Bookkeeping

- HW1 due 9/16 at 11:59 PM
 - Writing (please read the integrity page statement on GenAl use carefully)
 - Problem sets
 - Programming
- Today:
 - Last of uninformed search
 - · Uniform-Cost, Iterative Deepening, Bidirectional
 - · Informed search, heuristics
- Soon: Constraint satisfaction

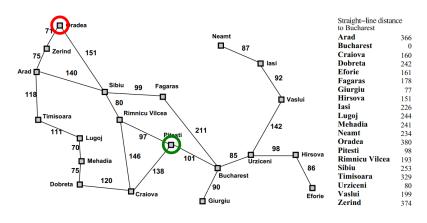
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Uninformed Search: Uniform-Cost (UCS)

- Enqueue nodes by path cost:
 - Let $g(n) = \frac{\cos t}{\sin n}$ of path from start node to current node n
 - Sort nodes by increasing value of q
 - Identical to breadth-first search if all operators have equal cost
- "Dijkstra's Algorithm" in algorithms literature
- "Branch and Bound Algorithm" in operations research literature
- Complete (*)
- Optimal/Admissible (*)
 - Admissibility depends on the goal test being applied when a node is removed from the nodes list, not when its parent node is expanded and the node is first generated
- Exponential time and space complexity, O(b^d)

Example: Path Costs

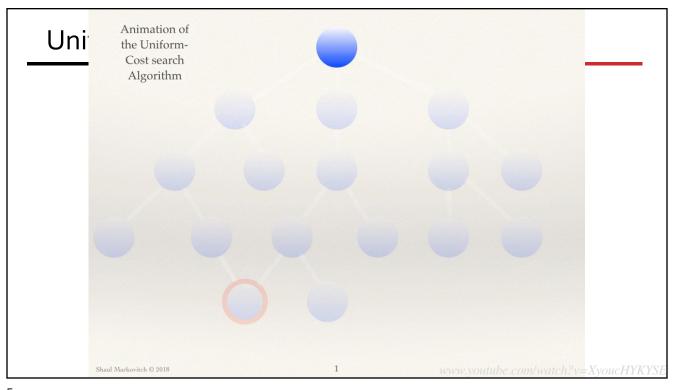
Romania with step costs in km



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UCS Implementation

- For each frontier node, save the total cost of the path from the initial state to that node
- Expand the frontier node with the lowest path cost
- Equivalent to breadth-first if step costs all equal
- Equivalent to Dijkstra's algorithm in general



Depth-First Iterative Deepening (DFID)

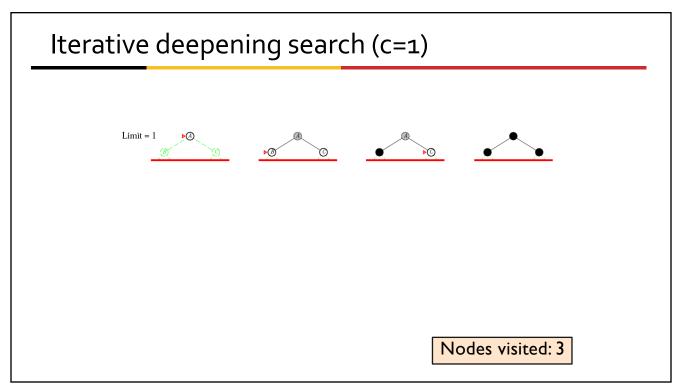
- 1. DFS to depth 0 (i.e., treat start node as having no successors)
- 2. Iff no solution, do DFS to depth 1

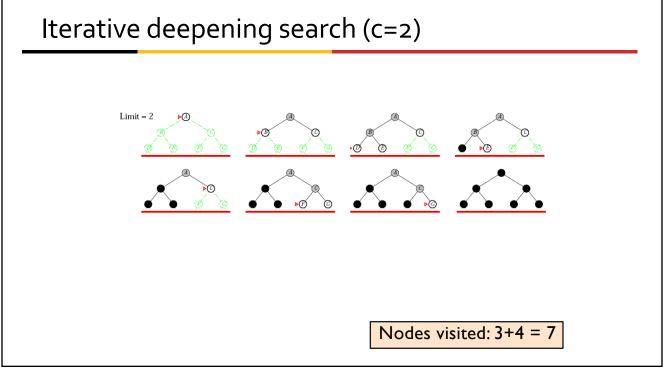
until solution found do:

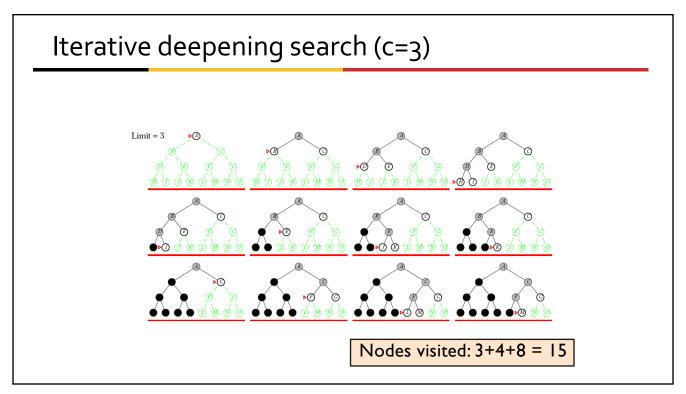
DFS with depth cutoff c;

