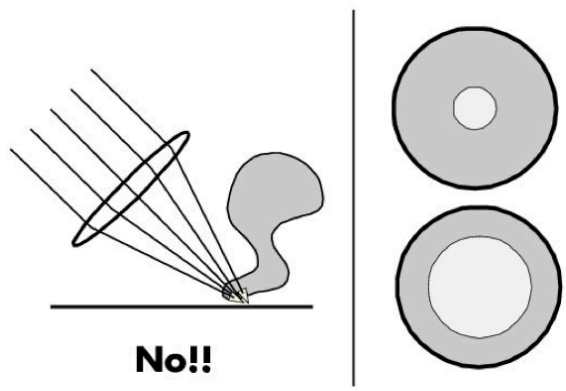
$$d\Phi_1 = L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2 = d\Phi_2$$

$$d\omega_1 = dA_2 / r^2 \qquad d\omega_2 = dA_1 / r^2$$

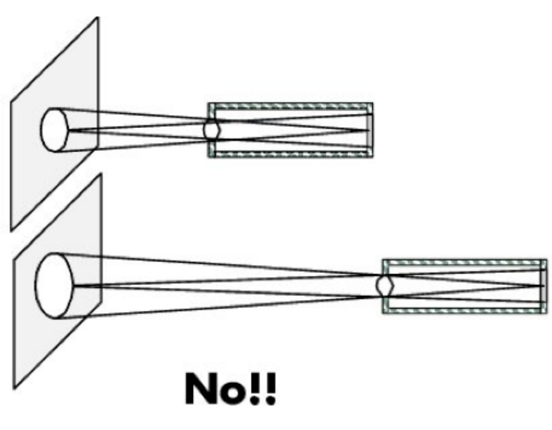
$$d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2$$

$$\therefore L_1 = L_2$$

- Does radiance increase under a magnifying glass?



- Does the brightness that a wall appears to the eye depend on the distance of the viewer to the wall?



Radiance properties

- Sensor response proportional to radiance (constant of proportionality is throughput)
  - Far away surface: See more, but subtends smaller angle
  - Wall equally bright across viewing distances
  - Radiance associated with rays in a ray tracer

Irradiance

- Irradiance E is the density (with respect to area, time, and wavelength) of the light energy arriving at a surface from all direction, or radiant power per unit area.
- When response is direction-dependent, irradiance is irrelevant.

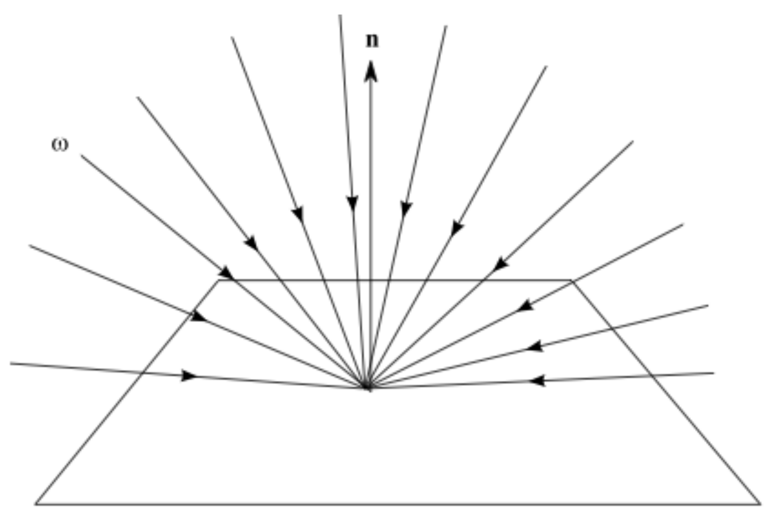
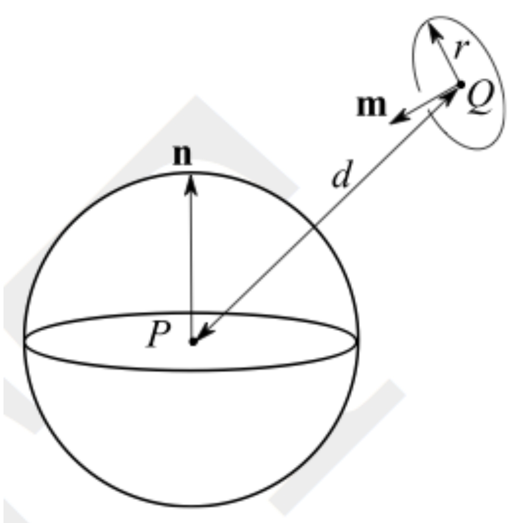


Figure 26.29: Notation for irradiance definition.

