Advanced Computer Graphics (Spring 2013)

Mesh representation, overview of mesh simplification

Many slides courtesy Szymon Rusinkiewicz

Motivation

- A polygon mesh is a collection of triangles
- We want to do operations on these triangles
  - E.g. walk across the mesh for simplification
  - Display for rendering
  - Computational geometry
- Best representations (mesh data structures)?
  - Compactness
  - Generality
  - Simplicity for computations
  - Efficiency

Mesh Data Structures

Desirable Characteristics 1

- Generality – from most general to least
  - Polygon soup
  - Only triangles
  - 2-manifold → ≤ 2 triangles per edge
  - Orientable → consistent CW / CCW winding
  - Closed → no boundary
- Compact storage

Mesh Data Structures

Desirable characteristics 2

- Efficient support for operations:
  - Given face, find its vertices
  - Given vertex, find faces touching it
  - Given face, find neighboring faces
  - Given vertex, find neighboring vertices
  - Given edge, find vertices and faces it touches
- These are adjacency operations important in mesh simplification, many other applications

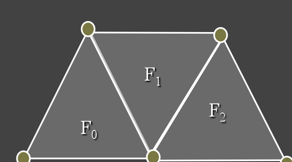
Outline

- *Independent faces*
- *Indexed face set*
- Adjacency lists
- Winged-edge
- Half-edge

Overview of mesh decimation and simplification

Independent Faces

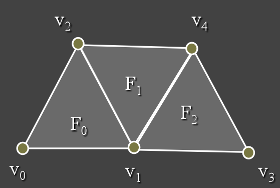
Faces list vertex coordinates
  - Redundant vertices
  - No topology information



Face Table
F_0: (x_0,y_0,z_0), (x_1,y_1,z_1), (x_2,y_2,z_2)
F_1: (x_3,y_3,z_3), (x_4,y_4,z_4), (x_5,y_5,z_5)
F_2: (x_6,y_6,z_6), (x_7,y_7,z_7), (x_8,y_8,z_8)

Indexed Face Set

- Faces list vertex references – “shared vertices”
- Commonly used (e.g. OFF file format itself)
- Augmented versions simple for mesh processing



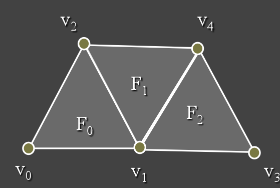
Vertex Table
v_0: (x_0,y_0,z_0)
v_1: (x_1,y_1,z_1)
v_2: (x_2,y_2,z_2)
v_3: (x_3,y_3,z_3)
v_4: (x_4,y_4,z_4)

Face Table
F_0: 0, 1, 2
F_1: 1, 4, 2
F_2: 1, 3, 4

Note CCW ordering

Indexed Face Set

- Storage efficiency?
- Which operations supported in O(1) time?



Vertex Table
v_0: (x_0,y_0,z_0)
v_1: (x_1,y_1,z_1)
v_2: (x_2,y_2,z_2)
v_3: (x_3,y_3,z_3)
v_4: (x_4,y_4,z_4)

Face Table
F_0: 0, 1, 2
F_1: 1, 4, 2
F_2: 1, 3, 4

Note CCW ordering

Efficient Algorithm Design

- Can *sometimes* design algorithms to compensate for operations not supported by data structures
- Example: per-vertex normals
  - Average normal of faces touching each vertex
  - With indexed face set, vertex → face is $O(n)$
  - Naive algorithm for all vertices: $O(n^2)$
  - Can you think of an $O(n)$ algorithm?

