Assignment 3

Problem 1: A Hairy Problem
Prove: There exist at least two distinct non-bald humans on earth who have exactly the same number of hairs on their heads. Carefully state all of your assumptions.

Problem 2: Countability

a) Prove: The set of polynomials over $\mathbb{Z}$ is countable.

b) Prove: The sets $\mathbb{R}$ and $[0,1] \cap \mathbb{R}$ have the same cardinality.

c) Prove: The number of functions mapping $\mathbb{N}$ to $\mathbb{N}$ is uncountable.

Problem 3 (Injective, surjective, and bijective functions).

a) Prove: *Let A be any nonempty set, and let f be any endomorphism on A.*[1] *If A is finite, then the following three statements are equivalent:*

   (i) f is injective

   (ii) f is surjective

   (iii) f is bijective.

b) Give an example of an endomorphism that is injective but not bijective. Justify your answer.

c) Give an example of an endomorphism that is surjective but not bijective. Justify your answer.

[1] An *endomorphism* is a function whose domain and target are the same set.