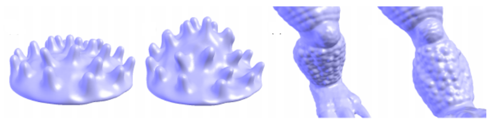
Geometric Morphometrics



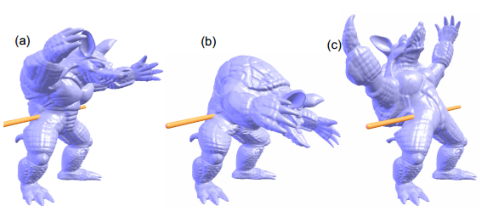
Lecturer: Dr. Keqin Wu

To do

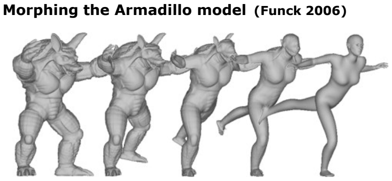
- work on programming assignment 2
- Continue to think about final project
- Form final group

Geometric Morphology What is it?

- Deals with the study of shape



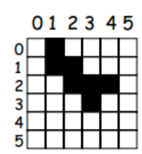
Morphing the Armadillo model **(Funck 2006)**



Set Operations

Example

Binary Image



A = { (1,0), (1,1), (2,1), (2,2), (3,2), (4,2), (3,3) }

In practice, we use the image itself to represent this set.

Set Operations

Basic Definitions

- Let A and B be sets in Z^2
  - with components a=(a1, a2) and b=(b1,b2), respectively
  - The *translation* of A by x = (x1,x2), denoted by $(A)_x$ is defined by:

$$(A)_x = \{c \mid c = a + x, \quad \text{for } a \in A\}$$

Set Operations

Basic Definitions

- The *reflection* of B, denoted by $\hat{B}$ is defined as:

$$\hat{B} = \{x \mid x = -b, \quad \text{for } b \in B\}$$

- The *complement* of set A is:

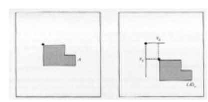
$$A^c = \{x \mid x \notin A\}$$

Basic Definitions

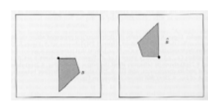
- The *difference* of two sets A and B, denoted by A-B, is defined by:

$$A - B = \{x \mid x \in A, x \notin B\} = A \cap B^c$$

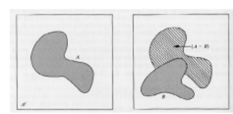
Example



Translation



Reflection



Difference

Set Operations

Dilation Operator

- With A and B as sets in Z^2 and ϕ denoting the empty set, the *dilation* of A by B, denoted by $A \oplus B$ is defined as:

$$A \oplus B = \{x \mid (\hat{B})_x \cap A \neq \phi\}$$

or

$$A \oplus B = \{x \mid (\hat{B})_x \cap A \subseteq A\}$$

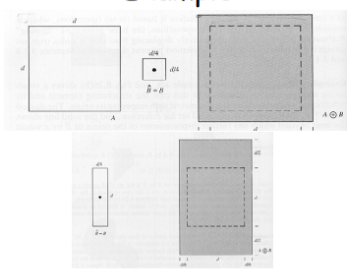
Set Operations

Dilation

- Process consists of obtaining the reflection of B, about its origin
- Then shifting this reflection, $\hat{B}$, by x
- The dilation of A by B is the set of all x displacements such that $\hat{B}$ and A overlap by at least *one* element
- B is often called the "structuring element"

Set Operations

Example



Set Operations

Erosion

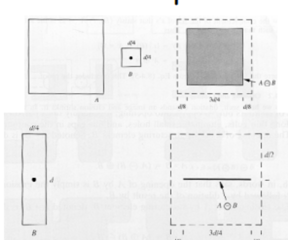
- For sets A and B in Z^2, the erosion of A by B, denoted by $A \ominus B$, is defined by:

$$A \ominus B = \{x \mid (B)_x \subseteq A\}$$

- $A \ominus B$ is the set of all points x, such that B, translated by x, is contained in A.