Efficient Algorithm Design

- Can *sometimes* design algorithms to compensate for operations not supported by data structures
- Example: per-vertex normals
  - Average normal of faces touching each vertex
  - With indexed face set, vertex → face is $O(n)$
  - Naive algorithm for all vertices: $O(n^2)$
  - Can you think of an $O(n)$ algorithm?
- Useful to augment with vertex → face adjacency
  - For all vertices, find adjacent faces as well
  - Can be implemented while simply looping over faces

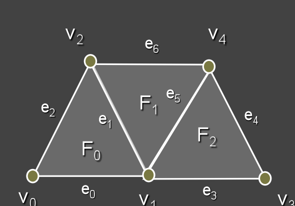
Outline

- Independent faces
- Indexed face set
- *Adjacency lists*
- Winged-edge
- Half-edge

Overview of mesh decimation and simplification

Full Adjacency Lists

- Store all vertex, face, and edge adjacencies



Edge Adjacency Table
e_0: v_0, v_1; $F_0,\varnothing$; $\varnothing,e_2,e_1,\varnothing$
e_1: v_1,v_2; F_0,F_1; e_5,e_0,e_2,e_6
⋮

Face Adjacency Table
F_0: v_0,v_1,v_2; $F_1,\varnothing,\varnothing$; e_0,e_2,e_0
F_1: v_1,v_4,v_2; $\varnothing,F_0,F_2$; e_6,e_1,e_5
F_2: v_1,v_3,v_4; $\varnothing,F_1,\varnothing$; e_4,e_5,e_3

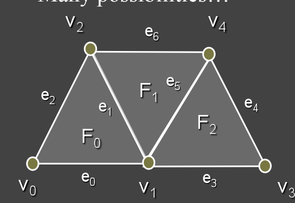
Vertex Adjacency Table
v_0: v_1,v_2; F_0; e_0,e_2
v_1: v_3,v_4,v_2,v_0; F_2,F_1,F_0; e_3,e_5,e_1,e
⋮

Full adjacency: Issues

- Garland and Heckbert claim they do this
- Easy to find stuff
- Issue is storage
- And updating everything once you do something like an edge collapse for mesh simplification
- I recommend you implement something simpler (like indexed face set plus vertex to face adjacency)

Partial Adjacency Lists

- Store some adjacencies, use to derive others
- Many possibilities…



Edge Adjacency Table

e_0: v_0, v_1; F_0,∅; ∅,e_2,e_1,∅

e_1 : v_1,v_2; F_0,F_1; e_5,e_0,e_2,e_6

⋮

Face Adjacency Table

F_0: v_0,v_1,v_2; F_1,∅,∅; e_0,e_2,e_0

F_1 : v_1,v_4,v_2; ∅,F_0,F_2; e_6,e_1,e_5

F_2: v_1,v_3,v_4; ∅,F_1,∅; e_4,e_5,e_3

Vertex Adjacency Table

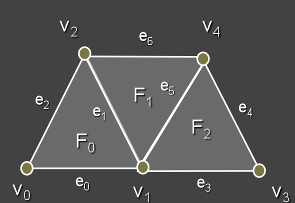
v_0: v_1,v_2; F_0; e_0,e_2

v_1 : v_3,v_4,v_2,v_0; F_2,F_1,F_0; e_3,e_5,e_1,e_0

⋮

Partial Adjacency Lists

- Some combinations only make sense for closed manifolds



Edge Adjacency Table

e_0: v_0, v_1; F_0,∅; ∅,e_2,e_1,∅

e_1 : v_1,v_2; F_0,F_1; e_5,e_0,e_2,e_6

⋮

Face Adjacency Table

F_0: v_0,v_1,v_2; F_1,∅,∅; e_0,e_2,e_0

F_1 : v_1,v_4,v_2; ∅,F_0,F_2; e_6,e_1,e_5

F_2: v_1,v_3,v_4; ∅,F_1,∅; e_4,e_5,e_3

Vertex Adjacency Table

v_0: v_1,v_2; F_0; e_0,e_2

v_1: v_3,v_4,v_2,v_0; F_2,F_1,F_0; e_3,e_5,e_1,e

⋮

Outline

- Independent faces
- Indexed face set
- Adjacency lists
- *Winged-edge*
- *Half-edge*

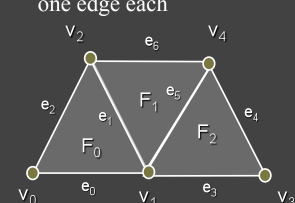
Overview of mesh decimation and simplification

