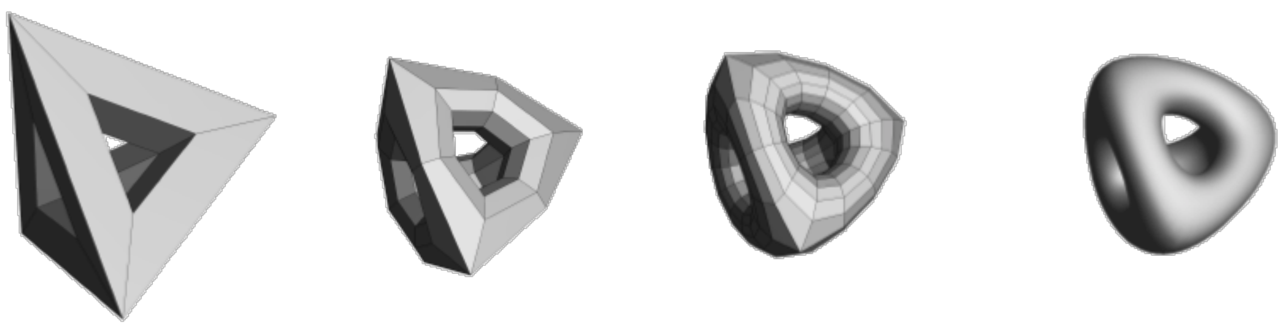
Subdivision

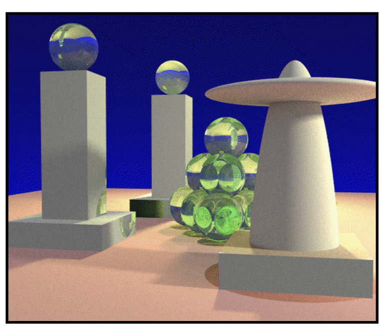


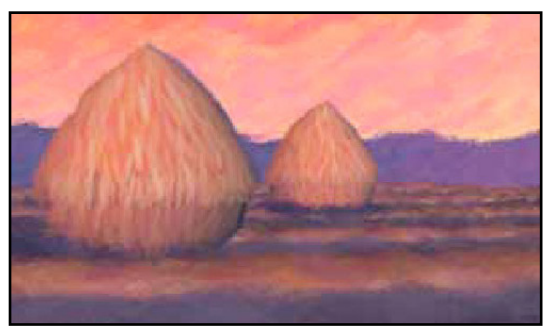
Materials from Denis Zorin, Peter Schroder et al. siggraph presentations

Motivation

- Still a hot topic today in computer graphics
- Advantages: simple (only need subdivision rule); local (only look at nearby vertices); arbitrary topology (since only local); no seams (unlike joining spline patches)
- Detailed study quite sophisticated
  - See online materials

In this class, we will briefly survey literature and discuss some ideas



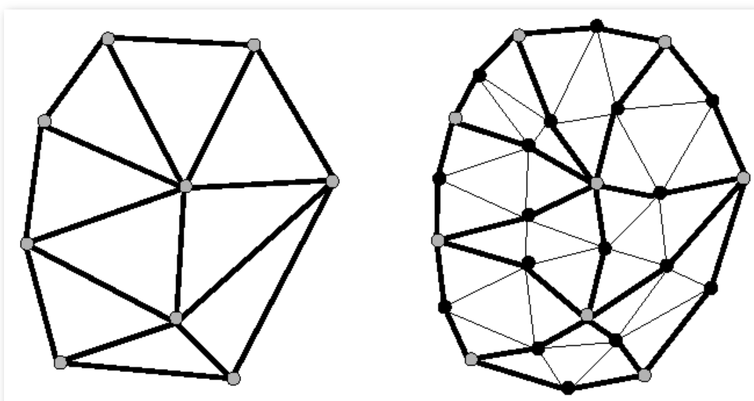


Motivation

- E.g., Gari's Game created using subdivision: http://www.youtube.com/watch?v=1m7dcbIKvlw

