UMBC | CMSC–603
Spring 1994 | Advanced Discrete Structures

Assignment 7

Problem 1: VLSI Chips

For any positive integer n, let $V(n)$ denote the number of essentially different VLSI chips that can be made on an $n \times n$ regular square grid in the plane. Each chip consists of zero or more wires. Each wire connects two adjacent grid points, horizontally or vertically (not diagonally). At most one wire can connect any two grid points. Two chips are essentially the same if and only if some sequence of flips or rotations transforms one chip into the other. Compute $V(3)$ and $V(4)$. Carefully justify your answers.

Problem 2: Binary Functions

Definition. Let n and m be any positive integers. A *binary function from n inputs to m inputs* is any function $f : \{0,1\}^n \to \{0,1\}^m$. Let $\mathcal{F}_{n,m}$ denote the set of all binary functions from n inputs to m inputs. □

Definition. Let $f, g \in \mathcal{F}_{n,m}$. We say that *$f$ and g are equivalent* iff there exist permutations $\pi_1 \in \mathcal{S}_n$ and $\pi_2 \in \mathcal{S}_m$ such that $f = \pi_2 g \pi_1$. (Here, $\mathcal{S}_n$ is the symmetric group on n letters. The permutation π_1 permutes the order of bits in the input string of g; the permutation π_2 permutes the order of the bits in the output string of g.) □

Definition. For any $n, m \in \mathbb{Z}^+$, let $\psi(n,m)$ denote the number of equivalency classes in $\mathcal{F}_{n,m}$. □

a) Roughly estimate $\psi(n,m)$ by giving upper and lower bounds. Each bound should be a closed formula in terms of n and m.

b) Compute $\psi(2,2)$ exactly.

Problem 3: Necklaces

An (n,k)-*necklace* is a ring of n beads, where each bead is colored with one of k different colors. Two necklaces are equivalent if and only if some rotation of the beads of one necklace transforms that necklace into the other. Note that flips do not necessarily yield equivalent necklaces. For any positive integers n and k, let $L(n,k)$ denote the number of essentially different (n,k)-necklaces.

a) Compute $L(8,2)$. Carefully justify your answer.

b) Give a general formula for $L(n,k)$ in terms of n and k. Carefully justify your answer.