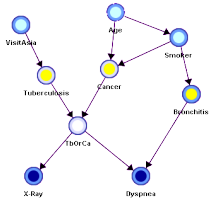
Reasoning with Bayesian Belief Networks

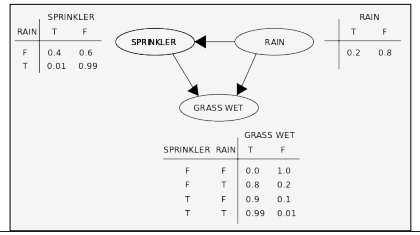


Overview

- Bayesian Belief Networks (BBNs) can reason with networks of propositions and associated probabilities
- Useful for many AI problems
  - Diagnosis
  - Expert systems
  - Planning
  - Learning

BBN Definition

- AKA Bayesian Network, Bayes Net
- A graphical model (as a DAG) of probabilistic relationships among a set of random variables
- Links represent direct influence of one variable on another



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Recall Bayes Rule

$$P(H,E) = P(H \mid E)P(E) = P(E \mid H)P(H)$$

$$\boxed{P(H \mid E) = \frac{P(E \mid H)P(H)}{P(E)}}$$

Note the symmetry: we can compute the probability of a hypothesis given its evidence and vice versa.

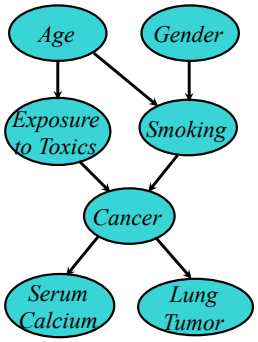
Simple Bayesian Network

$S \in \{no, light, heavy\}$ Smoking → Cancer $C \in \{none, benign, malignant\}$

P(S=no)	0.80
P(S=light)	0.15
P(S=heavy)	0.05

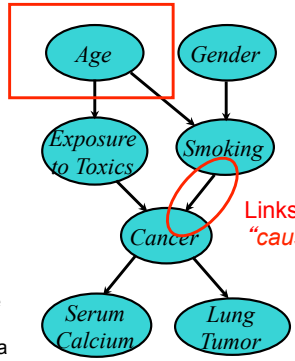
Smoking=	no	light	heavy
P(C=none)	0.96	0.88	0.60
P(C=benign)	0.03	0.08	0.25
P(C=malig)	0.01	0.04	0.15

More Complex Bayesian Network



More Complex Bayesian Network

Nodes represent variables

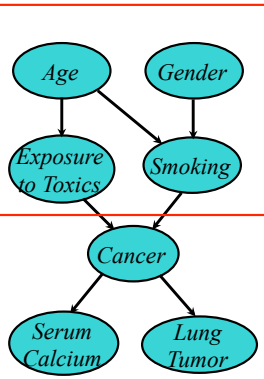


Links represent *"causal"* relations

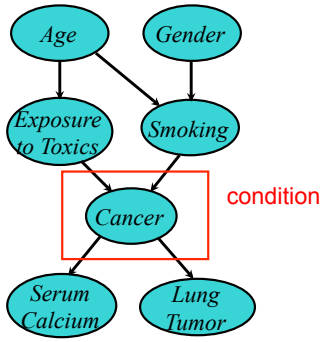
- Does gender cause smoking?
- Influence might be a more appropriate term

More Complex Bayesian Network

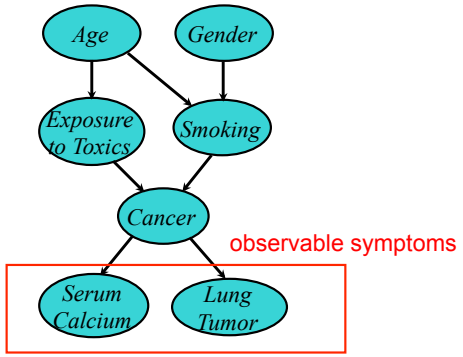
predispositions



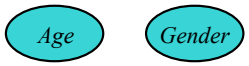
More Complex Bayesian Network



More Complex Bayesian Network



Independence



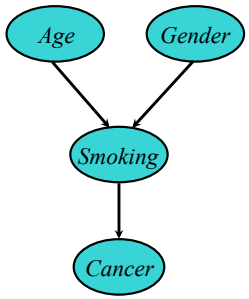
Age and *Gender* are independent.