Irradiance

- Irradiance due to a single source
  - Show that if the distance from a disk-shaped uniform light source of radius r, center Q, normal vector m, and radiance L0 to the point P is more than 5r, and the source is completely visible from P, then the irradiance at P from the light is well-approximated by



$$\pi r^2 L_0 \frac{(Q-P)\cdot \mathbf{n}\,(P-Q)\cdot \mathbf{m}}{\|Q-P\|^4}.$$

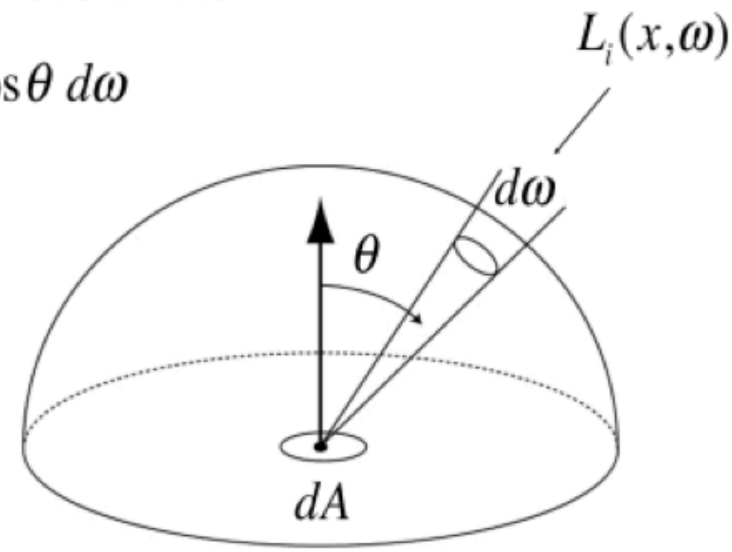
Figure 26.30: The rule of five.

Irradiance from the environment

$$d^2\Phi_i(x,\omega) = L_i(x,\omega)\cos\theta \, dA \, d\omega$$

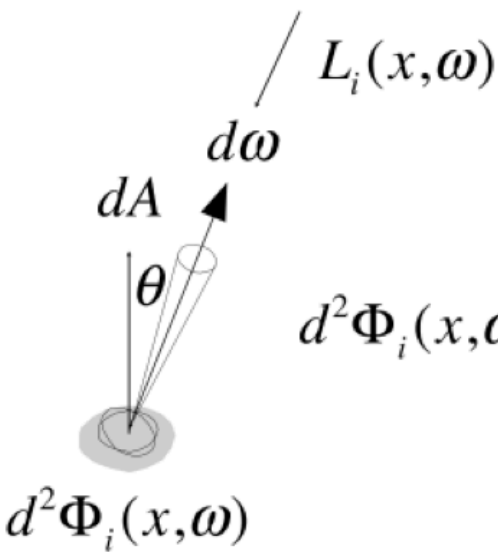
$$dE(x,\omega) = L_i(x,\omega)\cos\theta \; d\omega$$

Light meter



$$E(x) = \int_{H^2} L_i(x,\omega)\cos\theta \; d\omega$$

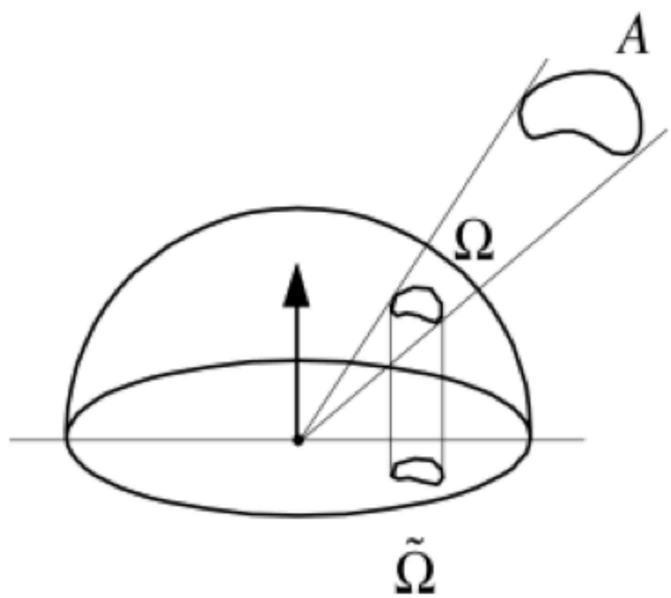
Directional power arriving at a surface



$$d^2\Phi_i(x,\omega) = L_i(x,\omega)\cos\theta \, dA \, d\omega$$

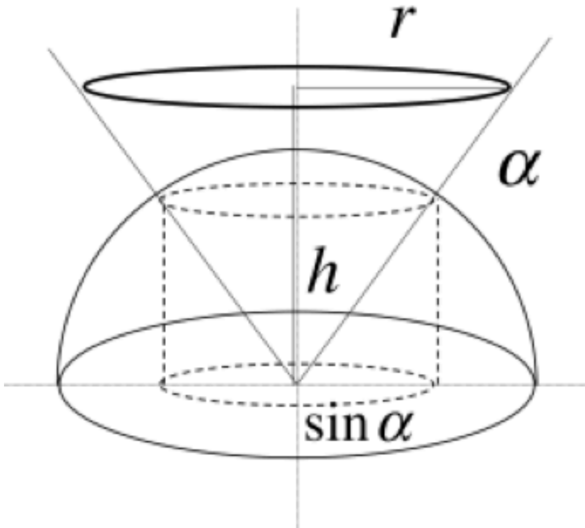
Uniform area source

$$E(x) = \int_{H^2} L \cos\theta \, d\omega$$
$$= L \int_{\Omega} \cos\theta \, d\omega$$
$$= L\tilde{\Omega}$$