Set Operations

Example



Set Operations

Dilation and Erosion Relationship

- Are duals of each other with respect to complementation and reflection, that is:

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

Set Operations

Dilation and Erosion

- Dilation
  - expands an image
- Erosion
  - shrinks an image
- From these two operators, we can construct several new operators

Set Operations

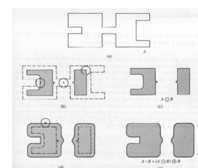
Opening

- The *opening* of set A, by structuring element B is:

$$A \circ B = (A \ominus B) \oplus B$$

- in other words, *opening* of A by B is simply the *erosion* of A by B, followed by a *dilation* by B

Example



Set Operations

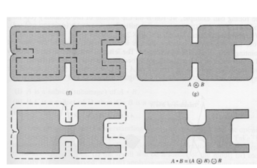
Closing

- The *closing* of set A, by structuring element B is:

$$A \bullet B = (A \oplus B) \ominus B$$

- in other words, *closing* of A by B is simply the *dilation* of A by B, followed by an *erosion* by B

Example



Set Operations

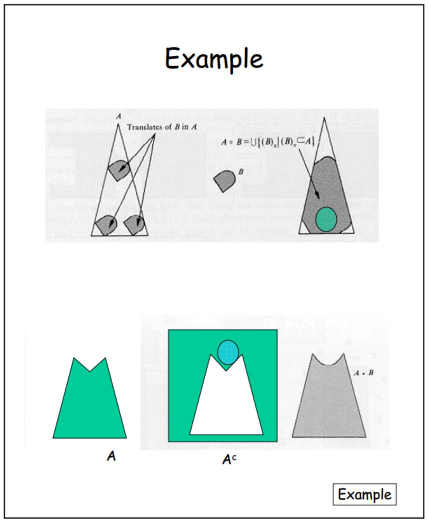
Geometric Interpretation

- Opening
  - can be considered a geometric *fitting* problem
  - it is the union of all translates of B that fit into A

$$A \circ B = \bigcup \{(B)_x \mid (B)_x \subset A\}$$

- *Closing*
  - If you are painting the outside of A (ie, you are paint A^c), it is all the points you cannot paint.

Set Operations



As-Rigid-As-Possible Shape Manipulation

T. Igarashi, T. Mascovich, J. F. Hughes

Presented in SIGGRAPH 2005

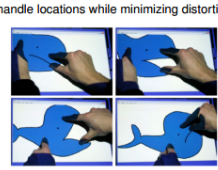
Introduction

- Interactive shape manipulation technique for 2D shapes, without using a skeleton or FFD.
- Introduces internal model rigidity into shape manipulation.
- Uses a quadractic error metric so the minimization problem is formulated as a set of simultaneous linear equations.
- The problem is split into two least-squares minimization problems (rotation and scale) that are solved sequentially.

T. Igarashi, T. Mascovich, J. F. Hughes As-Rigid-As-Possible Shape Manipulation

Introduction

- The user chooses handle points within the shape and moves each handle to a desired location.
- The system the moves, rotates and deforms the shape to match the new handle locations while minimizing distortions.



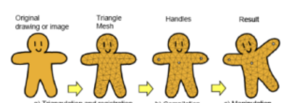
Triangulation

- The boundary of the shape should be represented as a simple closed polygon.
- The system generates a triangulated mesh inside the boundary of the shape.
- Near equilateral triangles are used for better results.
- Mesh should not be too large.
- Resulting triangulation is the **rest shape**.

T. Igarashi, T. Mascovich, J. F. Hughes As-Rigid-As-Possible Shape Manipulation

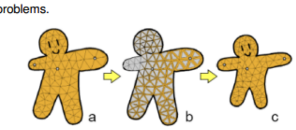
Registration and Compilation

- The system performs a pre-computation, called **registration**, to accelerate computation during the interaction.
- When handles are added or removed, additional pre-computations are performed, called **compilation**.
- Compilation allows computation of resulting shape, given handle configurations.



Algorithm

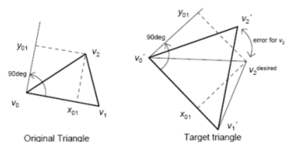
- **Input**: xy-coordinates of the handles.
- **Output**: xy-coordinates of the free vertices that minimize distortion.
- Uses two independent quadratic error functions for the rotation and scaling.
- Final result is obtained by sequentially solving two least-squares problems.