$P(A,G) = P(G)\ P(A)$

$P(A\ |G) = P(A)$
$P(G\ |A) = P(G)$

$P(A,G) = P(G|A)\ P(A) = P(G)P(A)$
$P(A,G) = P(A|G)\ P(G) = P(A)P(G)$

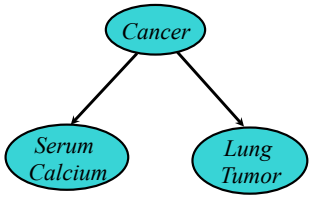
Conditional Independence



Cancer is independent of *Age* and *Gender* given *Smoking*

$P(C\ |\ A,G,S) = P(C|S)$

Conditional Independence: Naïve Bayes



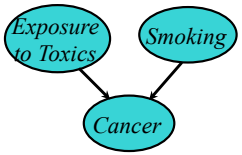
Serum Calcium and *Lung Tumor* are dependent

Serum Calcium is independent of *Lung Tumor*, given *Cancer*

$P(L \mid SC,C) = P(L|C)$
$P(SC \mid L,C) = P(SC|C)$

Naïve Bayes assumption: evidence (e.g., symptoms) is independent given the disease. This make it easy to combine evidence

Explaining Away



Exposure to Toxics and *Smoking* are independent

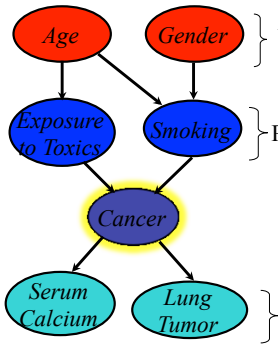
Exposure to Toxics is **dependent** on *Smoking*, given *Cancer*

$P(E=heavy|C=malignant) >$
$P(E=heavy|C=malignant, S=heavy)$

- Explaining away: reasoning pattern where confirmation of one cause of an event reduces need to invoke alternatives
- Essence of Occam's Razor

Conditional Independence

A variable (node) is conditionally independent of its non-descendants given its parents



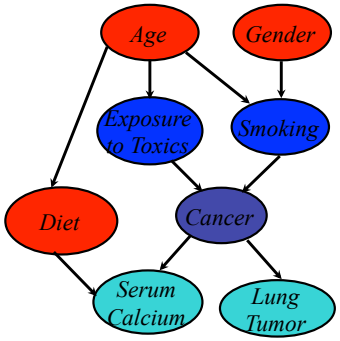
Non-Descendants

Parents

Descendants

Cancer is independent of *Age* and *Gender* given *Exposure to Toxics* and *Smoking*.

Another non-descendant



A variable is conditionally independent of its non-descendants given its parents

Cancer is independent of *Diet* given *Exposure to Toxics* and *Smoking*

BBN Construction

The knowledge acquisition process for a BBN involves three steps

- Choosing appropriate variables
- Deciding on the network structure
- Obtaining data for the conditional probability tables