Uniform disk source

Geometric Derivation



$$\tilde{\Omega} = \pi \sin^2 \alpha$$

Algebraic Derivation

$$\tilde{\Omega} = \int_{1}^{\cos\alpha} \int_{0}^{2\pi} \cos\theta \, d\phi \, d\cos\theta$$

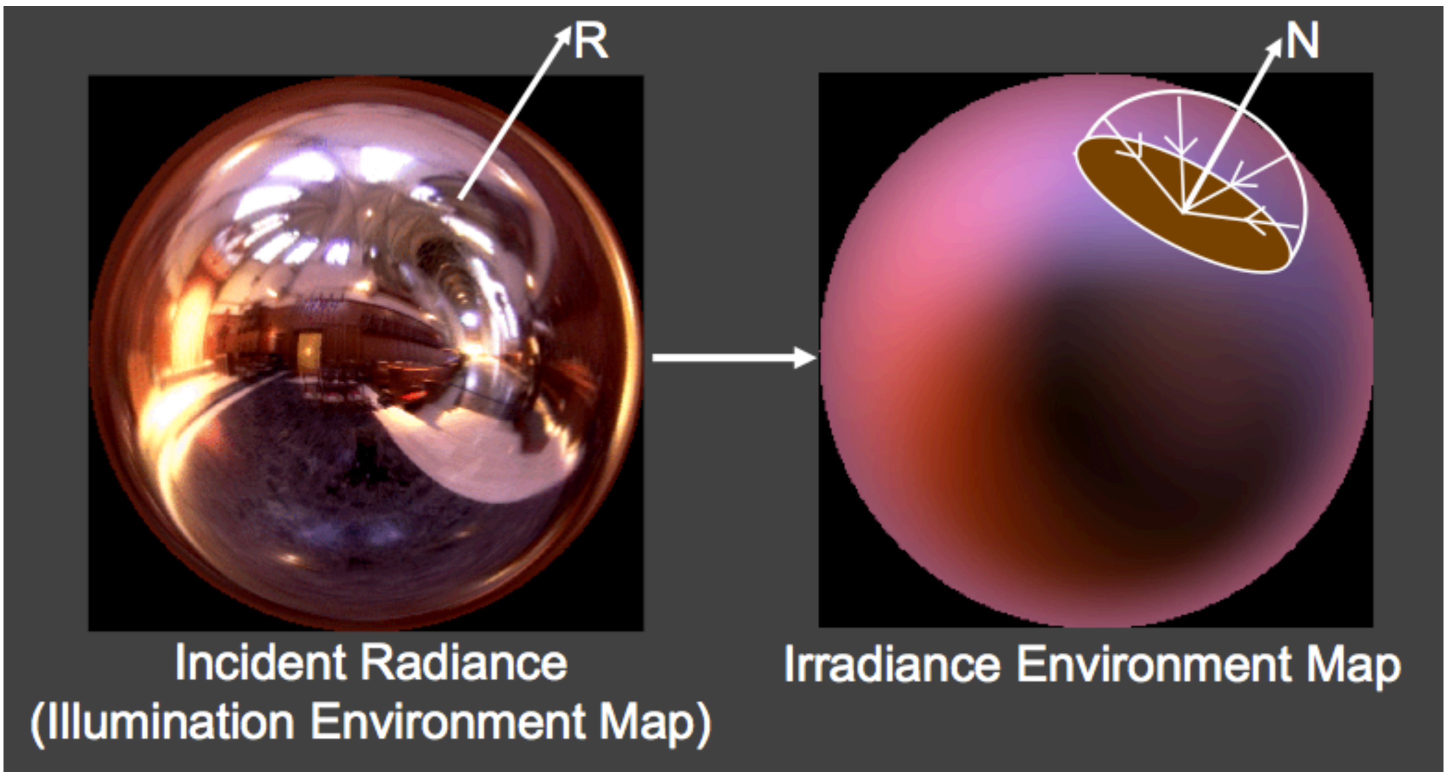
$$= 2\pi \left. \frac{\cos^2\theta}{2} \right|_{1}^{\cos\alpha}$$

$$= \pi \sin^2 \alpha$$

$$= \pi \frac{r^2}{r^2 + h^2}$$

See Book chapter 26.9

Irradiance Environment Maps



Radiosity

- The corresponding measure of light *leaving* a surface in all possible directions is called spectral radiant existance or radiosity
  - Power per unit area leaving surface (just like irradiance with the opposite direction of light)
- Radiant power or radiant flux
  - Computed by integrating again
  - Power measured in Joules per second.

