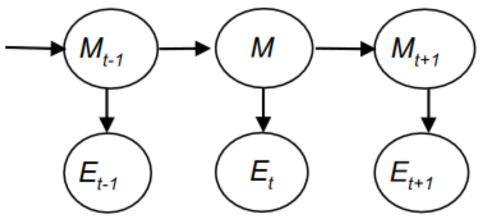
Part 2 – Filtering

Preface to Part 2 Response

The prompt describes a scenario in which a movie theater worker determines the popularity of a movie based on whether the popcorn machine is empty and the number of nights that the movie has been shown (Matuszek, 2024). A diagram of the dependency relationship network and conditional probability tables can be found in Figure 2 below.



M_{t-1}	$P(M_t \mid M_{t-1})$
T	0.75
F	0.12

E_t	$P(E_t \mid M_t)$
T	0.9
F	0.1

Figure 2: Dependency network and conditional probability tables for Homework 3, Part 2. (Matuszek, 2024)

Response to Part 2, Question 4

The prompt requests the calculation of the probability that a movie is popular on day 3 based on finding a full popcorn hopper on day 2 and finding an empty popcorn hopper on day 3 (Matuszek, 2024).

First, one must translate the prompt into a conditional probability statement $P(M_3|\overline{E_2}, E_3)$ where M_3 represents a movie being popular on day 3, $\overline{E_2}$ representing finding a full hopper on day 2, and E_3 representing finding an empty hopper on day 3.

Given the relationship between the popularity of a movie on a given day and the state of the popcorn hopper for the same day, one can calculate the desired probability using a sequence of transition and sensor updates to the initial state of movie popularity.

Movies are claimed to have a probability of being popular at 75%. This can be represented as a vector

$$\langle P(M_0), P(\overline{M_0})\rangle = \langle 0.75, 0.25\rangle$$

Day 1 Transition Update

One can update the movie popularity probabilities using a transition update from day 0 to day 1.

$$\langle P(M_1), P(\overline{M_1})\rangle = \sum_{m_0 \in \{M_0, \overline{M_0}\}} \langle P(M_1|m_0), P(\overline{M_1}|m_0)\rangle \cdot P(m_0)$$

$$\langle P(M_1), P(\overline{M_1})\rangle = \langle P(M_1|M_0), P(\overline{M_1}|M_0)\rangle \cdot P(M_0) + \langle P(M_1|\overline{M_0}), P(\overline{M_1}|\overline{M_0})\rangle \cdot P(\overline{M_0})$$

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Using values from the state transition tables and the prior probabilities, one can substitute values for $P(M_1|M_0)$, $P(\overline{M_1}|M_0)$, $P(M_1|\overline{M_0})$, $P(\overline{M_1}|\overline{M_0})$, $P(M_0)$, and $P(\overline{M_0})$.

$$\langle P(M_1), P(\overline{M_1})\rangle = \langle 0.75, 0.25\rangle \cdot 0.75 + \langle 0.12, 0.88\rangle \cdot 0.25$$
$$\langle P(M_1), P(\overline{M_1})\rangle = \langle 0.5925, 0.4075\rangle$$

Since the state of the hopper is not known for day 1, there is no sensor update to these prior probabilities for $\langle P(M_1), P(\overline{M_1})\rangle$. Therefore, $\langle P(M_1|e_1), P(\overline{M_1}|e_1)\rangle = \langle P(M_1), P(\overline{M_1})\rangle$.

Day 2 Transition Update

One can update the movie popularity probabilities using a transition update from day 1 to day 2. Note that some looseness in notation is used: $P(m_1|e_1)$ in this context represents the prior probability from day 1 given all known evidence up to day 1.

$$\langle P(M_2), P(\overline{M_2})\rangle = \sum_{m_1 \in \{M_1, \overline{M_1}\}} \langle P(M_2|m_1), P(\overline{M_2}|m_1)\rangle \cdot P(m_1|e_1)$$
$$\langle P(M_2), P(\overline{M_2})\rangle = \langle P(M_2|M_1), P(\overline{M_2}|M_1)\rangle \cdot P(M_1|e_1) + \langle P(M_2|\overline{M_1}), P(\overline{M_2}|\overline{M_1})\rangle \cdot P(\overline{M_1}|e_1)$$

Using values from the state transition tables and the prior probabilities, one can substitute values for $P(M_2|M_1)$, $P(\overline{M_2}|M_1)$, $P(M_2|\overline{M_1})$, $P(\overline{M_2}|\overline{M_1})$, $P(M_1)$, and $P(\overline{M_1})$.