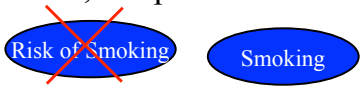
KA1: Choosing variables

Variables should be collectively exhaustive, mutually exclusive values

$$x_1 \vee x_2 \vee x_3 \vee x_4$$

$$\neg (x_i \wedge x_j) \quad i \neq j$$

They should be values, not probabilities

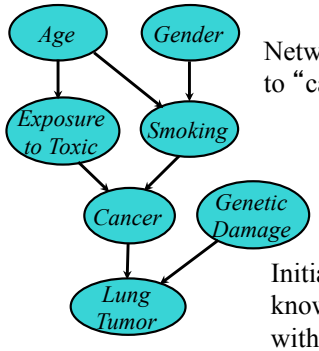


Heuristic: Knowable in Principle

Example of good variables

- Weather {Sunny, Cloudy, Rain, Snow}
- Gasoline: Cents per gallon
- Temperature { ≥ 100F , < 100F}
- User needs help on Excel Charting {Yes, No}
- User's personality {dominant, submissive}

KA2: Structuring

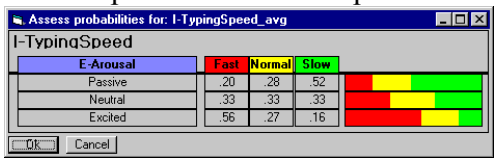


Network structure corresponding to "causality" is usually good.

Initially this uses the designer's knowledge but can be checked with data

KA3: The numbers

- Second decimal usually doesn't matter
- Relative probabilities are important



- Zeros and ones are often enough
- Order of magnitude is typical: 10^{-9} vs 10^{-6}
- Sensitivity analysis can be used to decide accuracy needed

Three kinds of reasoning

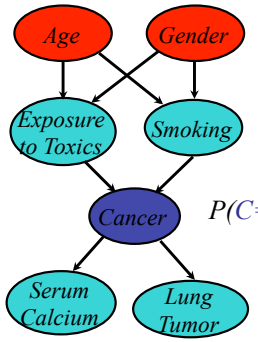
BBNs support three main kinds of reasoning:

- **Predicting** conditions given predispositions
- **Diagnosing** conditions given symptoms (and predisposing)
- **Explaining** a condition in by one or more predispositions

To which we can add a fourth:

- **Deciding** on an action based on the probabilities of the conditions

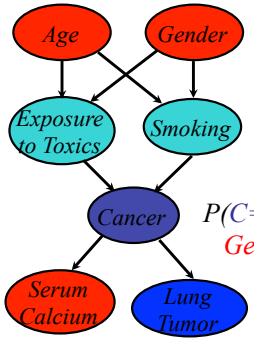
Predictive Inference



How likely are elderly males to get malignant cancer?

P(C=malignant | Age>60, Gender=male)

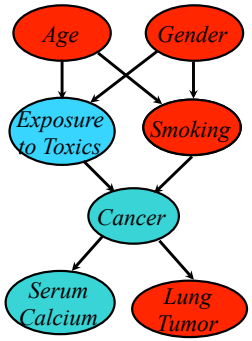
Predictive and diagnostic combined



How likely is an elderly male patient with high Serum Calcium to have malignant cancer?

P(C=malignant | Age>60, Gender= male, Serum Calcium = high)

Explaining away



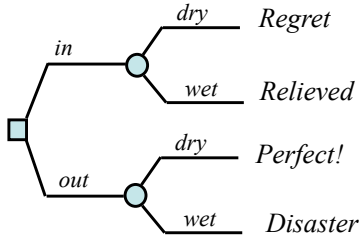
- If we see a lung tumor, the probability of heavy smoking and of exposure to toxics both go up.
- If we then observe heavy smoking, the probability of exposure to toxics goes back down.

Decision making

- Decision - an irrevocable allocation of domain resources
- Decision should be made so as to maximize expected utility.
- View decision making in terms of
  - Beliefs/Uncertainties
  - Alternatives/Decisions
  - Objectives/Utilities

A Decision Problem

Should I have my party inside or outside?