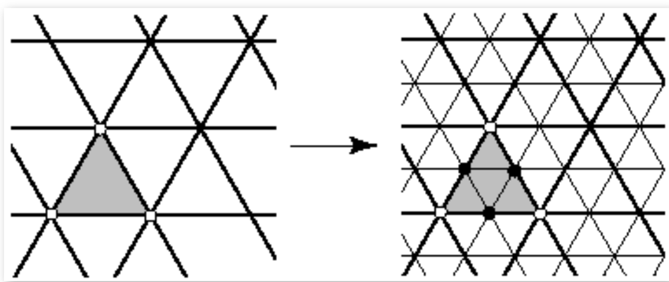
Key Questions

- How to refine mesh?
- Where to place new vertices?
  - Provable properties about limit surface

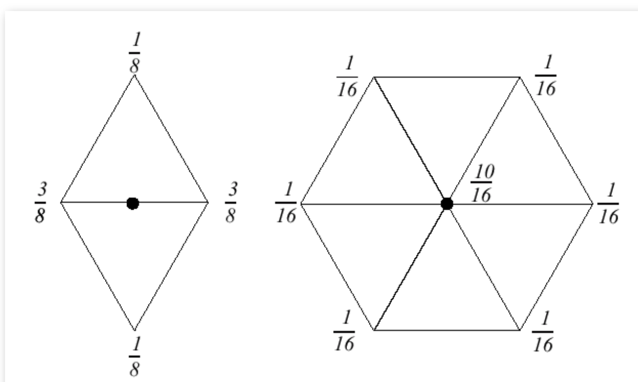


Loop Subdivision Scheme

- How to refine mesh?
  - Refine each triangle into 4 triangles by splitting each edge and connecting new vertices

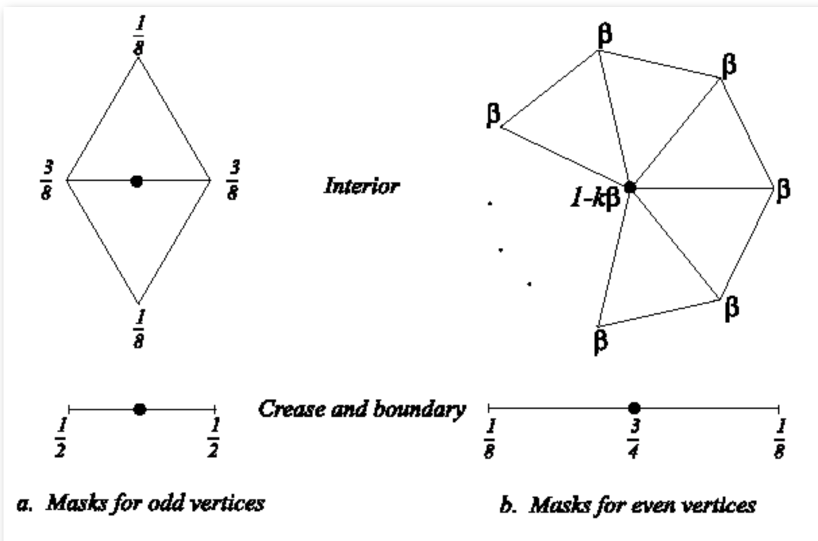


- Where to place new vertices?
  - Choose locations for new vertices as weighted average of original vertices in local neighborhood



Loop Subdivision Scheme

- Where to place new vertices?
  - Rules for extraordinary vertices and boundaries