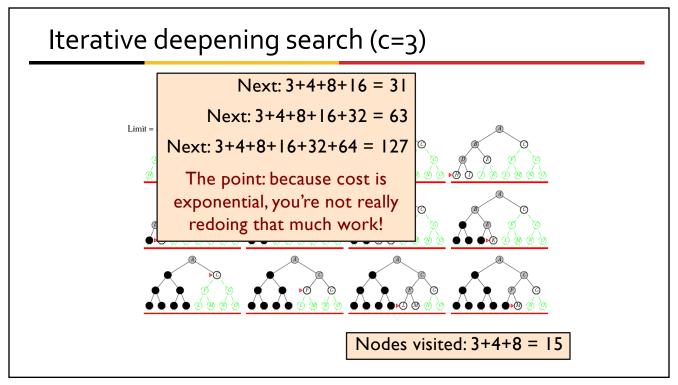
c = c+1

- Complete
- Optimal/Admissible if all operators have the same cost
 - Otherwise, not optimal, but guarantees finding solution of shortest length
- Time complexity is a little worse than BFS or DFS
- Nodes near the top of the tree are generated multiple times
 - Because most nodes are near the bottom of a tree, worst case time complexity is still exponential, O(bd)







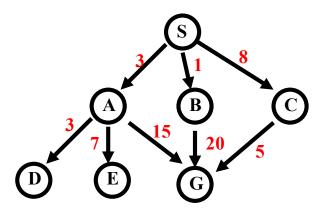


Depth-First Iterative Deepening

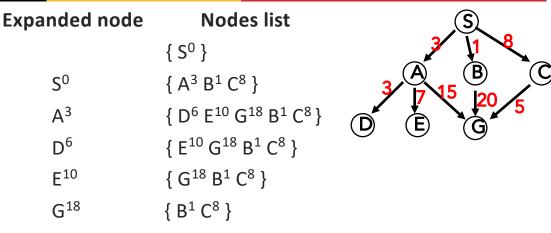
- If branching factor is b and solution is at depth d, then nodes at depth d are generated once, nodes at depth d-1 are generated twice, etc.
 - Hence $b^d + 2b^{(d-1)} + ... + db \le b^d / (1 1/b)^2 = O(b^d)$.
 - If b=4, then worst case is 1.78 * 4^d, i.e., 78% more nodes searched than exist at depth d (in the worst case).
- Linear space complexity, O(bd), like DFS
- Has advantage of both BFS (completeness) and DFS (limited space, finds longer paths more quickly)
- Generally preferred for large state spaces where solution depth is unknown

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Example for Illustrating Search Strategies



Depth-First Search



Solution path found is $S \rightarrow A \rightarrow G$, cost 18

Number of nodes expanded (including goal node) = 5

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Breadth-First Search

Expanded node Nodes list { S⁰ } $\{A^3 B^1 C^8\}$ S^0 $\{ B^1 C^8 D^6 E^{10} G^{18} \}$ A^3 $\{ C^8 D^6 E^{10} G^{18} G^{21} \}$ B^1 $\{ D^6 E^{10} G^{18} G^{21} G^{13} \}$ C₈ $\{ E^{10} G^{18} G^{21} G^{13} \}$ D_{6} E^{10} $\{ G^{18} G^{21} G^{13} \}$ $\{ G^{21} G^{13} \}$ G^{18}

Solution path found is $S \rightarrow A \rightarrow G$, cost 18

Number of nodes expanded (including goal node) = 7

Uniform-Cost Search

Expanded node	Nodes list	(S)
	{ S ⁰ }	3/11-8
S^0	$\{ B^1 A^3 C^8 \}$	A B C
B^1	$\{ A^3 C^8 G^{21} \}$	3/17/15 120
A^3	$\{ D^6 C^8 E^{10} G^{18} G^{21} \}$	5
D^6	$\{ C^8 E^{10} G^{18} G^1 \}$	
C ₈	$\{ E^{10} G^{13} G^{18} G^{21} \}$	
E ¹⁰	$\{ G^{13} G^{18} G^{21} \}$	
G ¹³	$\{ G^{18} G^{21} \}$	

Solution path found is S \rightarrow C \rightarrow G, cost 13

Number of nodes expanded (including goal node) = 7

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How they Perform

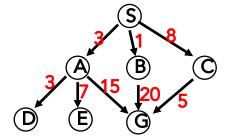
- Depth-First Search:
 - Expanded nodes: S A D E G
 - Solution found: S A G (cost 18)
- Breadth-First Search:
 - Expanded nodes: S A B C D E G
 - Solution found: S A G (cost 18)



- Expanded nodes: S A D B C E G
- Solution found: S C G (cost 13)
- This is the only **uninformed** search that worries about costs.

Iterative-Deepening Search:

- nodes expanded: SSABCSADEG
- Solution found: S A G (cost 18)



Comparing Search Strategies

	Co	mplete	Optimal	Time complexity	Space complexity
Breadth f	irst search:	yes	yes	$O(b^d)$	$O(b^d)$
Depth fir	st search	no	no	$O(b^{m})$	O(bm)
Depth lin	nited search	if $l \ge d$	no	$O(b^l)$	O(bl)
depth firs	t iterative g search	yes	yes	$O(b^d)$	O(bd)
bi-directi	onal search	yes	yes	$O(b^{d/2})$	$O(b^{d/2})$

b is branching factor, d is depth of the shallowest solution, m is the maximum depth of the search tree, l is the depth limit

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Blind Search (Redux)

- Last time:
 - Search spaces
 - Problem states
 - Goal-based agents
 - Breadth-first
 - Depth-first
 - Uniform-cost
 - · Iterative deepening