(see books and papers for the equations)

Radiometry

- Physical measurement of electromagnetic energy

- *Measure spatial (and angular) properties of light*
  - Radiance, Irradiance
  - *Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF (section 26.10)*
  - *Reflection Equation*
  - Simple BRDF models
- Environment Maps

Reflectance models

- Light striking a bit of surface from some distant point may scatter in many different directions, and that light from many different directions may therefore all contribute to the light leaving in some particular direction.

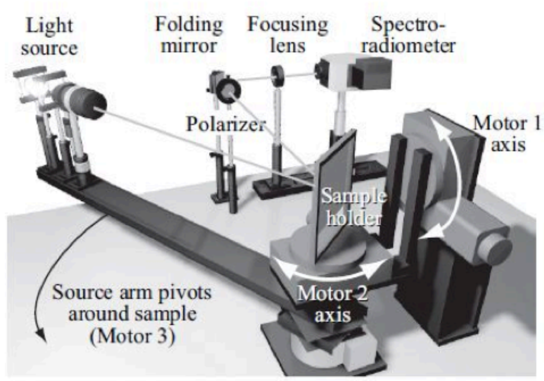


Figure 26.31: A modern gonioreflectometry system. (Courtesy of Steve Westin.)

- Sum of each single scattered light.
- Now we need to computer lights for each single direction.

BRDF

- Bidirectional reflectance distribution function fr [Nicodemus 77]
- The unit of fr is 1/sr, which is independent of time.

$$f_{\mathrm{r}}(P, \boldsymbol{\omega}_{\mathrm{i}}, \boldsymbol{\omega}_{\mathrm{o}}, \lambda) = \frac{L(t, P, \boldsymbol{\omega}_{\mathrm{o}}, \lambda)}{L(t, P, -\boldsymbol{\omega}_{\mathrm{i}}, \lambda)\cos(\phi)\, m(\Omega)} \tag{26.78}$$

where

- Ω is the solid angle subtended by the light source at the sample, and we've written $m(\Omega)$ to indicate its measure, and
- $\boldsymbol{\omega}_{\mathrm{i}}$ and $\boldsymbol{\omega}_{\mathrm{o}}$ are respectively the directions from the sample *to* the source (the subscript "i" is for "incoming") and to the sensor (the subscript "o" is for "outgoing").

$$L^{\mathrm{r}}(t, P, \boldsymbol{\omega}_{\mathrm{o}}, \lambda) = \int_{\boldsymbol{\omega}_{\mathrm{i}} \in \mathbf{S}^2_+(P)} L^{\mathrm{i}}(t, P, -\boldsymbol{\omega}_{\mathrm{i}}, \lambda) f_{\mathrm{r}}(P, \boldsymbol{\omega}_{\mathrm{i}}, \boldsymbol{\omega}_{\mathrm{o}}, \lambda)\, \boldsymbol{\omega}_{\mathrm{i}} \cdot \mathbf{n}(P)\, d\boldsymbol{\omega}_{\mathrm{i}} \tag{26.80}$$

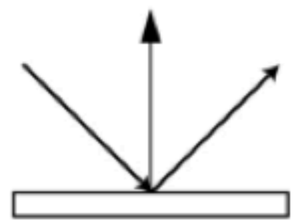
Building up the BRDF

- Function based on incident, view direction
- Relates incoming light energy to outgoing
  - The ratio of outgoing radiance to incoming irradiance
- Unifying framework for many materials

Types of reflection functions

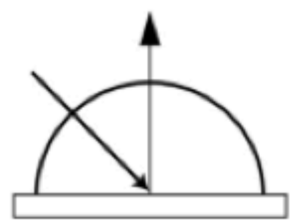
Ideal Specular

- Reflection Law
- Mirror



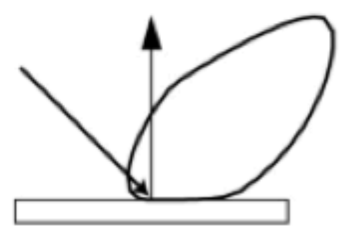
Ideal Diffuse

- Lambert's Law
- Matte



Specular

- Glossy
- Directional diffuse



Materials

Plastic Metal Matte

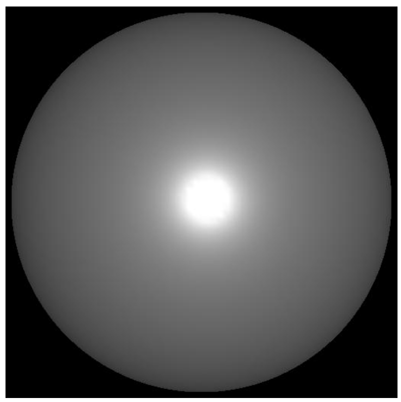
From Apodaca and Gritz, *Advanced RenderMan*

Spheres [Matusik et al.]