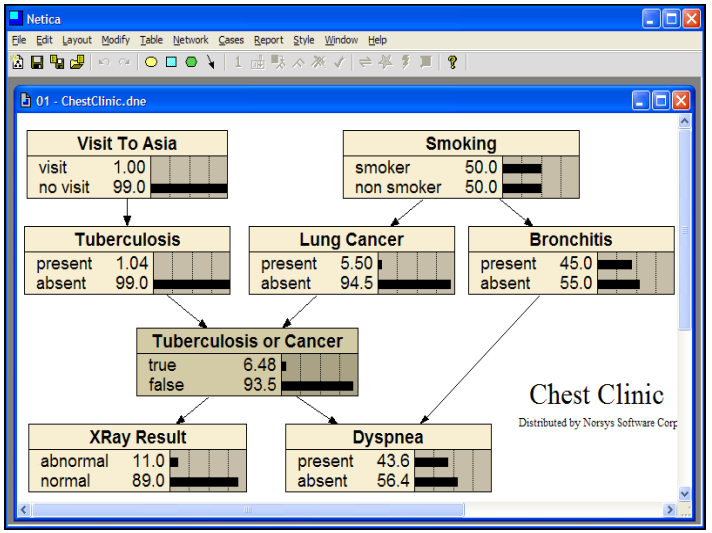
Value Function

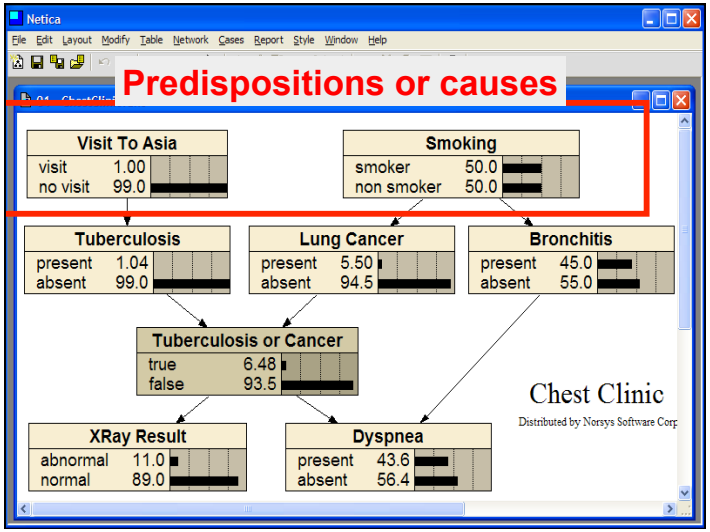
A numerical score over all possible states of the world allows BBN to be used to make decisions

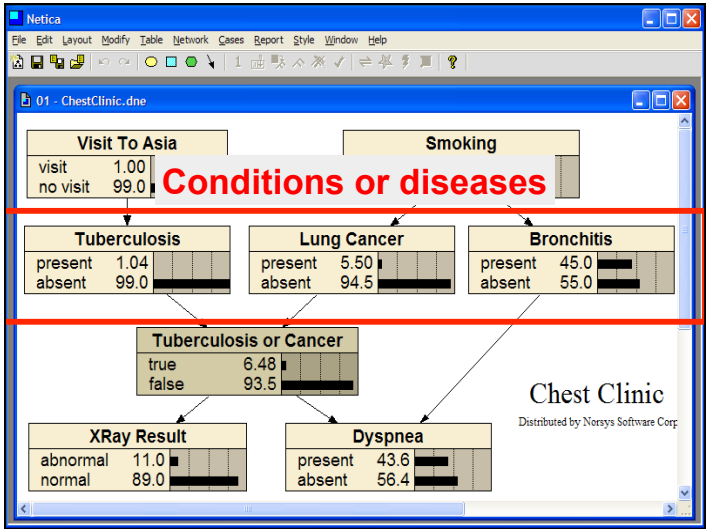
Location?	Weather?	Value
in	dry	$50
in	wet	$60
out	dry	$100
out	wet	$0