- From the book:
 - Bidirectional
 - Holy Grail Search

Comparing Search Strategies

• b is branching factor, d is depth of the shallowest solution, m is the maximum depth of the search tree, 1 is the depth limit

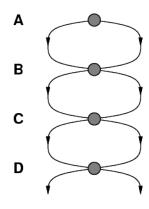
(Complete	Optimal '	Time complexity	Space complexity
Breadth first search:	yes	yes	O(b ^d) Give	$\mathrm{O}(b^\mathrm{d})$ n unit
Depth first search	no	no	$O(b^m)^{arcc}$	oosts O(bm)
Depth limited search	if 1 >= d	no	$O(b^l)$	O(bl)
depth first iterative deepening search	yes	yes	O(b ^d)	O(bd)
bi-directional search	yes	yes	$O(b^{d/2})$	$O(b^{d/2})$

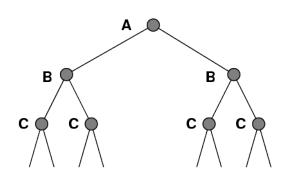
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Avoiding Repeated States

- Ways to reduce size of state space (with increasing computational costs)
- In increasing order of effectiveness and cost:
 - Do not return to the state you just came from.
 - Do not create paths with cycles in them.
 - Do not generate any state that was ever created before.
- Effect depends on frequency of loops in state space.
 - Worst case, storing as many nodes as exhaustive search!

State Space → An Exponentially Growing Search Space





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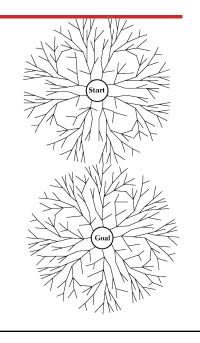
Bi-directional Search

- Alternate searching from
 - start state → goal
 - goal state → start
- Works well only goal states
- Requires ability t

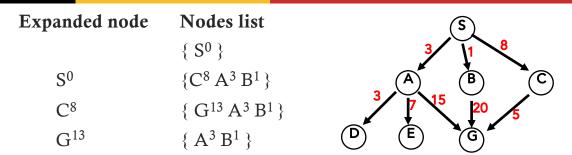
Stop when the free world problem where you can generate hd predecessors?

What's a problem where you cannot?

Can (sometimes) find a solution fast



Holy Grail Search



Solution path found is S C G, cost 13 (optimal)

Number of nodes expanded (including goal node) = 3

(minimum possible!)

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Holy Grail Search

- Why not go straight to the solution, without any wasted detours off to the side?
- If we knew where the solution was we wouldn't be searching!

If only we knew where we were headed...

"Satisficing"

- Wikipedia: "Satisficing is ... searching until
 an acceptability threshold is met"

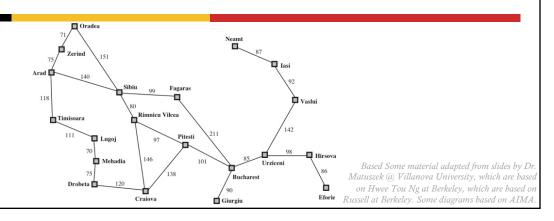
 Another piece of problem definition

 definition
- Contrast with **optimality**
 - Satisficable problems do not get more benefit from finding an optimal solution
- Ex: You have an A in the class. Studying for 8 hours will get you a 98 on the final. Studying for 16 hours will get you a 100 on the final. What to do?
- A combination of satisfy and suffice
- Introduced by Herbert A. Simon in 1956

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Informed Search (Ch. 3.5-3.7)

"An informed search strategy—one that uses problem specific knowledge... can find solutions more efficiently then an uninformed strategy." – R&N pg. 92



Overview of Informed Search

- Heuristic search
- Heuristic functions
- Admissibility
- Best-first search
 - Greedy search, beam search, A*
 - Examples
- Memory-conserving variations of A*

Questions?

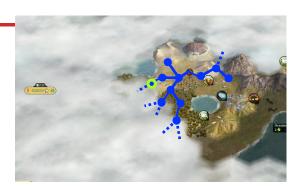
"An informed search strategy—one that uses problem specific knowledge... can find solutions more efficiently then an uninformed strategy."

- R&N pg. 92

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The Core Idea

- How can we make search smarter?
 - Use problem-specific knowledge beyond the definition of the problem
 - Specifically, incorporate knowledge of how good a non-goal state is



- Informed or Best-First Search
 - Node selected for expansion is based on an evaluation function f(n)
 - I.e., expand the node that appears to be the best bet
 - Node with lowest evaluation is selected for expansion
 - Uses a priority queue

Slide from Dr. Rebecca Hutchinson @ Oregon Sta Image: medium.com/blockchain-gaming/fog-of-war-7dba2b7faa/

Definition: Heuristic

- Free On-line Dictionary of Computing*: A rule of thumb, simplification, or educated guess
- WordNet (r) 1.6*: Commonsense rule (or set of rules) intended to increase the probability of solving some problem
- · Reduces, limits, or guides search in particular domains
- Does not guarantee feasible solutions; often with no theoretical guarantee
 - Playing chess: try to take the opponent's queen
 - Getting someplace: head in that compass direction when possible

*Heavily edited for clarity

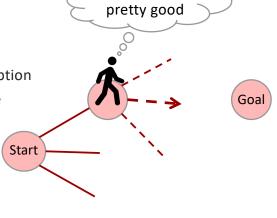
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Heuristic Search

- Uninformed search is **generic**
 - · Node selection depends only on shape of tree and node expansion strategy
- Domain knowledge* → better decisions (sometimes)
 - Knowledge about the specific problem
 - · Often calculated based on state
- * Domain knowledge is a general term in Al. A domain is a specific problem space, which you may or may not know something about. Examples: game playing; chess; medicine; perfumery; ...