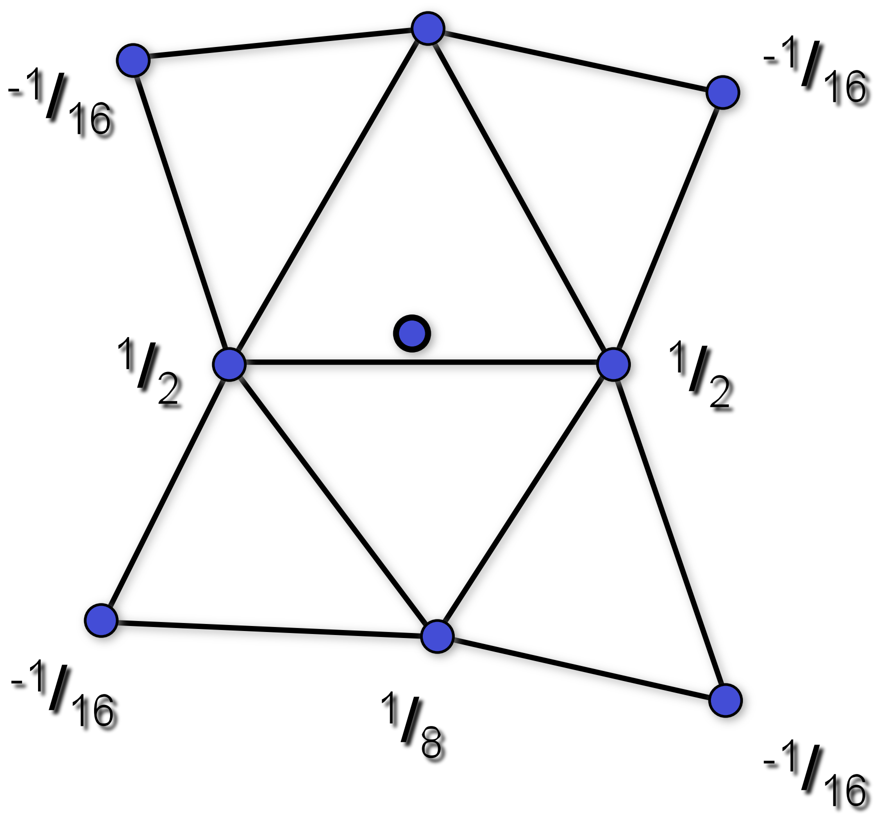
Choose β by analyzing continuity of limit surface

Original Loop: $\beta = \frac{1}{n}\left(\frac{5}{8} - \left(\frac{3}{8} + \frac{1}{4}\cos\frac{2\pi}{n}\right)^2\right)$

Warren: $\beta = \begin{cases} \frac{3}{8n} & n > 3 \\ \frac{3}{16} & n = 3 \end{cases}$

Butterfly Subdivision

- Interpolating subdivision: Larger neighborhood

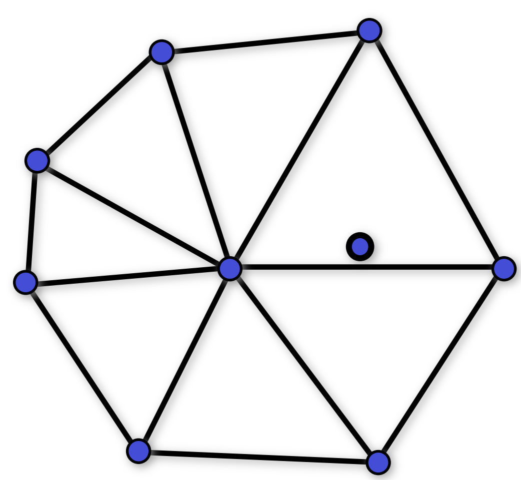


Modified Butterfly Subdivision

- Need special weights near extraodinary vertices
  - For n=3, weights are 5/12, -1/12, -1/12
  - For n=4, weights are 3/8, 0, -1/8, 0
  - For n lt.eq 5, weights are,

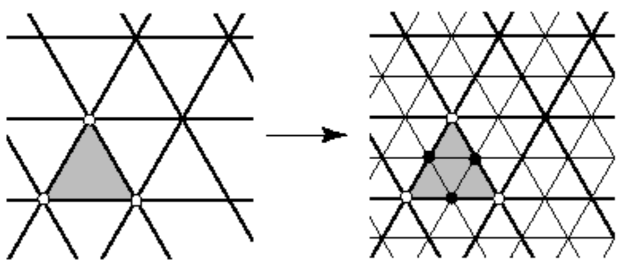
$$\frac{1}{n}\left(\frac{1}{4}+\cos\frac{2\pi j}{n}+\frac{1}{2}\cos\frac{4\pi j}{n}\right), j=0..n-1$$