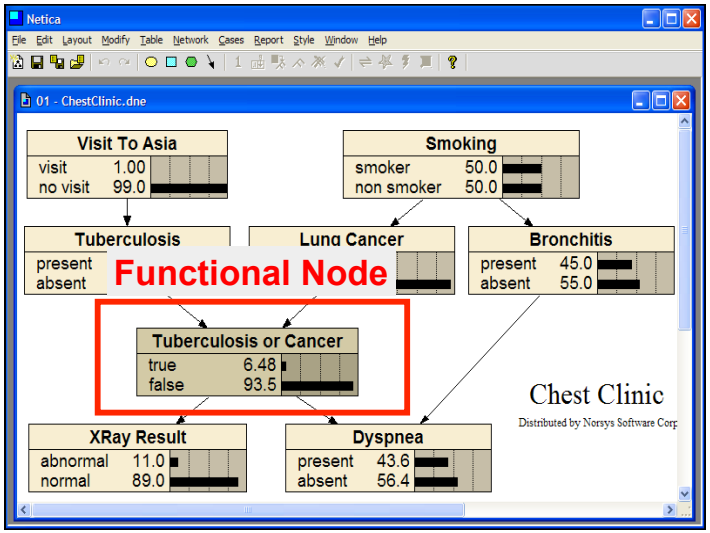
Two software tools

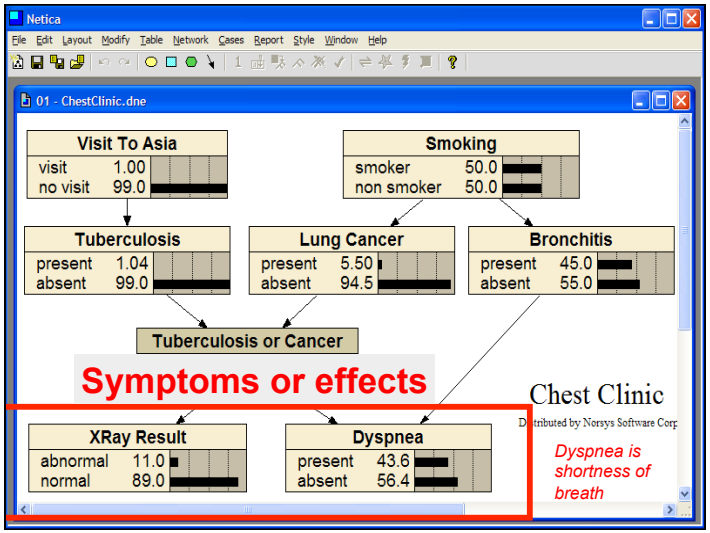
- Netica: Windows app for working with Bayesian belief networks and influence diagrams
  - A commercial product but free for small networks
  - Includes a graphical editor, compiler, inference engine, etc.
- Samiam: Java system for modeling and reasoning with Bayesian networks
  - Includes a GUI and reasoning engine