Is It A Heuristic?

- A heuristic function is:
 - An estimate of how close we are to a goal
 - · We don't assume perfect knowledge
 - That would be holy grail search
 - So, the estimate can be wrong
 - Based on domain-specific information
 - Computable from the current state description
 - · A function over nodes that returns a value
 - Node = particular problem state

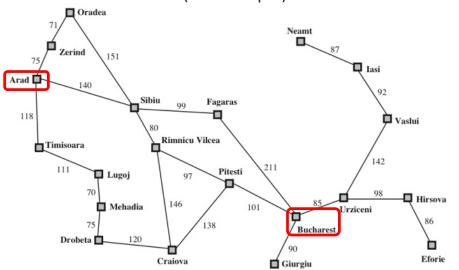


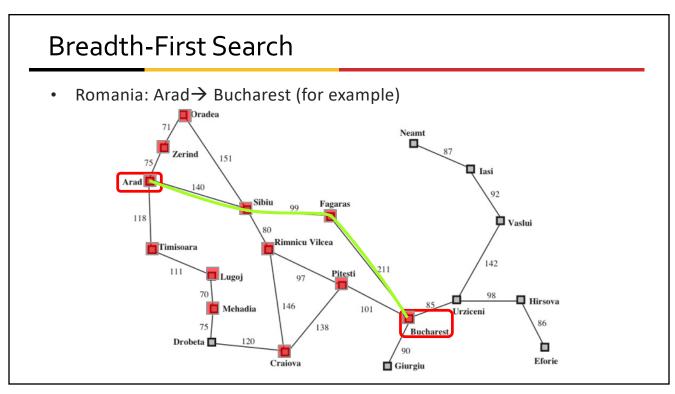
This way seems

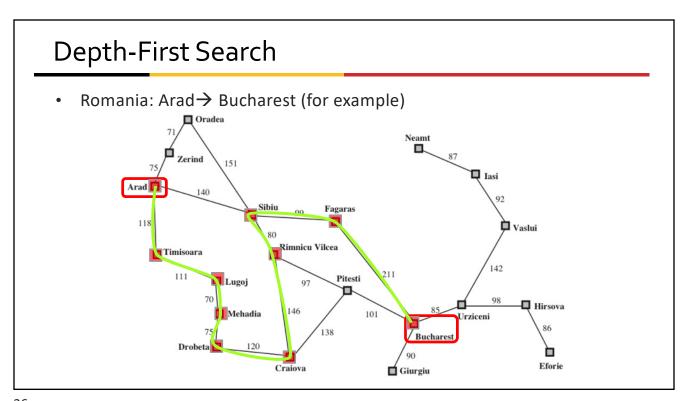
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Heuristic Search

• Romania: Arad→ Bucharest (for example)

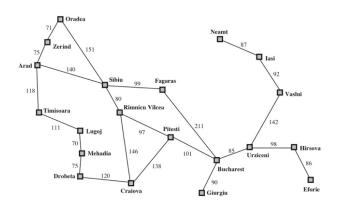






Heuristic Search

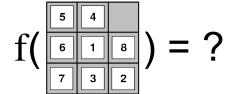
- Romania:
 - Eyeballing it → certain cities first
 - They "look closer" o where we are going
- Can domain knowledge be captured in a heuristic?

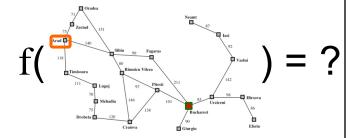


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Heuristics Examples

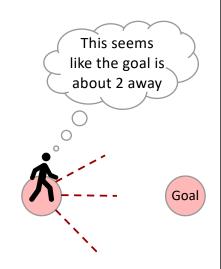
- 8-puzzle:
 - # of tiles in wrong place
- 8-puzzle (better):
 - Sum of distances from goal
 - Captures distance and number of nodes
- Romania:
 - Straight-line distance from current node to goal
 - Captures "closer to Bucharest"





Heuristic Function

- All domain-specific knowledge is encoded in heuristic function h
- h is some **estimate** of how desirable a move is
 - How "close" (we think, maybe) it gets us to our goal
- Usually:
 - h(n) ≥ 0: for all nodes n
 h(n) = 0: n is a goal node
 - $h(n) = \infty$: n is a dead end (no goal can be reached from n)



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Example Search Space Revisited start state A 8 B 4 C 3 h value D & E & G 0 goal state

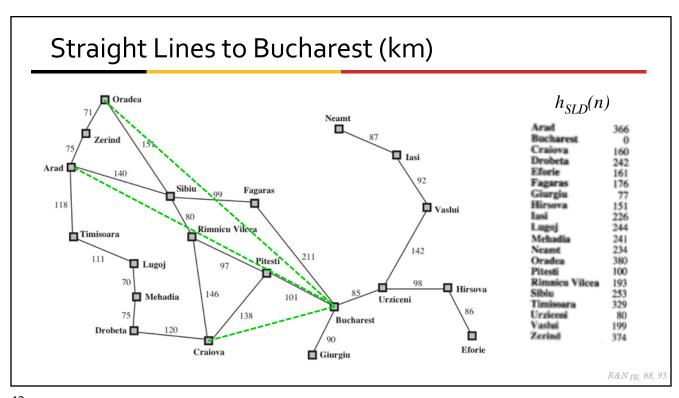
Weak vs. Strong Methods

- Weak methods:
 - Extremely general, not tailored to a specific situation
- Examples
 - Subgoaling: split a large problem into several smaller ones that can be solved one at a time.
 - **Space splitting:** try to list possible solutions to a problem, then try to rule out *classes* of these possibilities
 - Means-ends analysis: consider current situation and goal, then look for ways to shrink the differences between the two
- Called "weak" methods because they do not take advantage of more powerful domain-specific heuristics