T. Igarashi, T. Mascovich, J. F. Hughes As-Rigid-As-Possible Shape Manipulation

Scale-Free Construction

- Generates an intermediate result by minimizing an error function that allows rotation and uniform scaling.
- For a triangle in rest shape $\{v_0, v_1, v_2\}$, $v_2 = v_0 + x_{01}\overrightarrow{v_0v_1} + y_{01}R_{90}\overrightarrow{v_0v_1}$.
- Given a deformed triangle $\{v_0', v_1', v_2'\}$, $v_2^{desired}$ can be computed similarly.



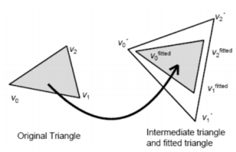
- Then, $E_{v_2} = \|v_2^{desired} - v_2'\|^2$.
- And, $E_{\{v_0,v_1,v_2\}} = \sum_{i=1,2,3}\|v_i^{desired} - v_i'\|^2$
- Error for the entire mesh is sum of error for all triangles, expressed in matrix form as: $E_{1_{\{v'\}}} = \mathbf{v'^T G v'}$
  $\Rightarrow \mathbf{G'u} + \mathbf{Bq} = \mathbf{0}$
- Only $\mathbf{q}$ changes during manipulation.
- $\mathbf{G}$ can be pre-computed during registration and $\mathbf{G'^{-1}B}$ during compilation.

Fitting triangles

- The fitted triangle should minize $E_{f_{\{v_0^{fitted},v_1^{fitted},v_2^{fitted}\}}} = \sum_{i=1,2,3}\|v_i^{fitted} - v_i'\|^2$.
- Approximate by first minimizing the error allowing uniform scaling.
- Like before, we have: $v_2^{fitted} = v_0^{fitted} + x_{01}\overline{v_0^{fitted}v_1^{fitted}} + y_{01}R_{90}\overline{v_0^{fitted}v_1^{fitted}}$
- So the fitting functional is a quadratic in the four free variables $w = (v_0x^{fitted}, v_0y^{fitted}, v_1x^{fitted}, v_1y^{fitted})$
- Minimize E_f by setting partial derivaties over w to zero.
- $\Rightarrow \frac{dE_f}{dw} = \mathbf{Fw} + \mathbf{C} = 0$
- F is fixed for a given mesh, so can be pre-computed and inverted during registration.
- C is defined by the result of Step 1 and is computed during manipulation.
- Scale the fitted triangle obtained by solving this equation by a factor of $\|v_0^{fitted} - v_1^{fitted}\| / \|v_0 - v_1\|$
- This makes the fitted triangle congruent to the corresponding triangle in the rest shape.

Scale Adjustment

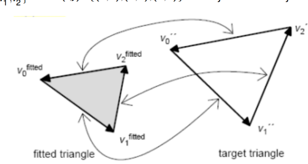
- Step 1 doesn capture changes in scale and is handled here.
- First each triangle in the rest shape is **fitted** in the intermediate result, allowing rotation and translation.
- For a triangle $\{v_0', v_1', v_2'\}$ in the intermediate result, we need to find a new triangle $\{v_0^{fitted}, v_1^{fitted}, v_2^{fitted}\}$ that is congruent to the corresponding triangle $\{v_0, v_1, v_2\}$ in the rest shape.



- The fitted triangle should minize $E_{f_{\{v_0^{fitted},v_1^{fitted},v_2^{fitted}\}}} = \sum_{i=1,2,3}\|v_i^{fitted} - v_i'\|^2$.
- Approximate by first minimizing the error allowing uniform scaling.
- Like before, we have: $v_2^{fitted} = v_0^{fitted} + x_{01}\overline{v_0^{fitted}v_1^{fitted}} + y_{01}R_{90}\overline{v_0^{fitted}v_1^{fitted}}$
- So the fitting functional is a quadratic in the four free variables $w = (v_0x^{fitted}, v_0y^{fitted}, v_1x^{fitted}, v_1y^{fitted})$