  - Weight of extraordinary vertex = 1 = the sum of other weights

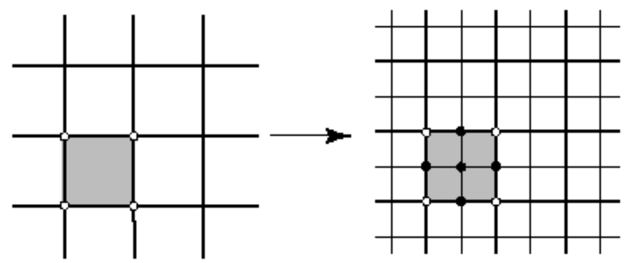


A Variety of Subdivision Schemes

- Triangles vs. quads
- Interpolating vs. approximating



Face split for triangles

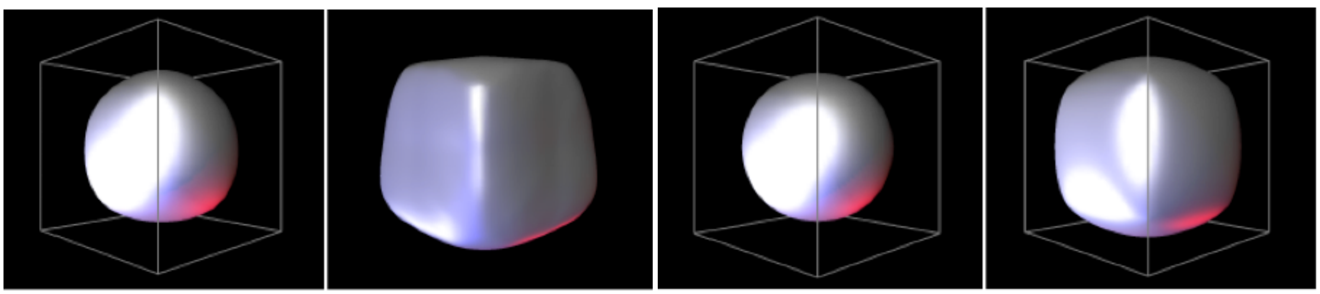


Face split for quads

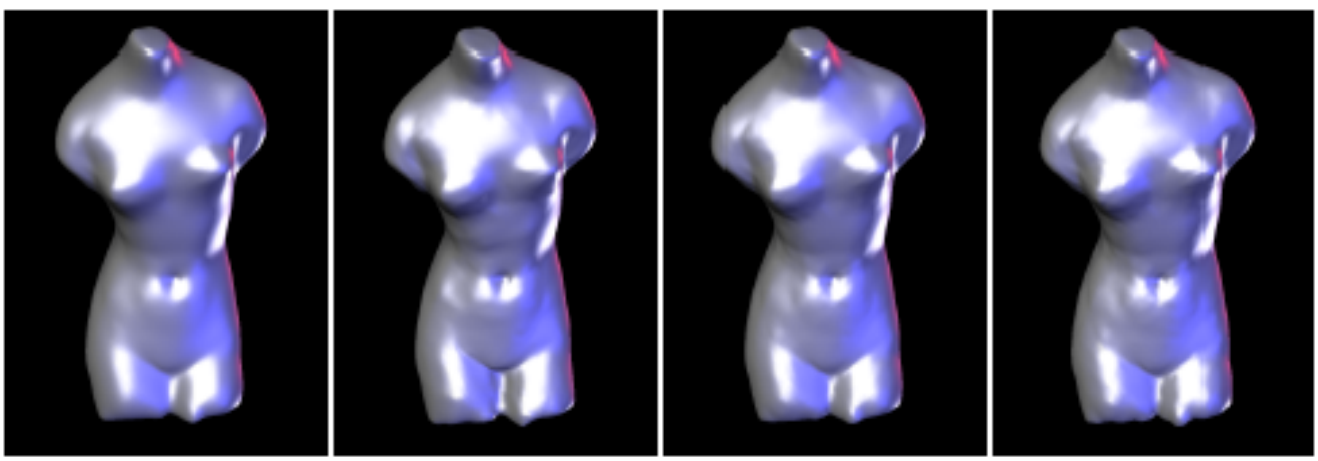
Face split		
	Triangular meshes	*Quad. meshes*
Approximating	Loop (C^2)	Catmull-Clark (C^2)
Interpolating	Mod. Butterfly (C^1)	Kobbelt (C^1)