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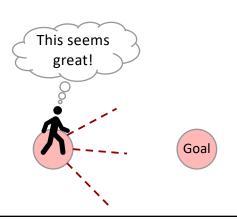
Domain Information

- Informed methods add domain-specific information!
- Goal: select the best path to continue searching
 - Uninformed methods (BFS, DFS, UCS) push nodes onto the search list based only
 on the order in which they are encountered and the cost of reaching them
 - Informed methods try to explore the best ("most likely looking") nodes first
- Define h(n) to estimate the "goodness" of node n
 - h(n) =estimated cost (or distance) of minimal cost path from n to a goal state



Admissible Heuristics

- Admissible heuristics never overestimate cost
 - They are optimistic think goal is closer than it is
 - $h(n) \leq h^*(n)$
 - where h*(n) is true cost to reach goal from n
 - $h_{SLD}(Lugoj) = 244$
 - Can there be a shorter path?



Admissibility

- · Admissibility is a property of heuristics
 - They are *optimistic* think goal is closer than it is
 - (Or, exactly right)
- Is "∀n, h(n)=1 kilometer" admissible?
- · Admissible heuristics can be pretty bad!

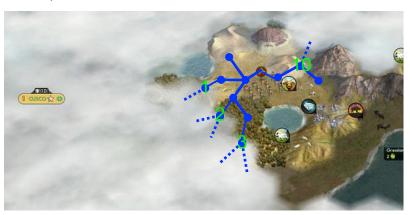
• Using admissible heuristics guarantees that the first solution found will be optimal, for some algorithms (A*).



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Best-First Search

- A generic way of referring to informed methods
- Use an **evaluation function** f(n) over nodes
 - Gives an estimate of "desirability"
 - f(n) incorporates domain-specific information
 - Different $f(n) \rightarrow$ Different searches
 - f(n) can incorporate knowledge from h(n)
- So let's estimate f(n) for these nodes...



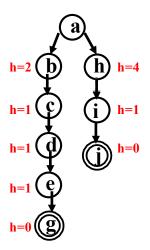
Best-First Search (more)

- Order nodes on the list by increasing value of f(n)
- Expand most desirable unexpanded node
 - Implementation:
 - · Order nodes in frontier in decreasing order of desirability
- Special cases:
 - · Greedy best-first search
 - A* search

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Greedy Best-First Search

- Idea: always choose "closest node" to goal
 - Most likely to lead to a solution quickly
- So, evaluate nodes based only on heuristic function
 - f(n) = h(n)
- Sort nodes by increasing values of f
- Select node believed to be closest to a goal node (hence "greedy")
 - That is, select node with smallest *f* value



Greedy Best-First Search

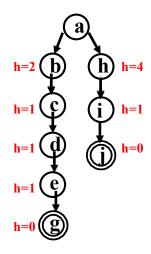
- Optimal?
 - · Why not?
- Example:
 - Greedy search will find:

$$a \rightarrow b \rightarrow c \rightarrow d \rightarrow e \rightarrow g$$
; cost = 5

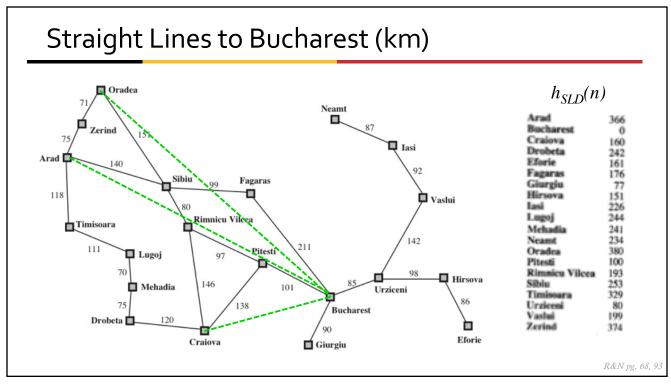
· Optimal solution:

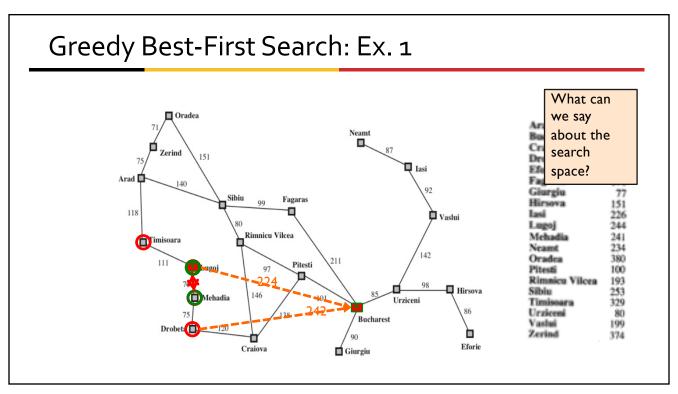
$$a \rightarrow h \rightarrow i \rightarrow j$$
; cost = 3