$$\langle P(M_2), P(\overline{M_2})\rangle = \langle 0.75, 0.25\rangle \cdot 0.5925 + \langle 0.12, 0.88\rangle \cdot 0.4075$$
$$\langle P(M_1), P(\overline{M_1})\rangle = \langle 0.493275, 0.506725\rangle$$

Day 2 Sensor Update

Evidence on day 2 shows the popcorn hopper is full for that day. This can be integrated into the prior probability for day 2 using a sensor update.

$$\langle P(M_2|\overline{E_2}), P(\overline{M_2}|\overline{E_2})\rangle = \alpha \cdot \langle P(\overline{E_2}|M_2), P(\overline{E_2}|\overline{M_2})\rangle \cdot \langle P(M_2), P(\overline{M_2})\rangle$$

Using values from the sensor tables and the prior probabilities, one can substitute values for $P(\overline{E_2}|M_2)$, $P(\overline{E_2}|\overline{M_2})$, $P(M_2)$, and $P(\overline{M_2})$.

$$\langle P(M_2|\overline{E_2}), P(\overline{M_2}|\overline{E_2})\rangle = \alpha \cdot \langle 0.1, 0.9\rangle \cdot \langle 0.493275, 0.506725\rangle$$
$$\langle P(M_2|\overline{E_2}), P(\overline{M_2}|\overline{E_2})\rangle = \alpha \cdot \langle 0.0493275, 0.4560525\rangle$$

Using an alpha value $\alpha = 1/0.50538$, one can normalize the probabilities of $P(M_2|\overline{E_2})$ and $P(\overline{M_2}|\overline{E_2})$.

$$\langle P(M_2|\overline{E_2}), P(\overline{M_2}|\overline{E_2})\rangle \approx \langle 0.0976, 0.9024\rangle$$

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Day 3 Transition Update

One can update the movie's popularity probabilities using a transition update from day 2 to day 3. Note that some looseness in notation is used: $P(m_2|e_{1:2})$ in this context represents the prior probability from day 2 given all known evidence up to day 2.

$$\langle P(M_3), P(\overline{M_3})\rangle = \sum_{m_2 \in \{M_2, \overline{M_2}\}} \langle P(M_3|m_2), P(\overline{M_3}|m_2)\rangle \cdot P(m_2|e_{1:2})$$

$$\langle P(M_3), P(\overline{M_3})\rangle = \langle P(M_3|M_2), P(\overline{M_3}|M_2)\rangle \cdot P(M_2|e_{1:2}) + \langle P(M_3|\overline{M_2}), P(\overline{M_3}|\overline{M_2})\rangle \cdot P(\overline{M_2}|e_{1:2})$$

Using values from the state transition tables and the prior probabilities, one can substitute values for $P(M_3|M_2)$, $P(\overline{M_3}|M_2)$, $P(M_3|\overline{M_2})$, $P(\overline{M_3}|\overline{M_2})$, $P(M_2|e_{1:2})$, and $P(\overline{M_2}|e_{1:2})$.

$$\langle P(M_3), P(\overline{M_3})\rangle = \langle 0.75, 0.25\rangle \cdot 0.0976 + \langle 0.12, 0.88\rangle \cdot 0.9024$$
$$\langle P(M_3), P(\overline{M_3})\rangle = \langle 0.181488, 0.818512\rangle$$

Day 3 Sensor Update

Evidence on day 3 shows the popcorn hopper is empty for that day. This can be integrated into the prior probability for day 2 using a sensor update.

$$\langle P(M_3|E_3), P(\overline{M_3}|E_3)\rangle = \alpha \cdot \langle P(E_3|M_3), P(E_3|\overline{M_3})\rangle \cdot \langle P(M_3), P(\overline{M_3})\rangle$$

Using values from the sensor tables and the prior probabilities, one can substitute values for $P(E_3|M_3)$, $P(E_3|\overline{M_3})$, $P(M_3)$, and $P(\overline{M_3})$.

$$\langle P(M_3|E_3), P(\overline{M_3}|E_3)\rangle = \alpha \cdot \langle 0.9, 0.1\rangle \cdot \langle 0.181488, 0.818512\rangle$$
$$\langle P(M_3|E_3), P(\overline{M_3}|E_3)\rangle = \alpha \cdot \langle 0.1633392, 0.0818512\rangle$$

Using an alpha value $\alpha = 1/0.2451904$, one can normalize the probabilities of $P(M_3|\overline{E_3})$ and $P(\overline{M_3}|\overline{E_3})$.

$$\langle P(M_3|E_3), P(\overline{M_3}|E_3)\rangle \approx \langle 0.6662, 0.3338\rangle$$

Therefore, the probability that a movie was popular on day 3 given a full hopper on day 2 and an empty hopper on day 3 is ~66.62%.