Vertex split
Doo-Sabin, Midedge (C^1)
Biquartic (C^2)

Results



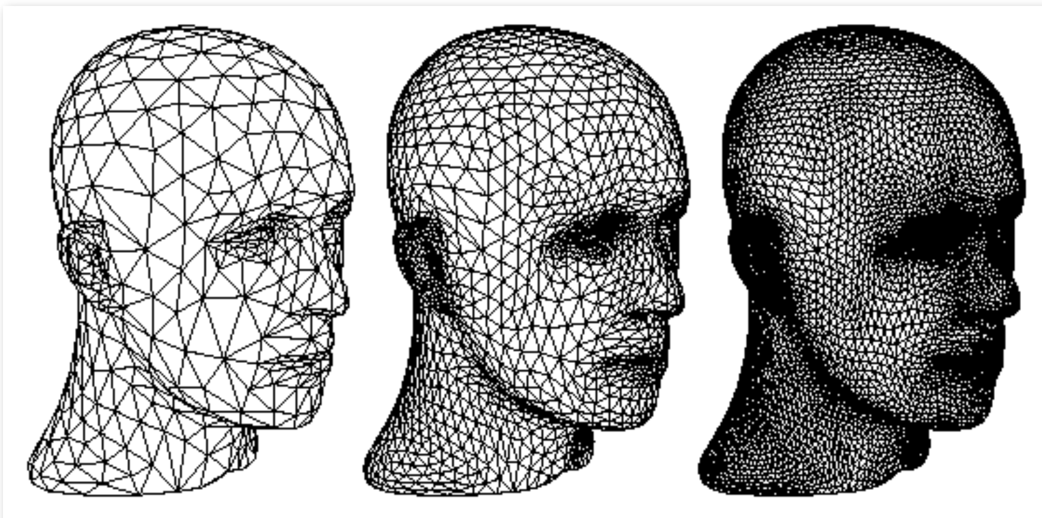
Loop | Butterfly | Catmull-Clark | Doo-Sabin



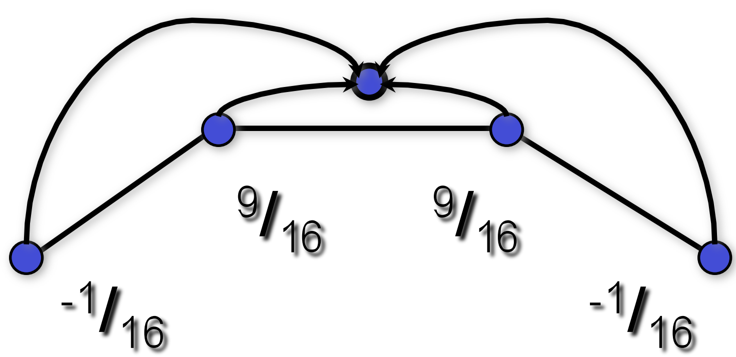
Loop | Butterfly | Catmull-Clark | Doo-Sabin

Analysis of Continuity

- Analyzing subdivision schemes
  - Smoothness properties

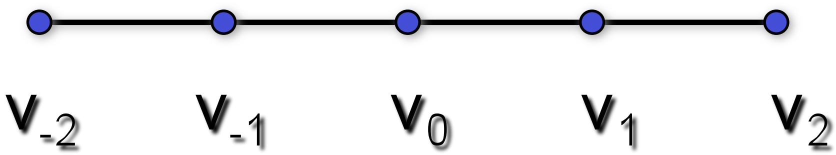


  - Start with curves: 4-point interpolating scheme (old points left where they are)

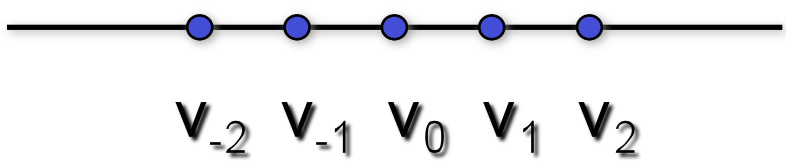


Calculate New Points

Step *i*:



Step *i+1*:



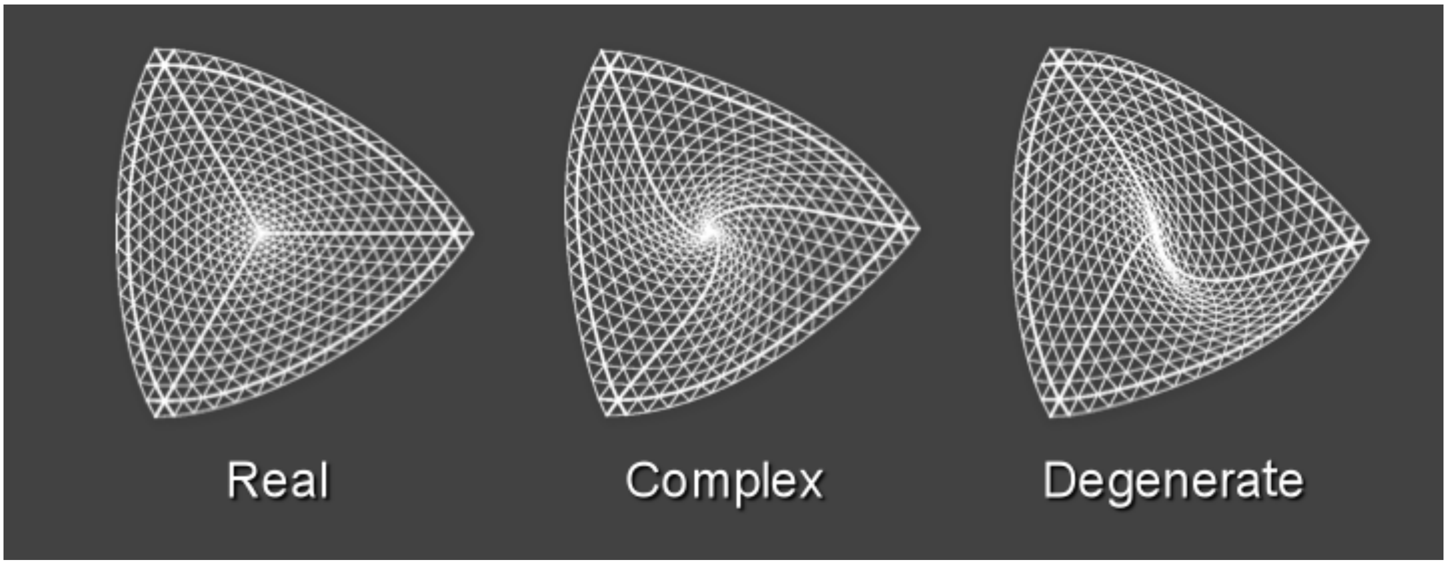
$$\begin{pmatrix} V_{-2}^{(i+1)} \\ V_{-1}^{(i+1)} \\ V_{0}^{(i+1)} \\ V_{1}^{(i+1)} \\ V_{2}^{(i+1)} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ -1/16 & 9/16 & 9/16 & -1/16 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1/16 & 9/16 & 9/16 & -1/16 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} V_{-2}^{(i)} \\ V_{-1}^{(i)} \\ V_{0}^{(i)} \\ V_{1}^{(i)} \\ V_{2}^{(i)} \end{pmatrix}$$

$$\vec{\mathbf{V}}^{(i+1)} = \mathbf{S}\vec{\mathbf{V}}^{(i)}$$

After *n* rounds: $\vec{\mathbf{V}}^{(n)} = \mathbf{S}^{n}\vec{\mathbf{V}}^{(0)}$

Fun with Subdivision Methods

- Behavior of surfaces depends on eigenvalues of the matrix



Practical Evaluation

- Problems with Uniform Subdivision
  - Exponential growth of control mesh
  - Need several subdivisions before error is small
  - OK if you are “drawing and forgetting”, otherwise…
- Exact Evaluation at arbitrary points
- Tangent and other derivative evaluation needed

- Jos Stam SIGGRAPH 98 efficient method
  - Exact evaluation (essentially take out “subdivision)
  - Smoothness analysis methods used to evaluate