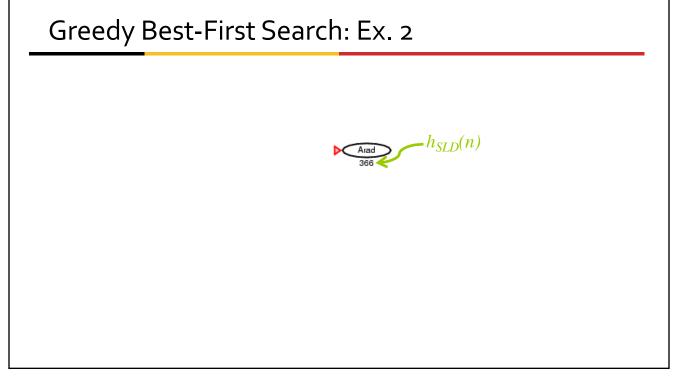
Not complete (why?)



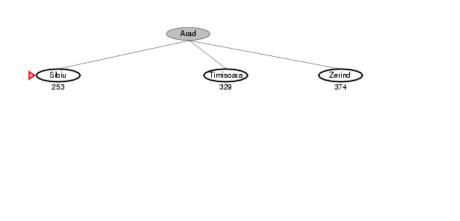
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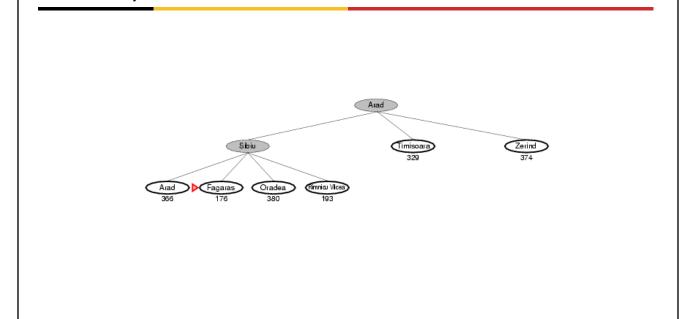




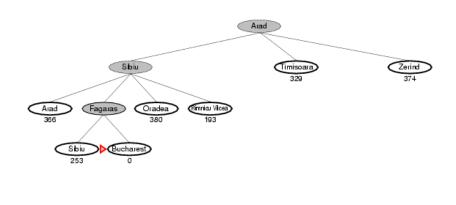




Greedy Best-First Search: Ex. 2



Greedy Best-First Search: Ex. 2



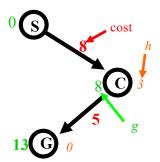
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Beam Search

- Use an evaluation function f(n) = h(n), but the maximum size of the nodes list is k, a fixed constant
- Only keeps *k* best nodes as candidates for expansion, and throws the rest away—can **never** explore those nodes
- More space-efficient than greedy search, but may throw away a node that is on a solution path
- Not complete
- Not admissible

A* Search

- Idea: Evaluate nodes by combining g(n), the cost of reaching a node, with h(n), the cost of getting from the node to the goal.
 - A* because $h(n) \le h^*(n)$
- Evaluation function: f(n) = g(n) + h(n)
 - $g(n) = \cos t$ so far to reach n
 - h(n) = estimated cost from n to goal
 - f(n) = estimated total cost of path through n to goal



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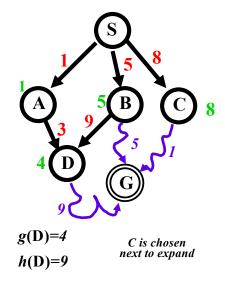
Quick Terminology Reminders

- What is *f*(*n*)?
 - An evaluation function that gives...
 - A cost estimate of...
 - The distance from n to G
- What is *h*(*n*)?
 - A heuristic function that...
 - Encodes domain knowledge about...
 - · The search space

- What is h*(n)?
 - A heuristic function that gives the...
 - True cost to reach goal from n
 - Why don't we just use that?
- What is g(n)?
 - The path cost of getting from S to n
 - describes the "already spent" costs of the current search

Algorithm A*

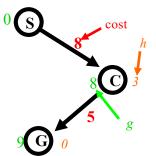
- Use evaluation function f(n) = g(n) + h(n)
- g(n) = minimal-cost path from S to state n
 - That is, the cost of getting to the node so far
- Ranks nodes on frontier by estimated cost of solution
 - From start node, through given node, to goal
- Not complete if h(n) can = ∞