Winged, Half Edge Representations

- Idea is to associate information with edges
- Compact Storage
- Many operations efficient
- Allow one to walk around mesh
- Usually general for arbitrary polygons (not triangles)
- But implementations can be complex with special cases relative to simple indexed face set++ or partial adjacency table

Winged Edge

- Most data stored at edges
- Verts, faces point to one edge each



Edge Adjacency Table

e_0: v_0, v_1; F_0,∅; ∅,e_2,e_1,∅

e_1 : v_1,v_2; F_0,F_1; e_5,e_0,e_2,e_6

⋮

Face Adjacency Table

F_0: v_0,v_1,v_2; F_1,∅,∅; e_0,e_2,e_0

F_1: v_1,v_4,v_2; ∅,F_0,F_2; e_6,e_1,e_5

F_2: v_1,v_3,v_4; ∅,F_1,∅; e_4,e_5,e_3

Vertex Adjacency Table

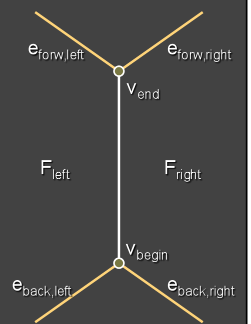
v_0: v_1,v_2; F_0; e_0,e_2

v_1: v_3,v_4,v_2,v_0; F_2,F_1,F_0; e_3,e_5,e_1,e

⋮

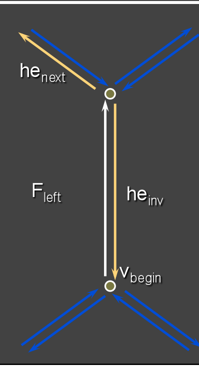
Winged Edge

- Each edge stores 2 vertices, 2 faces, 4 edges – fixed size
- Enough information to completely “walk around” faces or vertices
- Think how to implement
  - Walking around vertex
  - Finding neighborhood of face
  - Other ops for simplification



Half Edge

- Instead of single edge, 2 directed “half edges”
- Makes some operations more efficient
- Walk around face very easily (each face need only store one pointer)



Outline

- Independent faces
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Overview of mesh decimation and simplification

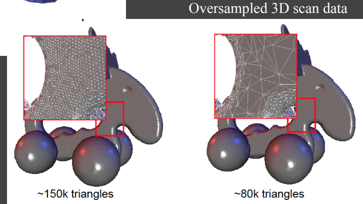
Mesh Decimation

Multi-resolution hierarchies for efficient geometry processing and level of detail rendering

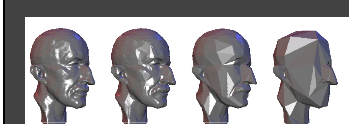
Triangles
: 41,855
27,970
20,922
12,939
8,385
4,766

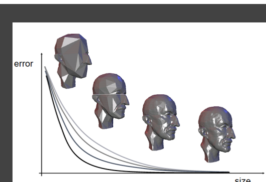
Division, Viewpoint, Cohen

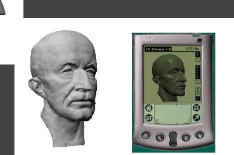
Oversampled 3D scan data



Adapt to hardware capabilities

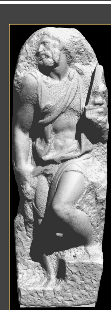






Mesh Decimation

- Reduce number of polygons
  - Less storage
  - Faster rendering
  - Simpler manipulation
- Desirable properties
  - Generality
  - Efficiency
  - Produces "good" approximation



Michelangelo's St. Matthew
Original model: ~400M polygons

Primitive Operations

Simplify model a bit at a time by removing a few faces (mesh simplification)

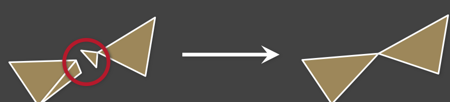
- Repeated to simplify whole mesh

Types of operations

- Vertex cluster
- Vertex remove
- Edge collapse (main operation used in assignment)