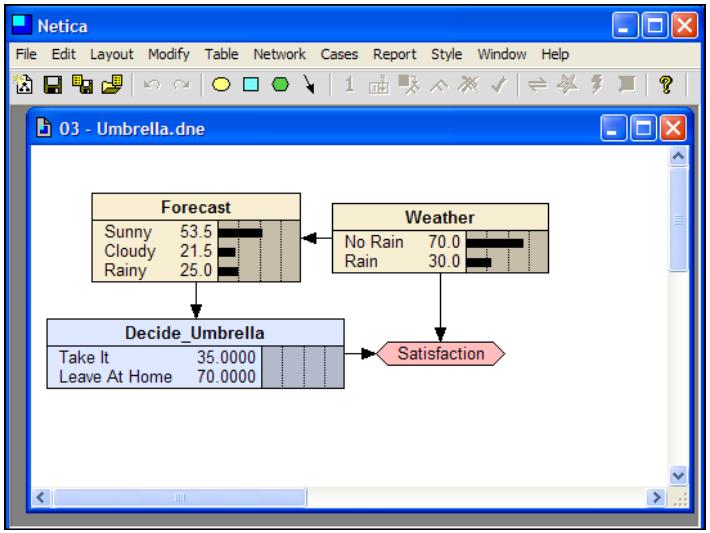


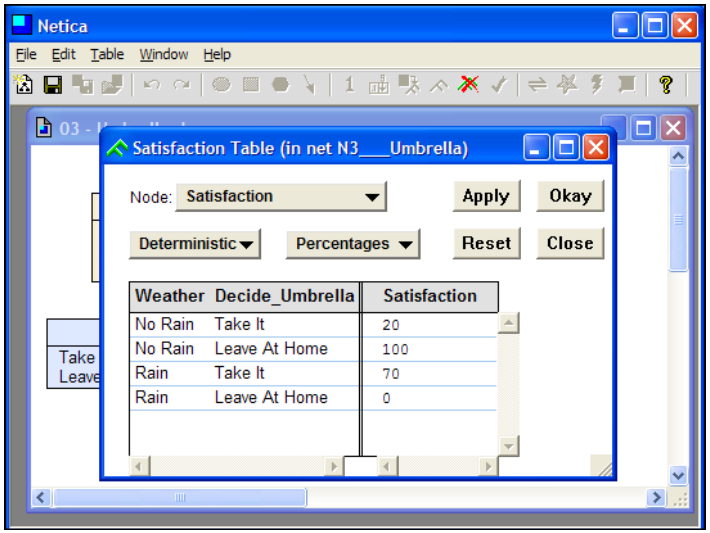
Decision Making with BBNs

- Today's weather forecast might be either sunny, cloudy or rainy
- Should you take an umbrella when you leave?
- Your decision depends only on the forecast
  - The forecast "depends on" the actual weather
- Your satisfaction depends on your decision and the weather
  - Assign a utility to each of four situations: (rain|no rain) x (umbrella, no umbrella)

Decision Making with BBNs

- Extend the BBN framework to include two new kinds of nodes: Decision and Utility
- A **Decision** node computes the expected utility of a decision given its parent(s), e.g., forecast, an a valuation
- A **Utility** node computes a utility value given its parents, e.g. a decision and weather
  - We can assign a utility to each of four situations: (rain|no rain) x (umbrella, no umbrella)
  - The value assigned to each is probably subjective


Netica
File Edit Layout Modify Table Network Cases Report Style Window Help
03 - Umbrella.dne
Forecast
Sunny 53.5
Cloudy 21.5
Rainy 25.0
Weather
No Rain 70.0
Rain 30.0
Decide_Umbrella
Take It 35.0000
Leave At Home 70.0000
Satisfaction