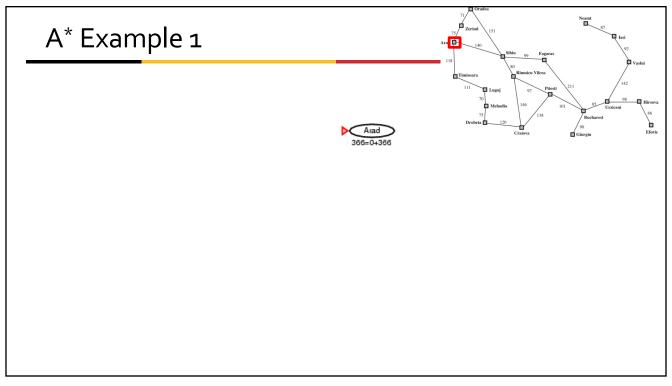


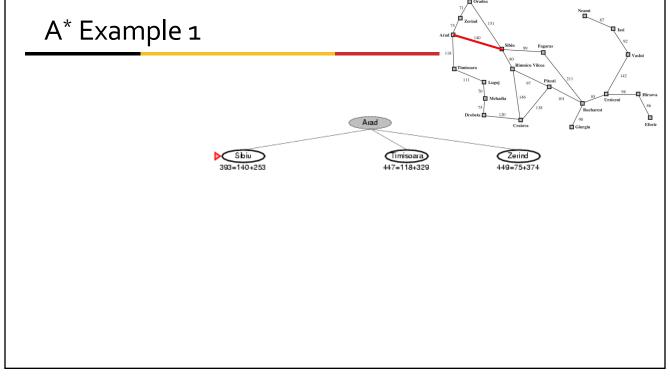
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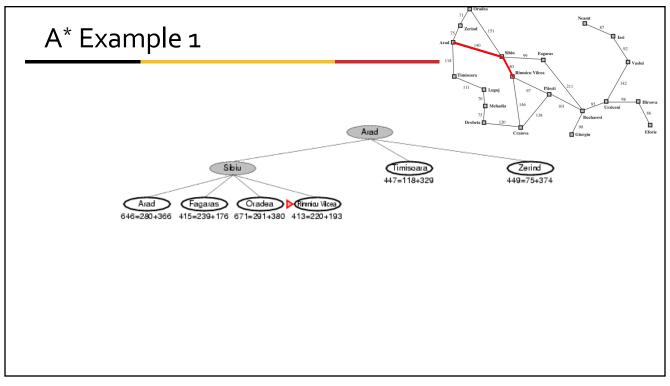
A* Search

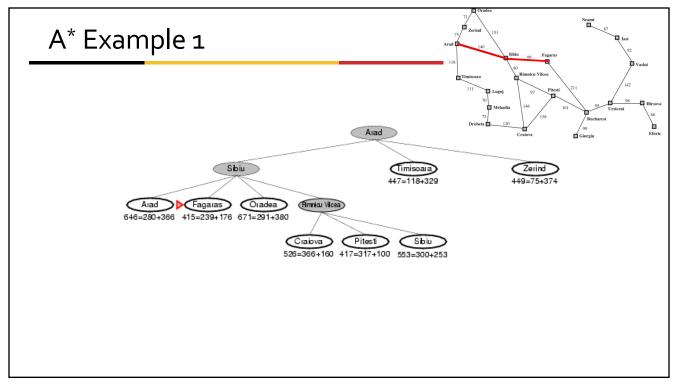
- Avoid expanding paths that are already expensive
 - · Combines costs-so-far with expected-costs
- Is complete iff
 - Branching factor is finite
 - Every operator has a fixed positive cost
- Is admissible iff
 - h(n) is admissible

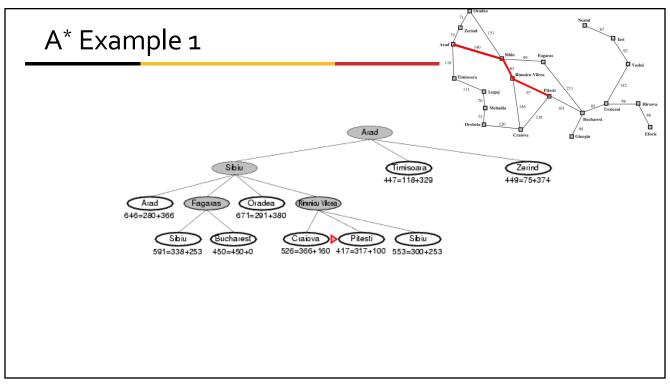


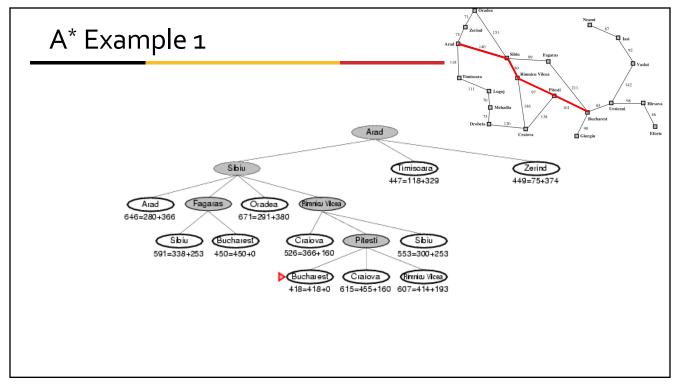










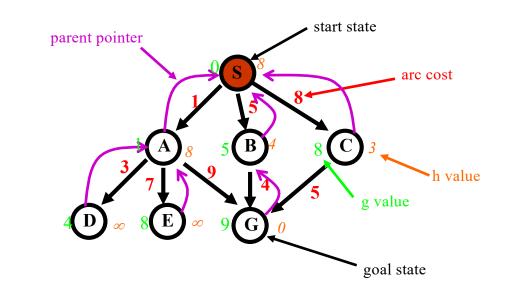


Algorithm A*

- Algorithm A with constraint that $h(n) \le h^*(n)$
 - $h^*(n)$ = true cost of the minimal cost path from n to a goal.
- Therefore, h(n) is an underestimate of the distance to the goal
- h() is admissible when $h(n) \le h^*(n)$
 - · Guarantees optimality
- A* is complete whenever the branching factor is finite, and every operator has a fixed positive cost
- A* is admissible

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Example Search Space Revisited