Vertex Cluster

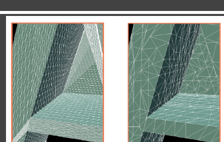
- Method
  - Merge vertices based on proximity
  - Triangles with repeated vertices can collapse to edges or points
- Properties
  - General and robust
  - Can be unattractive if results in topology change



Vertex Clustering

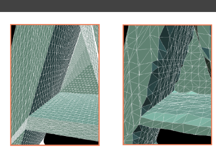
- Cluster generation
  - Hierarchical approach
  - Top-down or bottom up
- Computing a representative
  - Average / median vertex position
  - Error quadrics
- Mesh generation
- Topology changes

Further reading: Model simplification using vertex clustering, Low and Tan, I3D, 1997.

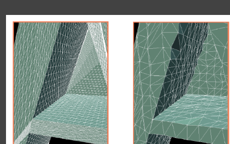


Best approach

Error quadrics



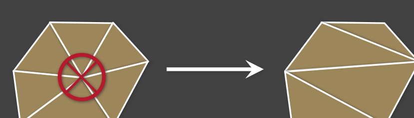
Average vertex position



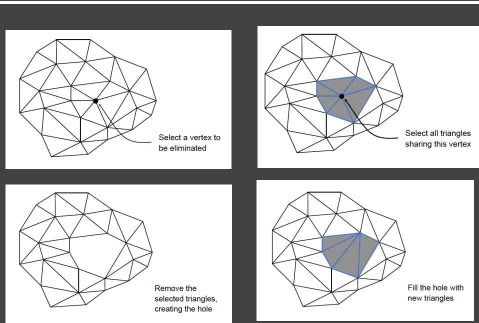
Median vertex position

Vertex Remove

- Method
  - Remove vertex and adjacent faces
  - Fill hole with new triangles (reduction of 2)
- Properties
  - Requires manifold surface, preserves topology
  - Typically more attractive
  - Filling hole well not always easy

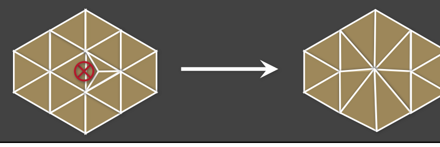


Vertex Removal

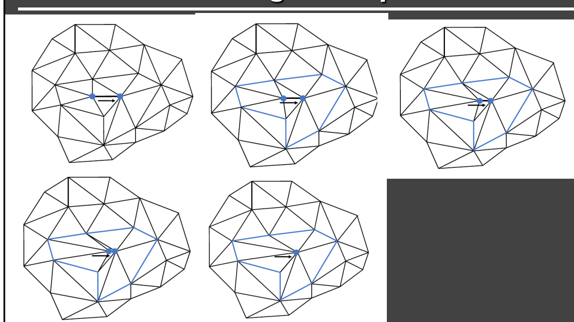


Edge Collapse

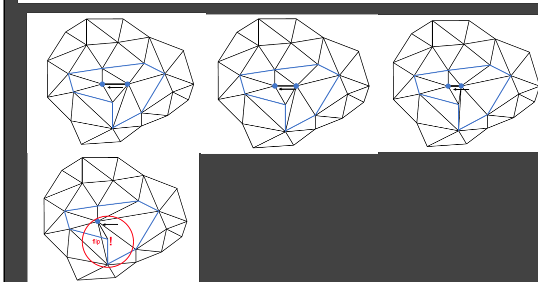
- Method
  - Merge two edge vertices to one
  - Delete degenerate triangles (triangle formed by three collinear points)
- Properties
  - Special case of vertex cluster
  - Allows smooth transition
  - Can change topology



Half-edge collapse



Half-Edge Collapse



Mesh Decimation/Simplification

Typical: greedy algorithm

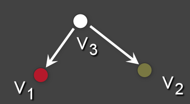
- Measure error of possible “simple” operations (primarily edge collapses)
- Place operations in queue according to error
- Perform operations in queue successively (depending on how much you want to simplify model)
- After each operation, re-evaluate error metrics