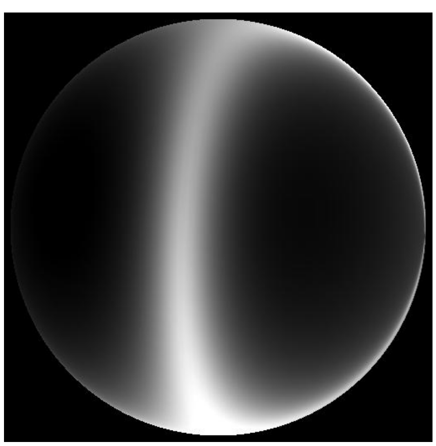
Many other model names to describe different materials.

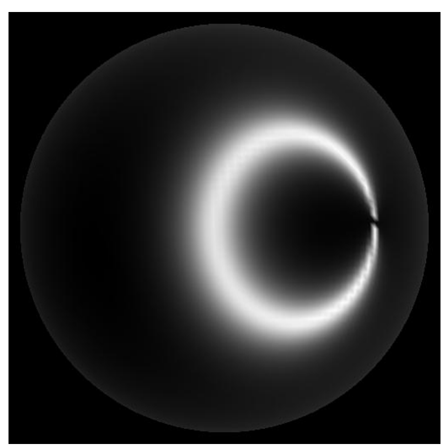
Isotropic vs. anisotropic

- Isotropic: Most materials (you can rotate about normal without changing reflections)

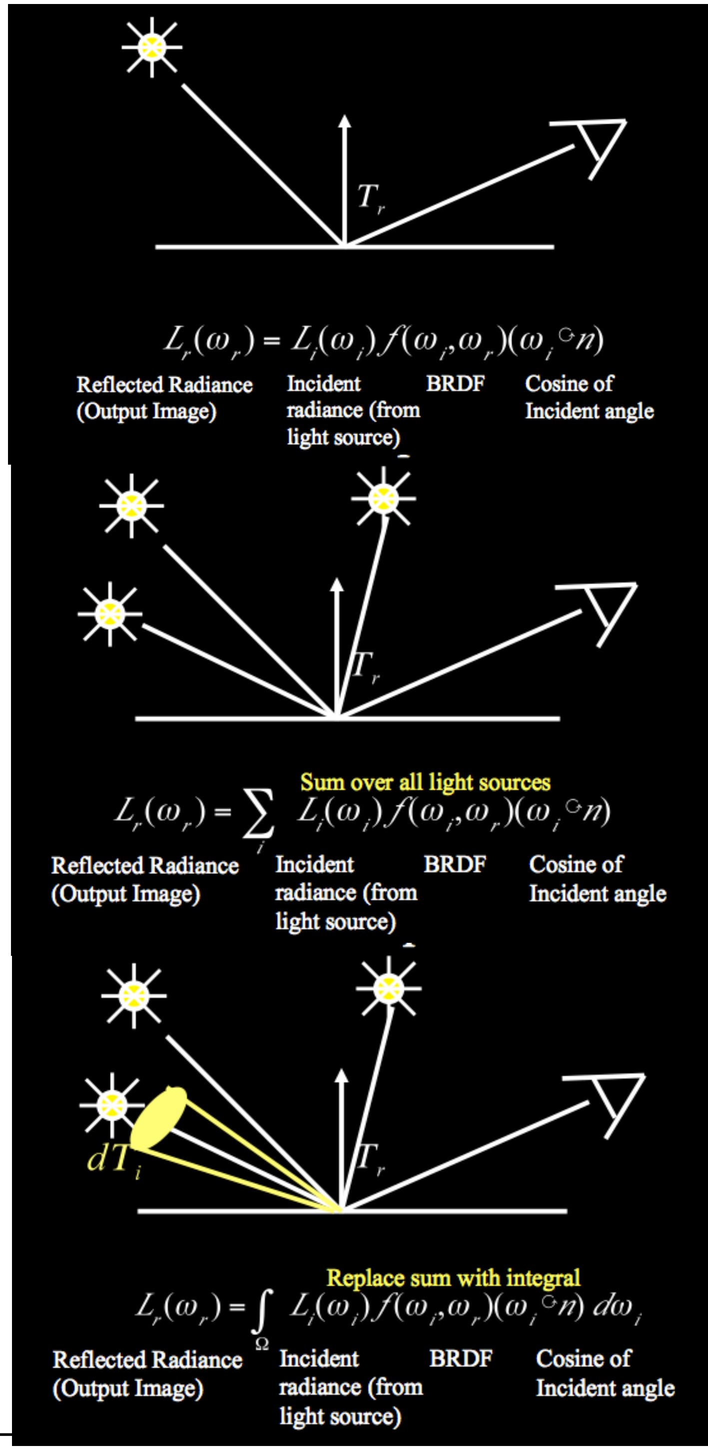


- Anisotropic: brushed metal etc. preferred tangential direction



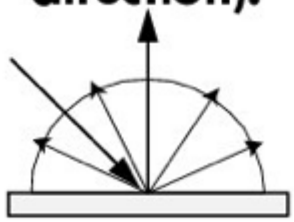


Reflection equation



Ideal diffuse reflection

Assume light is equally likely to be reflected in any output direction (independent of input direction).