Generating final result

- Need to reconcile vertices of several triangles that correspond to the same mesh vertex.
- For a triangle $\{v_0'', v_1'', v_2''\}$ and corresponding fitted triangle $\{v_0^{fitted}, v_1^{fitted}, v_2^{fitted}\}$, a quadratic edge error function is defined:
  $E_{2_{\{v_0'',v_1'',v_2''\}}} = \sum_{(i,j)\in\{(0,1),(1,2),(2,0)\}}\|\overrightarrow{v_i''v_j''} - \overrightarrow{v_i^{fitted}v_j^{fitted}}\|^2$



Generating final result

- The optimal position for a vertex is some average of the positions desired by each triangle in which it appears.
- The error for the entire mesh is given by: $E_{2_{v''}} = \mathbf{v''}^{\mathbf{T}}\mathbf{H}\mathbf{v''} + \mathbf{f}\mathbf{v''} + c$
- H is defined by the connectivity of the original mesh and, f and c are determined by the fitted triangles.

- To minimize E_2, setting partial derivatives over u to zero, we get: $\mathbf{H'u} + \mathbf{Dq} + \mathbf{f_0} = 0$
- H can be precomputed during registration and, H' and D during compilation.
- f_0 is computed during manipulation using fitted triangles.
- Final result obtained by solving the last equation using pre-computed LU factorization of H'.

ADVANCED COMPUTER GRAPHICS

Results

- Pentium III 1GHz processor, 756MB memory and Java implementation.
- Real-time performance.
- Interaction deteriorates starting at around 300 vertices.
- Fairly robust against uneven triangulation and irregularly spaced mesh.

Table 1: Sample running times (milliseconds) for the meshes in Figure 17

# of vertices	Registration		Compilation		Update	
	Step1	Step2	Step1	Step2	Step1	Step2
93	16	18	14	4	0.06	2.2
150	42	38	29	8	0.09	3.5
287	160	107	72	19	0.16	7.5

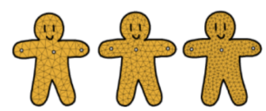


Figure 17: Example triangulations. The number of vertices is 85, 156, 298 respectively and three handles are attached to each.

ADVANCED COMPUTER GRAPHICS

Demo

- http://www.youtube.com/watch?v=1M_oyUEOHK8

CMSC 635 Feb 27, 2013 Geometric Morphology

ADVANCED COMPUTER GRAPHICS

Deformable Models in Image Analysis

CMSC 635 Feb 27, 2013 Geometric Morphology

Mathematical Foundations

- Geometry
- Physics
- Approximation theory

Mathematical Foundations

- Geometry
  - Broad shape coverage by employing geometric representations that involve many degrees of freedom, such as splines.
  - The model remains manageable because the degrees of freedom are generally not permitted to evolve independently

Mathematical Foundations

- Physics
  - The physical interpretation views deformable models as elastic bodies which respond naturally to applied forces and constraints.
  - The energy grows monotonically as the model deforms away from a specified natural or “rest shape”
  - Often includes terms that constrain the smoothness or symmetry of the model
  - The deformation energy gives rise to elastic forces internal to the model.

Deformable model geometry

- Approximation theory
  - In physics-based view of classical optimal approximation, external potential energy functions are defined in terms of the data of interest to which the model is to be fitted.
  - These potential energies give rise to external forces which deform the model such that it fits the data

Image Analysis with Deformable Models

- Medical image interpretation tasks include segmentation, matching, and motion analysis.

Snakes: Active Contour Models (MICHAEL KASS, 1988)

- A snake is an energy-minimizing spline guided by external constraint forces and influenced by image forces that pull it toward features such as lines and edges.
- The interface allows a user to select starting points and exert forces on snakes interactively as they minimize their energy.

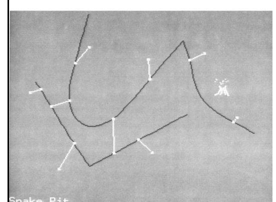


Fig. 2. The Snake Pit user-interface. Snakes are shown in black, springs and the volcano are in white.

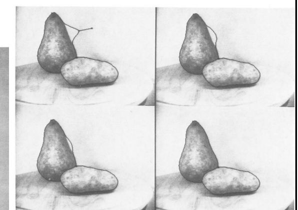


Fig. 3. Two edge snakes on a pear and potato. Upper-left: The user has pulled one of the snakes away from the edge of the pear. Others: After the user lets go, the snake snaps back to the edge of the pear.