Netica
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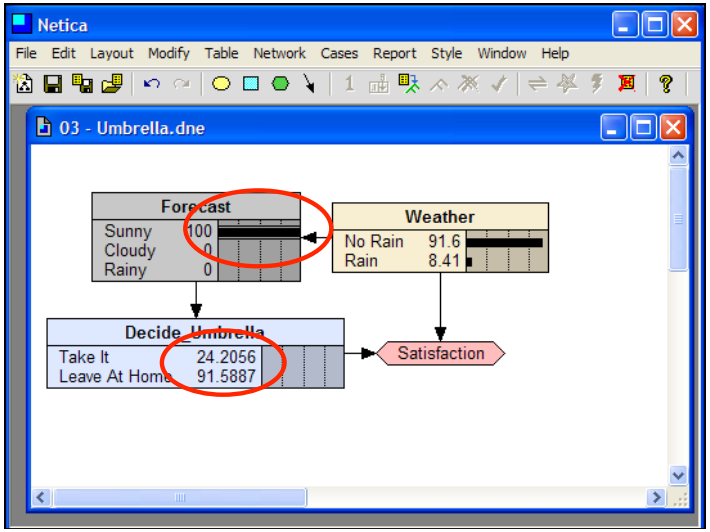
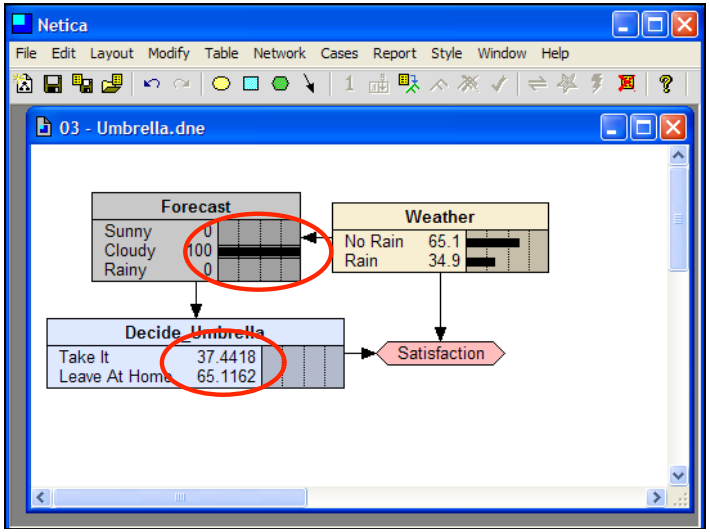
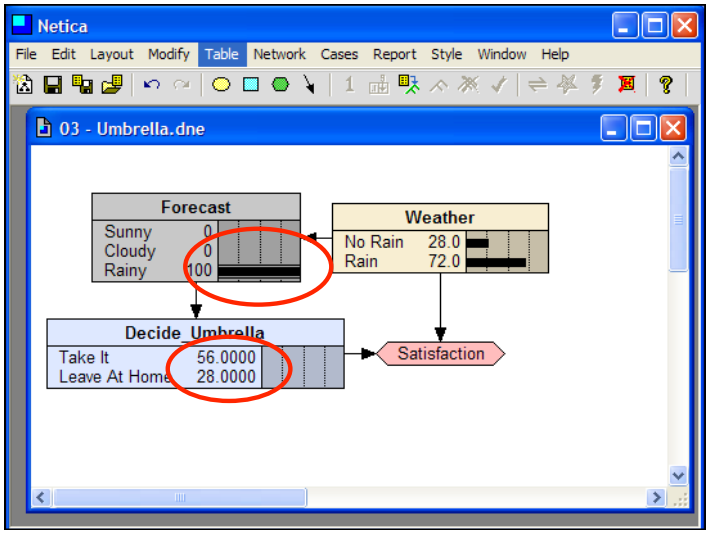
Satisfaction Table (in net N3___Umbrella)
Node: Satisfaction
Apply
Okay
Deterministic
Percentages
Reset
Close
Weather Decide_Umbrella Satisfaction
No Rain Take It 20
No Rain Leave At Home 100
Rain Take It 70
Rain Leave At Home 0


Netica
File Edit Layout Modify Table Network Cases Report Style Window Help
03 - Umbrella.dne
Forecast
Sunny 100
Cloudy 0
Rainy 0
Weather
No Rain 91.6
Rain 8.41
Decide_Umbrella
Take It 24.2056
Leave At Home 91.5887
Satisfaction


Netica
File Edit Layout Modify Table Network Cases Report Style Window Help
03 - Umbrella.dne
Forecast
Sunny 0
Cloudy 100
Rainy 0
Weather
No Rain 65.1
Rain 34.9
Decide_Umbrella
Take It 37.4418
Leave At Home 65.1162
Satisfaction


Netica
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03 - Umbrella.dne
Forecast
Sunny 0
Cloudy 0
Rainy 100
Weather
No Rain 28.0
Rain 72.0
Decide_Umbrella
Take It 56.0000
Leave At Home 28.0000
Satisfaction