$$L_{r,d}(\omega_r) = \int f_{r,d} L_i(\omega_i) \cos\theta_i \, d\omega_i$$
$$= f_{r,d} \int L_i(\omega_i) \cos\theta_i \, d\omega_i$$
$$= f_{r,d} E$$

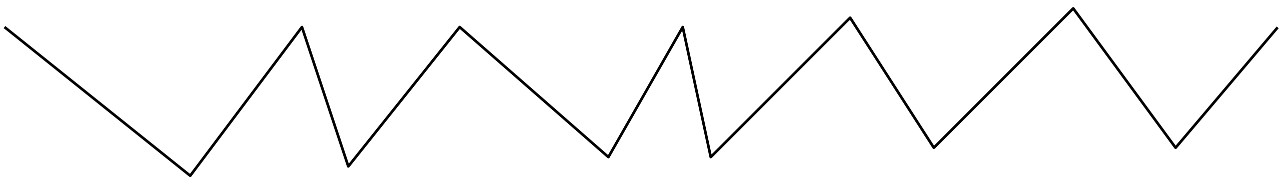
$$M = \int L_r(\omega_r) \cos\theta_r \, d\omega_r = L_r \int \cos\theta_r \, d\omega_r = \pi L_r$$

$$\rho_d = \frac{M}{E} = \frac{\pi L_r}{E} = \frac{\pi f_{r,d} E}{E} = \pi f_{r,d} \quad \Rightarrow \quad f_{r,d} = \frac{\rho_d}{\pi}$$

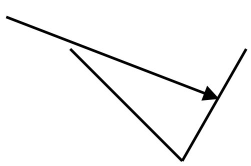
Lambert's Cosine Law $M = \rho_d E = \rho_d E_s \cos\theta_s$

Torrance-Sparrow

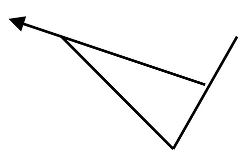
- Assume the surface is made up grooves at the microscopic level.



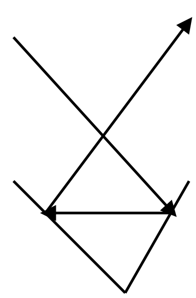
- Assume the faces of these grooves (called microfacets) are perfect reflectors.
- Take into account 3 phenomena



