



# Logical Inference 1 introduction

## Chapter 9

Some material adopted from notes by Andreas Geyer-Schulz, Chuck Dyer, and Mary Getoor

## Overview

- Model checking for propositional logic
- Rule based reasoning in first-order logic
  - Inference rules and generalized modes ponens
  - Forward chaining
  - Backward chaining
- Resolution-based reasoning in first-order logic
  - Clausal form
  - Unification
  - Resolution as search
- Inference wrap up

## PL Model checking

- Given KB, does sentence S hold?
- Basically generate and test:
  - Generate all the possible models
  - Consider the models M in which KB is TRUE
  - If  $\forall M S$ , then S is **provably true**
  - If  $\forall M \neg S$ , then S is **provably false**
  - Otherwise ( $\exists M1 S \wedge \exists M2 \neg S$ ): S is **satisfiable** but neither provably true or provably false

## From Satisfiability to Proof (1)

- To see if a satisfiable KB entails sentence S, see if  $\text{KB} \wedge \neg S$  is satisfiable
  - If it is not, then the KB entails S
  - If it is, then the KB does not entail S
  - This is a refutation proof
- Consider the KB with (P,  $P \Rightarrow Q$ ,  $\sim P \Rightarrow R$ )
  - Does the KB entail Q? R?

## Efficient PL model checking (1)

[Davis-Putnam algorithm](#) (DPLL) is [generate-and-test](#) model checking with several optimizations:

- *Early termination*: [short-circuiting](#) of disjunction/conjunction
- *Pure symbol heuristic*: symbols appearing only negated or unnegated must be FALSE/TRUE respectively  
e.g., in  $[(A \vee \neg B), (\neg B \vee \neg C), (C \vee A)]$  A & B are pure, C impure. Make pure symbol literal true: if there's a model for S, making pure symbol true is also a model
- *Unit clause heuristic*: Symbols in a clause by itself can immediately be set to TRUE or FALSE

## Using the AIMA Code

```
python> python
Python ...
>>> from logic import *
>>> expr('P & P==>Q & ~P==>R')
((P & (P >> Q)) & (~P >> R))

>>> dpll_satisfiable(expr('P & P==>Q & ~P==>R'))
{R: True, P: True, Q: True}

>>> dpll_satisfiable(expr('P & P==>Q & ~P==>R & ~R'))
{R: False, P: True, Q: True}

>>> dpll_satisfiable(expr('P & P==>Q & ~P==>R & ~Q'))
False

>>>
```

expr parses a string, and returns a logical expression

dpll\_satisfiable returns a model if satisfiable else False

The KB entails Q but does not entail R

## Efficient PL model checking (2)

- [WalkSAT](#) is a local search for satisfiability: Pick a symbol to flip (toggle TRUE/FALSE), either using min-conflicts *or* choosing randomly
- ...or you can use *any* local or global search algorithm!
- There are many model checking algorithms and systems
  - See for example, [MiniSat](#)
  - [International SAT Competition](#) (2003, ... 2012)

## AIMA KB Class

```
>>> kb1 = PropKB()
>>> kb1.clauses
[]
>>> kb1.tell(expr('P==>Q & ~P==>R'))
>>> kb1.clauses
[(Q | ~P), (R | P)]
>>> kb1.ask(expr('Q'))
False
>>> kb1.tell(expr('P'))
>>> kb1.clauses
[(Q | ~P), (R | P), P]
>>> kb1.ask(expr('Q'))
{}
>>> kb1.retract(expr('P'))
>>> kb1.clauses
[(Q | ~P), (R | P)]
>>> kb1.ask(expr('Q'))
False
```

PropKB is a subclass

A sentence is converted to CNF and the clauses added

The KB does not entail Q

After adding P the KB does entail Q

Retracting P removes it and the KB no longer entails Q

### **Reminder: Inference rules for FOL**

- Inference rules for propositional logic apply to FOL as well
  - Modus Ponens, And-Introduction, And-Elimination, ...
- New (sound) inference rules for use with quantifiers:
  - Universal elimination
  - Existential introduction
  - Existential elimination
  - Generalized Modus Ponens (GMP)