Example

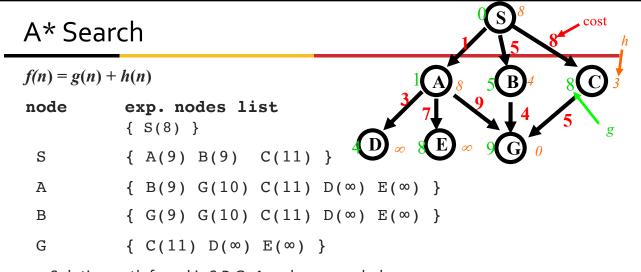
n	g(n)	h(n)	f(n)	$h^*(n)$	0(S) ⁸
S	0	8	8	9	1/15
A	1	8	9	9	
В	5	4	9	4	$1(A)_{8} 5(B)_{4}$
C	8	3	11	5	3/5/ 9 14
D	4	∞	∞	∞	\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
E	8	∞	∞	∞	4D ∞ 8E ∞ 9G 0
G	9	0	9	0	

- $h^*(n)$ is the (hypothetical) perfect heuristic.
- Since $h(n) \le h^*(n)$ for all n, h is admissible
- Optimal path = S B G with cost 9.

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Greedy Search

- Solution path found is S C G, 3 nodes expanded.
- Fast!! But NOT optimal.



- Solution path found is S B G, 4 nodes expanded..
- Still pretty fast, and optimal

Admissibility and Optimality

- Intuitively:
 - When A* finds a path of length k, it has already tried every other path which can have length ≤ k
 - Because all frontier nodes have been sorted in ascending order of f(n)=g(n)+h(n)
- Does an admissible heuristic guarantee optimality for greedy search?
 - Reminder: f(n) = h(n), always choose node "nearest" goal
 - No sorting beyond that

Admissible heuristics

- E.g., for the 8-puzzle:
 - h₁(n) = number of misplaced tiles
 - h₂(n) = total Manhattan distance
 - (i.e., # of squares each tile is from desired location)
- $h_1(S) = ?$
- $h_2(S) = ?$

7	2	4
5		6
8	3	1

Start

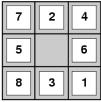


Goal

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Admissible heuristics

- E.g., for the 8-puzzle:
 - $h_1(n)$ = number of misplaced tiles
 - $h_2(n) = total Manhattan distance$
 - (i.e., # of squares each tile is from desired location)
- $h_1(S) = 8$
- $h_2(S) = 3+1+2+2+2+3+3+2 = 18$



Start



Goa

Dealing with Hard Problems

- For large problems, A* often requires too much space.
- Two variations conserve memory: IDA* and SMA*
- IDA* iterative deepening A*
 - uses successive iteration with growing limits on f. For example,
 - A* but don't consider any node n where f(n) > 10
 - A* but don't consider any node n where f(n) > 20
 - A* but don't consider any node n where f(n) > 30, ...
- SMA* Simplified Memory-Bounded A*
 - · Uses a queue of restricted size to limit memory use
 - · Throws away the "oldest" worst solution

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What's a Good Heuristic?

- If $h_1(n) < h_2(n) \le h^*(n)$ for all n, then:
 - · Both are admissible
 - h₂ is strictly better than ("dominates") h₁
- So... how do we find one?
- 1. Relaxing the problem:
 - Remove constraints to create a (much) easier problem
 - Use the solution cost for this problem as the heuristic function
- 2. Combining heuristics:
 - Take the max of several admissible heuristics
 - Still have an admissible heuristic, and it's better!

Finding a Good Heuristic (2)

- 3. Use statistical estimates to compute *h*
 - May lose admissibility
- 4. Identify good features, then use a learning algorithm to find a heuristic function
 - Also may lose admissibility
- Why are these a good idea, then?
 - Machine learning can give you answers you don't "think of"
 - Can be applied to new puzzles without human intervention
 - · Often works

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Some Examples of Heuristics?

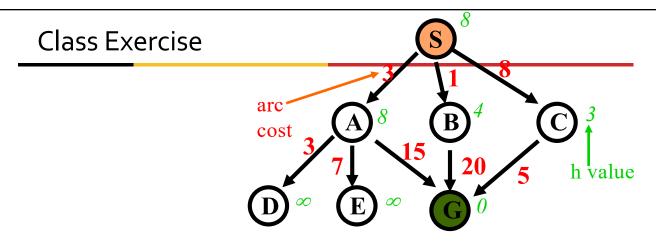
- 8-puzzle?
 - Manhattan distance
- · Driving directions?
 - Straight line distance
- Crossword puzzle?
- Making a medical diagnosis?

Summary: Informed Search

- **Best-first search:** general search where the *minimum-cost nodes* (according to some measure) are expanded first.
- Greedy search: uses minimal estimated cost h(n) to the goal state as measure. Reduces search time, but is neither complete nor optimal.
- A* search: combines UCS and greedy search
 - f(n) = g(n) + h(n)
 - A* is complete and optimal, but space complexity is high.
 - Time complexity depends on the quality of the heuristic function.
- IDA* and SMA* reduce the memory requirements of A*.

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Class Exercise: Creating Heuristics 8-Puzzle Boat Problems Sticks 1 2 3 8 4 7 6 5 Sunt Nate N-Queens Water Jug Problem Route Planning FRANCE



Apply the following to search this space. At each search step, show: the current node being expanded; g(n) (path cost so far); h(n) (heuristic estimate); f(n) (evaluation function); and $h^*(n)$ (true goal distance).

Depth-first search Breadth-first search A* search

Uniform-cost search Greedy search