Shadowing

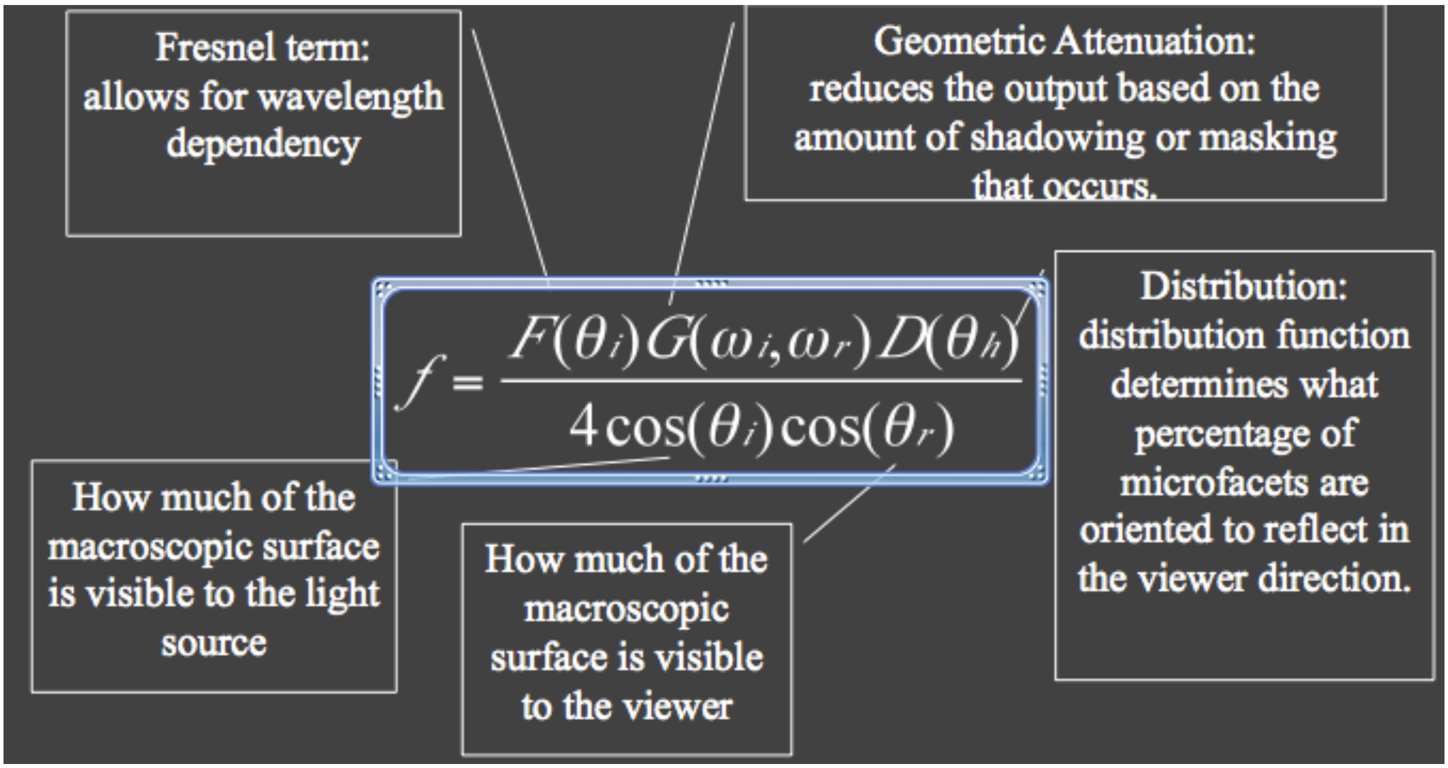


Masking



Inter-reflection

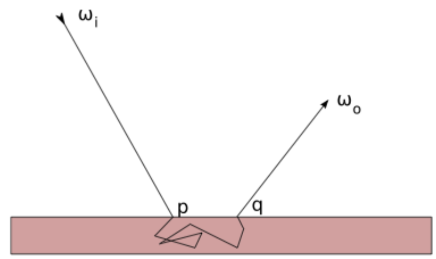
Torrance-Sparrow Result



$$f = \frac{F(\theta_i) G(\omega_i, \omega_r) D(\theta_h)}{4 \cos(\theta_i) \cos(\theta_r)}$$

Other BRDF models

- Empirical: Measure and build a 4D table
- Surface scattering



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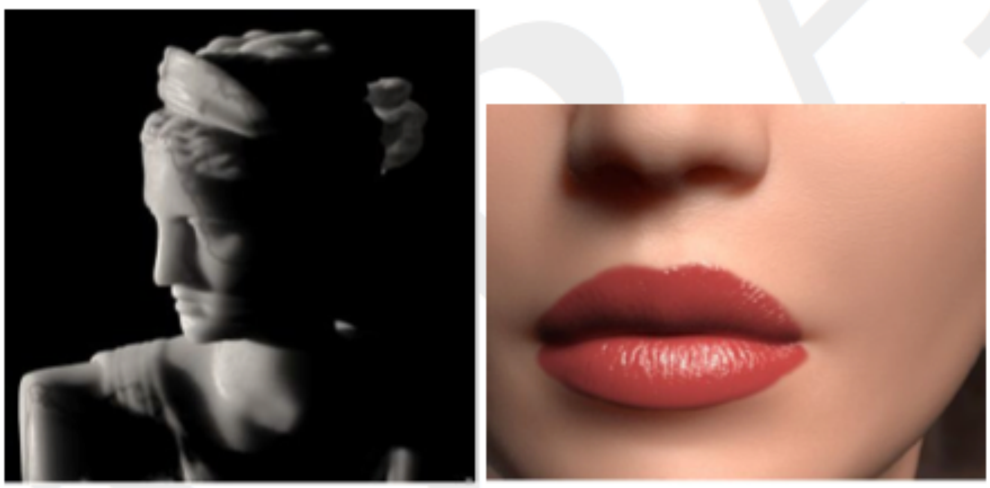


Figure 26.34: Subsurface scattering lets light pass through the marble and skin in these images. (Courtesy of Steve Marschner.) ©2001 ACM, Inc. Included here by permission.

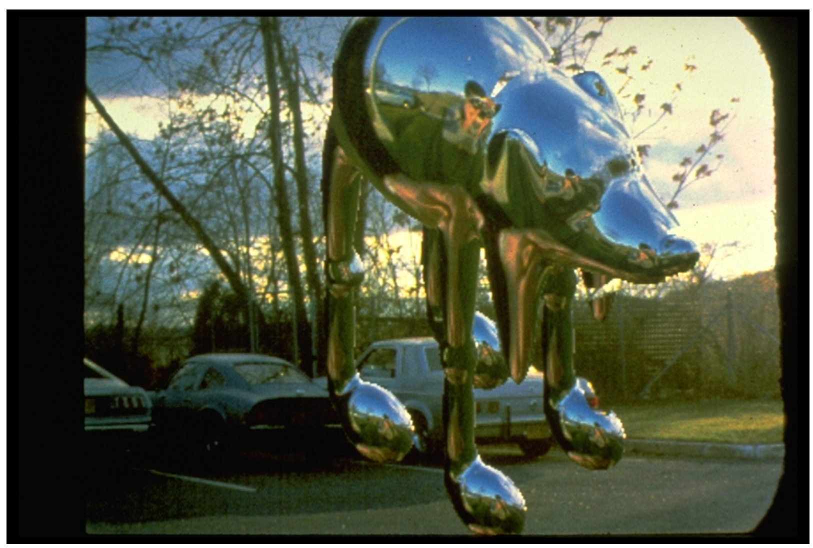
- Cartoon shaders, funky BRDFs
- Very active area of research
- Read 26.10 if you are interested