Image Segmentation with Deformable Curve

- Algorithm:

- Initial Points defined around Feature to be extracted
  - Explicitly defined
  - Approximation of an Ellipse
- Pre-defined number of Points generated
- Points are moved through an Iterative Process
- "Energy Function" for each point in the Local Neighbourhood is calculated
- Move to point with lowest Energy Function
- Repeat for every point
- Iterate until Termination Condition met
  - Defined number of iterations
  - Stability of the position of the points

Volume Image Segmentation with Deformable Surfaces

- Medical image interpretation tasks including segmentation, matching, and motion analysis.

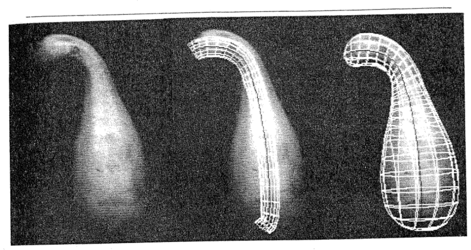


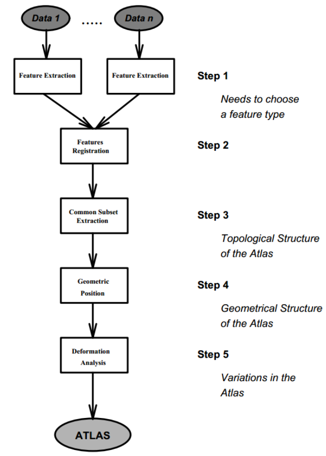
Figure 3. Reconstruction of a 3D model. Left to right: squash image; user initialized spine and shell; reconstructed squash.

On matching deformable models to images. (D. Terzopoulos, 1986)

Matching

- Matching of regions in images can be performed between
- the representation of a region and a model (labeling) or be- tween the representation of two distinct regions (registration).
- Incorporating Priori Knowledge
- Deformable atlas technique

Deformable atlas technique



A General Scheme for Automatically Building 3D Morphometric Anatomical Atlases: application to a Skull Atlas (Ge rard Subsol, 1995)

Deformable atlas technique

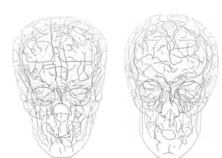


Figure 6: *Two sets of crest lines to register (left $\mathcal{B}$, right $\mathcal{C}$). Notice the difference in size and orientation and the variations in the lines shape, number and discretization. There are 591 lines and 19 302 points in the left set, 583 lines and 19 368 points in the right one.*

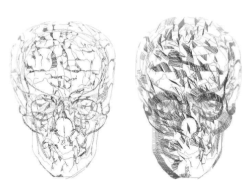


Figure 9: *Registration of $\mathcal{C}$ towards $\mathcal{B}$. Left, we see the deformed set $\mathcal{C}$ with $\mathcal{B}$. The matched points are linked. Notice how the two sets are well superimposed. Right, $\mathcal{C}$ is in its original position. It allows to estimate the extent of the deformation between the two sets.*

Deformable atlas technique

Figure 12: *The topological structure of the atlas is displayed for the skulls $\mathcal{B}$ and $\mathcal{C}$. Some subsets of common lines have been automatically labelled and highlighted: the mandible (bottom and up), the nose, the orbits, the cheekbones, the temples, the occipital foramen and the sphenoid and temporal bones.*

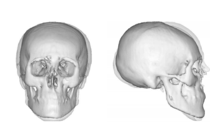


Figure 18: *Rigid and scale registration of the atlas (opaque) with $\mathcal{CR}$ (transparent) emphasizing the symptoms of the Crouzon syndrome as the low mandible, the flattened shape of the skull or the large orbits.*

Motion Tracking and Analysis

- Vector Field Based Shape Deformations
  (Wolfram von Funck 2006) Demo

Figure 1: Volume-preserving deformation of the hand model (36619 vertices): no skeletal hand model is involved, no self-intersections occur.

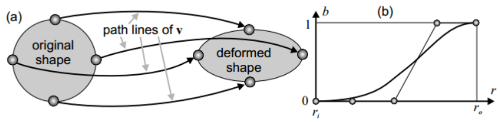


Figure 2: (a) Vector field based shape deformation: every vertex of the original shape undergoes a path line integration of **v** to find its new position. (b) Blending function $b(r)$.