Geometric Error Metrics

- Motivation
  - Promote accurate 3D shape preservation
  - Preserve screen-space silhouettes and pixel coverage
- Types
  - Vertex-Vertex Distance
  - Vertex-Plane Distance
  - Point-Surface Distance
  - Surface-Surface Distance

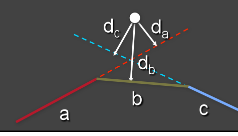
Vertex-Vertex Distance

- E = max(|v3-v1|, |v3-v2|)
- Appropriate during topology changes
  - Rossignac and Borrel 93
  - Luebke and Erikson 97
- Loose for topology-preserving collapses



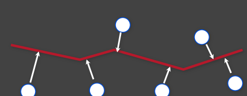
Vertex-Plane Distance

- Store set of planes with each vertex
  - Error based on distance from vertex to planes
  - When vertices are merged, merge sets
- Ronfard and Rossignac 96
  - Store plane sets, compute max distance
- Error Quadrics – Garland and Heckbert 96
  - Store quadric form, compute sum of squared distances



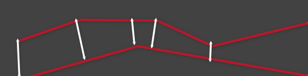
Point-Surface Distance

- For each original vertex,
  find closest point on simplified surface
- Compute sum of squared distances



Surface-Surface Distance

- Compute or approximate maximum distance between input and simplified surfaces
  - Tolerance Volumes - Guéziec 96
  - Simplification Envelopes - Cohen/Varshney 96
  - Hausdorff Distance - Klein 96
  - Mapping Distance - Bajaj/Schikore 96, Cohen et al. 97



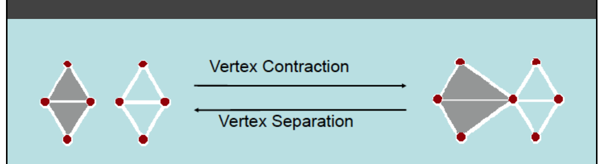
Geometric Error Observations

- Vertex-vertex and vertex-plane distance
  - Fast
  - Low error in practice, but not guaranteed by metric
- Surface-surface distance
  - Required for guaranteed error bounds

vertex-vertex ≠ surface-surface

Topology changes

- Marge vertices across non-edges
  - Changes mesh topology
  - Need spatial neighborhood information
  - Generates non-manifold meshes



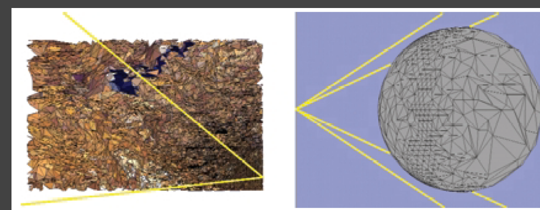
Mesh Simplification

Advanced Considerations

- Type of input mesh, Modifies topology, Continuous LOD, Speed vs. quality
- Vertex clustering is fast but difficult to control simplified mesh that will leads to the previously mentioned errors

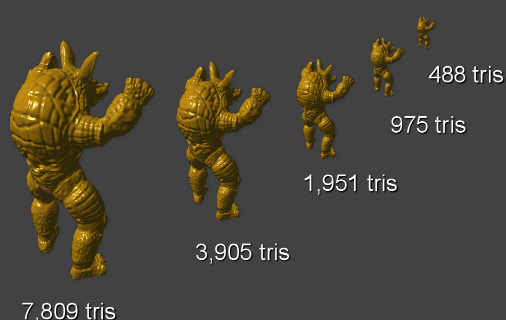
View-Dependent Simplification

- Simplify dynamically according to viewpoint
  - Visibility
  - Silhouettes
  - Lighting



Hoppe

Appearance Preserving



Caltech & Stanford Graphics Labs and Jonathan

Summary

- Many mesh data structures
  - Compact storage vs ease, efficiency of use
  - How fast and easy are key operations
- Mesh simplification
  - Reduce size of mesh in efficient quality-preserving way
  - Based on edge collapses mainly
- Choose appropriate mesh data structure
  - Efficient to update, edge-collapses are local
- Material covered in text
  - Classical approaches to simplification
  - Quadric metrics next week