Radiometry

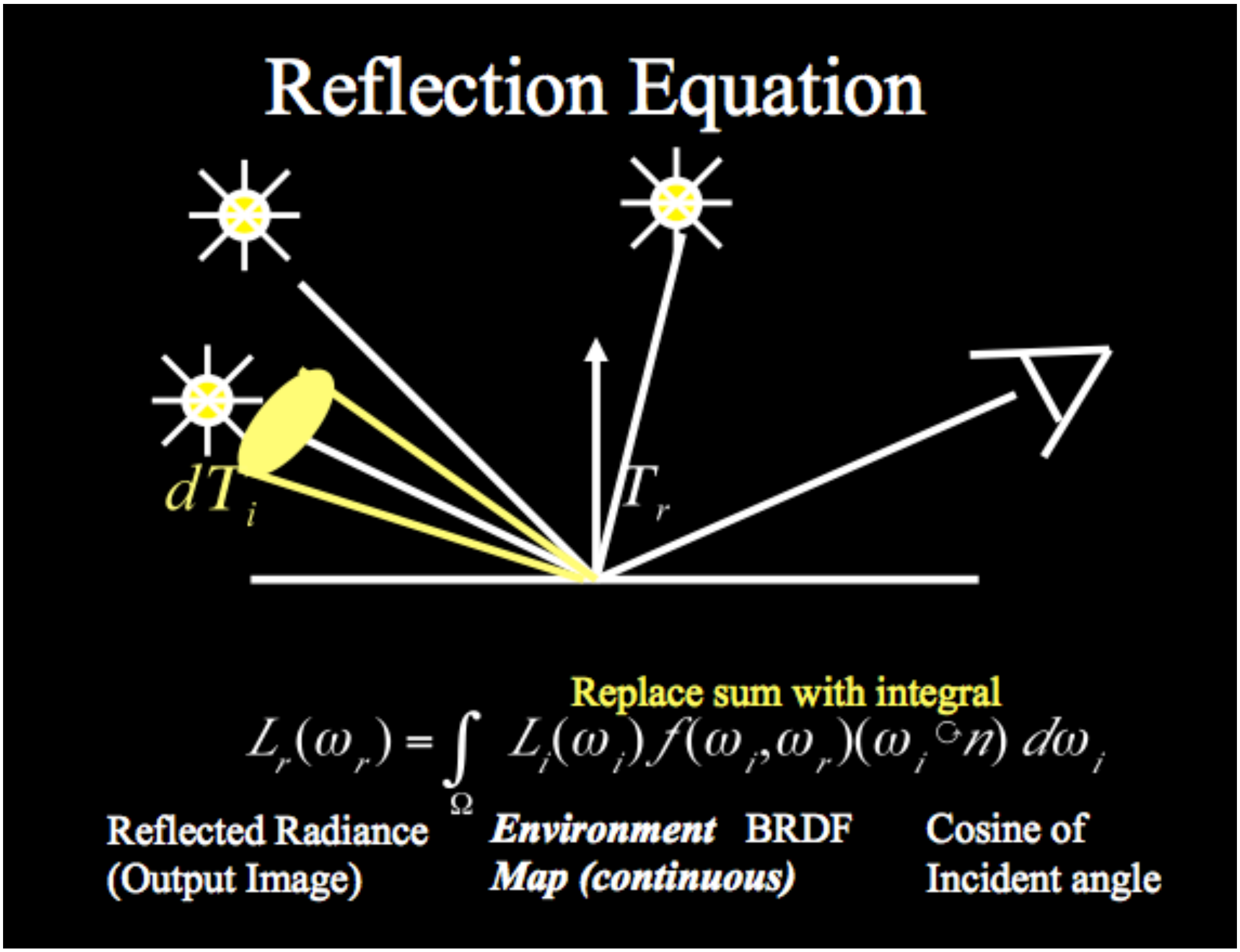
- Physical measurement of electromagnetic energy
- *Measure spatial (and angular) properties of light*
  - Radiance, Irradiance
  - Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF
  - Reflection Equation
  - Simple BRDF models
- Environment Maps

Environment maps

- Light as a function of direction, from entire environment
- Captured by photographing a chrome steel or mirror sphere
- Accurate only for one point, but distant lighting same at other scene locations (typically use only one env. map)

Blinn and Newell 1976, Miller and Hoffman, 1984
Later, Greene 86, Cabral et al. 87


Reflection Equation
dT_i
T_r
Replace sum with integral
$L_r(\omega_r) = \int_\Omega L_i(\omega_i) f(\omega_i, \omega_r)(\omega_i \cdot n)\, d\omega_i$
Reflected Radiance (Output Image)
Environment Map (continuous)
BRDF
Cosine of Incident angle

Environment maps

- Environment maps widely used as lighting representation
- Many modern methods deal with offline and real-time rendering with environment maps
- Image-based complex lighting + complex BRDFs